Topic: 3. 7 – PROPERTIES OF EVOLUTES

(8)

properties of Evolute:

1. The normal at any point of a curve is a tangent to its evolute touching at the corresponding centre of curvature.

2. The difference b/w the radius of curvature at two points of a curve is equal to the length of the arc of the evolute between the two corresponding points.

3. There is one evolute but an infinite number of involutes. (i) The normals to a curve are the tangents to its Evolute.

(ii) The evolute of a family of curves touches at each of its points the corresponding member of that family.

1. Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$ is another equal cycloid.

Soln: Given $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$

$$\frac{dx}{d\theta} = a(1-\cos\theta) \; ; \; \frac{dy}{d\theta} = a\sin\theta$$

$$\frac{dy}{dx} = \frac{a\sin\theta}{a(1-\cos\theta)} = \frac{\sin\theta}{1-\cos\theta}$$

$$y_1 = \frac{2\sin\theta/2 \cdot \cos\theta/2}{2\sin^2\theta/2} = \cot\theta/2$$

$$y_2 = \frac{d}{d\theta}\left[\cot\theta/2\right]\cdot\frac{d\theta}{dx} = \frac{-\operatorname{cosec}^2\theta/2\left[\frac{1}{2}\right]}{a[1-\cos\theta]}$$

$$= \frac{-\operatorname{cosec}^2\theta/2}{2a(2\sin^2\theta/2)} = -\frac{1}{4a\sin^4\theta/2}.$$

$$\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2)$$

$$= a(\theta - \sin\theta) - \left(\frac{\cos\theta/2}{\sin\theta/2}\right)\left(-4a\sin^4\theta/2\right)\left(\frac{1+\cos^2\theta/2}{\sin^2\theta/2}\right)$$

$$= a(\theta - \sin\theta) + 4a\sin^3\theta/2 \cdot \cos\theta/2\left(\frac{\sin^2\theta/2 + \cos^2\theta/2}{\sin^2\theta/2}\right)$$

$$= a(\theta - \sin\theta) + 4a \sin\tfrac{\theta}{2} \cdot \cos\tfrac{\theta}{2}.$$

$$= a(\theta - \sin\theta) + 2a\sin\theta$$

$$= a[\theta - \sin\theta + 2\sin\theta]$$

$$\bar{x} = a[\theta + \sin\theta] \rightarrow (1)$$

$$\bar{y} = y + \frac{1}{y_2}(1 + y_1^2)$$

$$= a(1-\cos\theta) + (-4a\sin^4\tfrac{\theta}{2})(1+\cot^2\tfrac{\theta}{2})$$

$$= a(1-\cos\theta) - 4a\sin^4\tfrac{\theta}{2}\left[\frac{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\right]$$

$$= a(1-\cos\theta) - 4a\sin^2\tfrac{\theta}{2}$$

$$= a[1-\cos\theta] - 4a\left[\frac{1-\cos\theta}{2}\right]$$

$$= a(1-\cos\theta) - 2a(1-\cos\theta)$$

$$= a[-1 + \cos\theta]$$

$$\bar{y} = -a[1-\cos\theta] \rightarrow (2)$$

From (1) & (2) we get.

$$x = a(\theta + \sin\theta); \quad y = a(1-\cos\theta)$$

this represents the equation of another cycloid.