TOPIC: 2.1 –SEQUENCES – DEFINTIONS AND EXAMPLES

Unit - II

Sequences and Series.

Sequence

An ordered set of real numbers $s_1, s_2, \ldots s_n$ is called a sequence and is denoted by $\{s_n\}$.

Example

(i) If $a_n = \frac{1}{n}$, the seq. is $1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}, \ldots$

(ii) If $a_n = n^3$, the seq. is $1^3, 2^3, \ldots n^3, \ldots$

(iii) If $a_n = k$, the seq. is $k, k, \ldots$

(iv) If $a_n = (-1)^n$, the seq. is $-1, 1, -1, 1 \ldots (-1)^n$

Finite sequence

A finite sequence has a finite number of terms.

Eg $1, 2, 3, 4, 5 \ldots\ldots 100$

Infinite sequence

A sequence, which is not finite is an infinite sequence.

Eg $1, 2, 3, \ldots\ldots$

Limit of a sequence

Let $\{S_n\}$ be the sequence of real numbers. Then S_n approaches the limit L as infinity, if for every $\epsilon > 0$, $\exists$ a +ve integer $N \ni$

$$|S_n - L| < \epsilon \ (n \geq N).$$

If S_n approaches the limit L, then

$$\lim_{n \to \infty} S_n = L$$

Convergence

A sequence $\{S_n\}$ is said to be convergent if it has a finite limit

i.e., $\lim_{n \to \infty} S_n = L$

Eg: $\left\{\frac{1}{n^2}\right\}$

$$\lim_{n \to \infty} \frac{1}{n^2} = 0.$$

Divergence

If $\lim_{n \to \infty} S_n = \infty$, then $\{S_n\}$ is divergent

Eg. $\{n\}$.

$$\lim_{n \to \infty} n = \infty.$$

Oscillatory

(iii) If $\lim_{n\to\infty} S_n$ is not unique (oscillates finitely) or $\pm\infty$ (oscillates infinitely) then $\{S_n\}$ is called an oscillatory sequence.

Eg

(i) $\{(-1)^n\}$.
oscillates finitely because $\lim_{n\to\infty} (-1)^n = \begin{cases} 1, & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$

(ii) $\{(-1)^n . n^2\}$.
oscillates infinitely because. $\lim_{n\to\infty} (-1)^n . n^2 = \pm\infty$

Bounded sequence:

A sequence $\{S_n\}$ is said to be bounded if $\exists$ a number k such that $S_n < k, \forall n$.

Eg 1, 2, 3, , 1, 2, 3

Monotonic sequence

A sequence $\{S_n\}$ is said to increase steadily or to decrease steadily according as $S_{n+1} \geq S_n$ or $S_{n+1} \leq S_n$ for all values of n.

Both increasing & decreasing sequence are called monotonic sequence.

Eg 1, 5, 10, 15

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.