SNS College of Engineering
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Asymptotic notations

AP/IT

- O notation: asymptotic “less than”: f(n) “≤” g(n)
- Ω notation: asymptotic “greater than”: f(n) “≥” g(n)
- Θ notation: asymptotic “equality”: f(n) “=” g(n)

Big-O

$$f(n) = O(g(n)): \text{there exist positive constants } c \text{ and } n_0 \text{ such that}$$
$$0 \le f(n) \le cg(n) \text{ for all } n \ge n_0$$

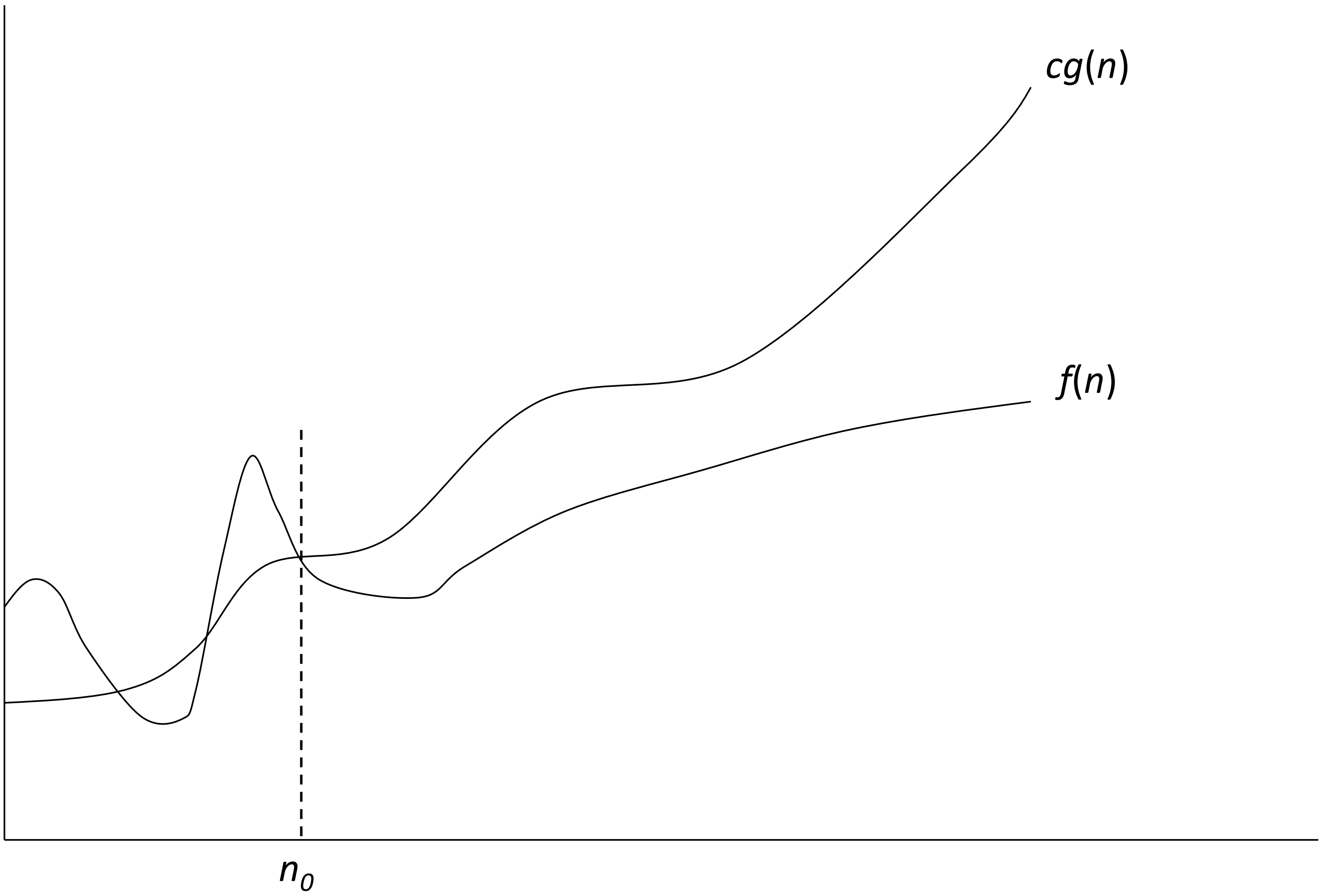
If $f(n) = O(n^2)$, then:

- $f(n)$ can be larger than n^2 sometimes, **but…**

- I can choose some constant c and some value n_0 such that for **every** value of n larger than n_0 : $f(n) < cn^2$

