

SNS COLLEGE OF ENGINEERING Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

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Topic: 3.8 – ENVELOPES

Erwelope: A curve which touches each member of a family of worke is called the envelope of that family of curves, The envelope of a family of curves is the locus of the ultimate points of intersection of the consecutive members of the family. Method 1; for finding envelope:

1) If the family of curves is expressed as a quadratic equation of the parameter, say, All 3/1+c=0 where A, B, c are functions of x and y and I is the parameter then the envelope of this family as given by B-4AC=0 Analytic method to find the Envelope the family of across. 1. Differentiate f(x,y,c)=0 partially w.r. + the Parameters c. 2. Eliminate 'c' from f(x, y, c) = 0 to df(x, y, c)=0 " We get the envelope of the family.



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Problems: 1. Find the envelope of the family of straight lines Y=mx+am²; m being the parameter, Boln: Given y=mx+am², ant+mx-y=0, This is quadratic in m, so the envelope is B-4AC=0, here A=9 x2-4a(-y)=0 > x+ +ay=0 2. Find the envelope of the family of $y = mx + a_m +$ Univer y=mx+a 4= m2+19 my = m2 + a. m2 n - my + a = 0. This is a quadratic in'm'. so the envelope is B- HAC = 0 1-42)-4xa=0 (i.e) y= 4ax.



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3. Find the envelope of the family of straight Lines. x coso + y sino = a, where o being parameter Adni Given x coso + y sin 0 = a >0 ndiffer. T. t => -x sin 0 +ycos0 = 0 -> @ 8 quaring and adding 10 +10 the envelope x (cos20+sin20)+y2(sin20+cos20)= 2 (ie) x2+y2=a2, Which is a circle. H. Find the envelope of the family of straight Lines y=mx+ Vaint+b2 where 'm' is the parameter. Moln: 4-mx = Va m + 12 (9-mx)2= 22m2+ 62=) y2-2mxy+mx=201 m2 (x2-2)-2mxy+y2-b=0 Which is quadratic in 'm' here $A = x^2 - a^2$; B = -axy; $C = y^2 - b^2$ B2- 4AC = 4x2y2- 4(x2-a2)(y2-b2)=0 4x2y2- 4(x2y2- x2b2 -a2y2+a2b2)=0 4x2y2 - 4x2y2 + 4x2 b2 + 4a2y2 - 4a2b2=0 + Hab = + x2 b2 + 4a2y2 = 4a2b2

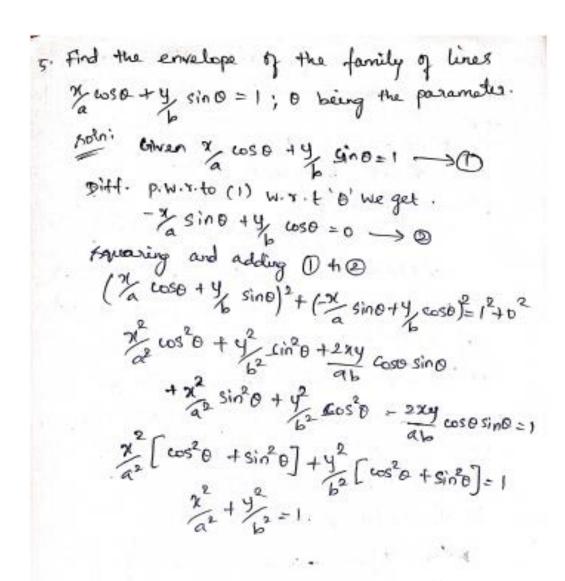


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b. Find the envelope of x seco - ytan 0 = a where "O' being the parameter. Given X sec 0 - 4+an 0 = a -> 1) x seco = a +y +ano. squaring both sides. x sec = a + 2ay +an 0 +y +an 0 x2(1+ten20) = a2 + 2 ay tano + y2+an20 x2 + x2 +ano = y2 +ano + 2ay +ano + a2 (y2-x2)+ano + 2ay tano + (a2-x2)=0 (i.e) (y2-x2) m2+ 2aym ++ (a2+x2)=0. Where 'm = tand which is a quadratic form in m Here A = y - x2, B = day; C = a2 - x2 The envelope is B-4AC=0. 4242 - 4(42-x2)(22-x2)=0 4a2y - 4[a2y2-x2y2-x2a2+x4]=0 42y2- 42y2 +x2y2++4x22-4x4=0 > by x => y2+ a2-x2=0 x2-42 = a2.