

SNS COLLEGE OF ENGINEERING Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

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Topic: 1.7 – DIAGONALIZATION OF MATRICES

Working Pule For Diagonalisation:

* To find the characteristic equation.

* To find the Eigen values

* To find the Eigen vectors

* If the Eigen vectors are orthogonal, then form a normalized model N.

* Normalised form is
$$N = \begin{bmatrix} x_1/2 \\ x_2/2 \\ x_3/2 \end{bmatrix}$$
 where $l = \sqrt{x_1^2 + x_2^2 + x_3^2}$

* Find N^T

* Calculate AN

* Calculate $D = N^T AN$

Problems:

1. Diagonalise the maturi
$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ +2 & -4 & 3 \end{bmatrix}$$
Let $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$
Characteristic equation $\Rightarrow \lambda^{8} - 18\lambda^{2} + 45\lambda = 0$
Figur values are $0,3,15$

Figur vectors i) If $\lambda = 0$, then Figur vector is $\begin{bmatrix} 1 & 2 & 2 \end{bmatrix}^{T}$
ii) If $\lambda = 3$, then Figur vector is $\begin{bmatrix} 2 & 1 & -2 \end{bmatrix}^{T}$
ii) If $\lambda = 15$, then Figur vector is $\begin{bmatrix} 2 & -2 & 1 \end{bmatrix}^{T}$



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To find the Figer vectors are orthogonal to each other.
$$X_1^T X_2 \implies 2+2-4 = 0$$

$$X_2^T X_3 \implies 4-2-2=0$$

$$X_1^T X_3 \implies 2-4+2=0$$
 They are orthogonal to each other.

To form the Normalized Matrix:

$$N = \begin{bmatrix} y_3 & \frac{1}{2} & \frac{2}{2} \\ \frac{1}{2} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \implies \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$
To find the transpare of normalized matrix

$$N^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \frac{1}{3}$$
Calculate AN

$$AN = \frac{1}{3} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 10 \\ 0 & 1 & -10 \\ 0 & -2 & 5 \end{bmatrix}$$
Calculate Diagonized Matrix D

$$D = NAN^T$$

$$= \frac{1}{3} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \cdot \frac{1}{3}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 10 \\ 0 & 1 & -10 \\ 0 & -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$