- That is, for values larger than n_0, $f(n)$ is never more than a constant multiplier greater than n^2

Visualization of $O(g(n))$



Big Omega – Notation

$\Omega()$ = A **lower** bound

$f(n) = \Omega(g(n))$: there exist positive constants c and n_0 such that

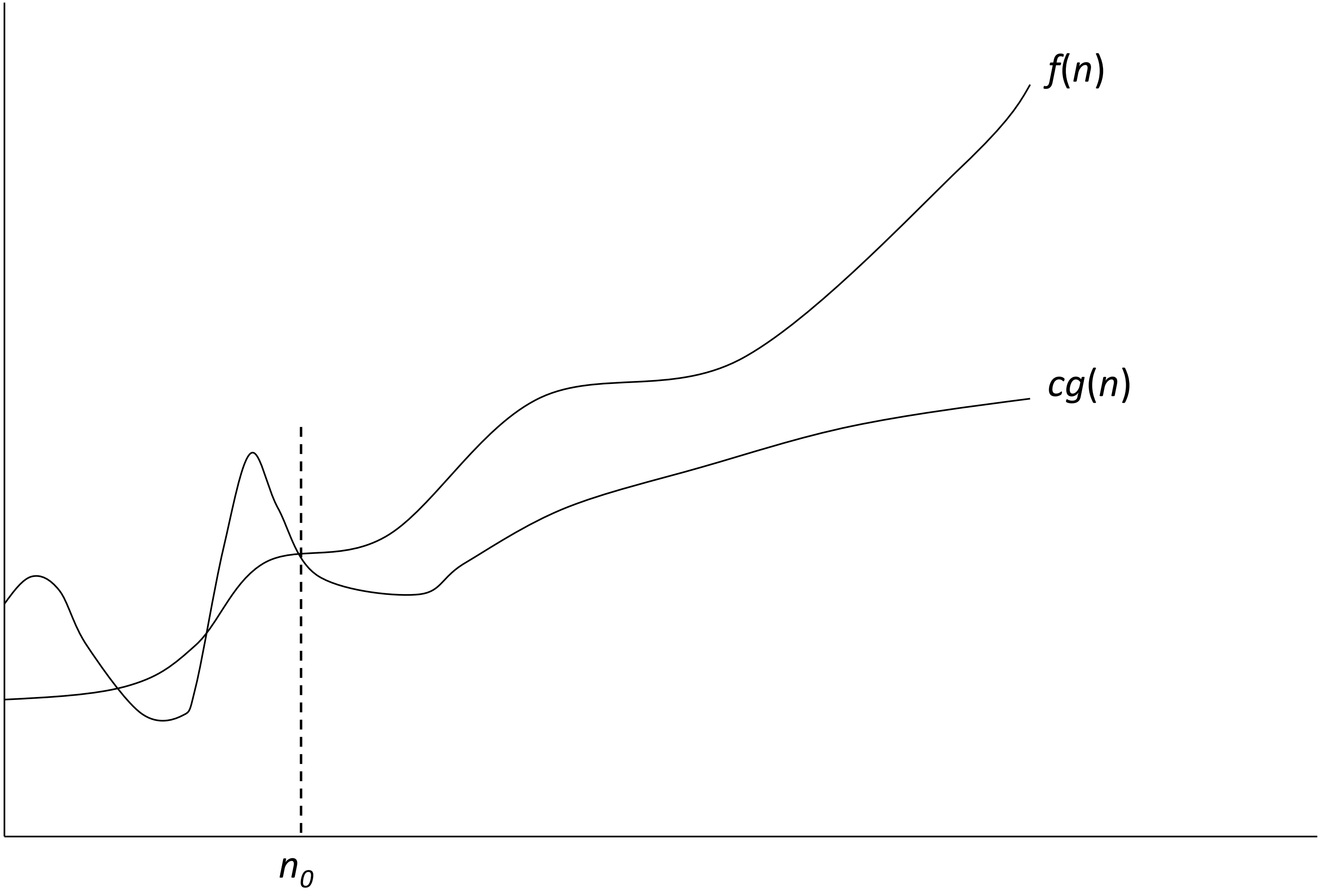
$$0 \leq f(n) \geq cg(n) \text{ for all } n \geq n_0$$

$n^2 = \Omega(n)$

Let $c = 1$, $n_0 = 2$

For all $n \geq 2$, $n^2 > 1 \times n$

Visualization of $\Omega(g(n))$



Θ-notation

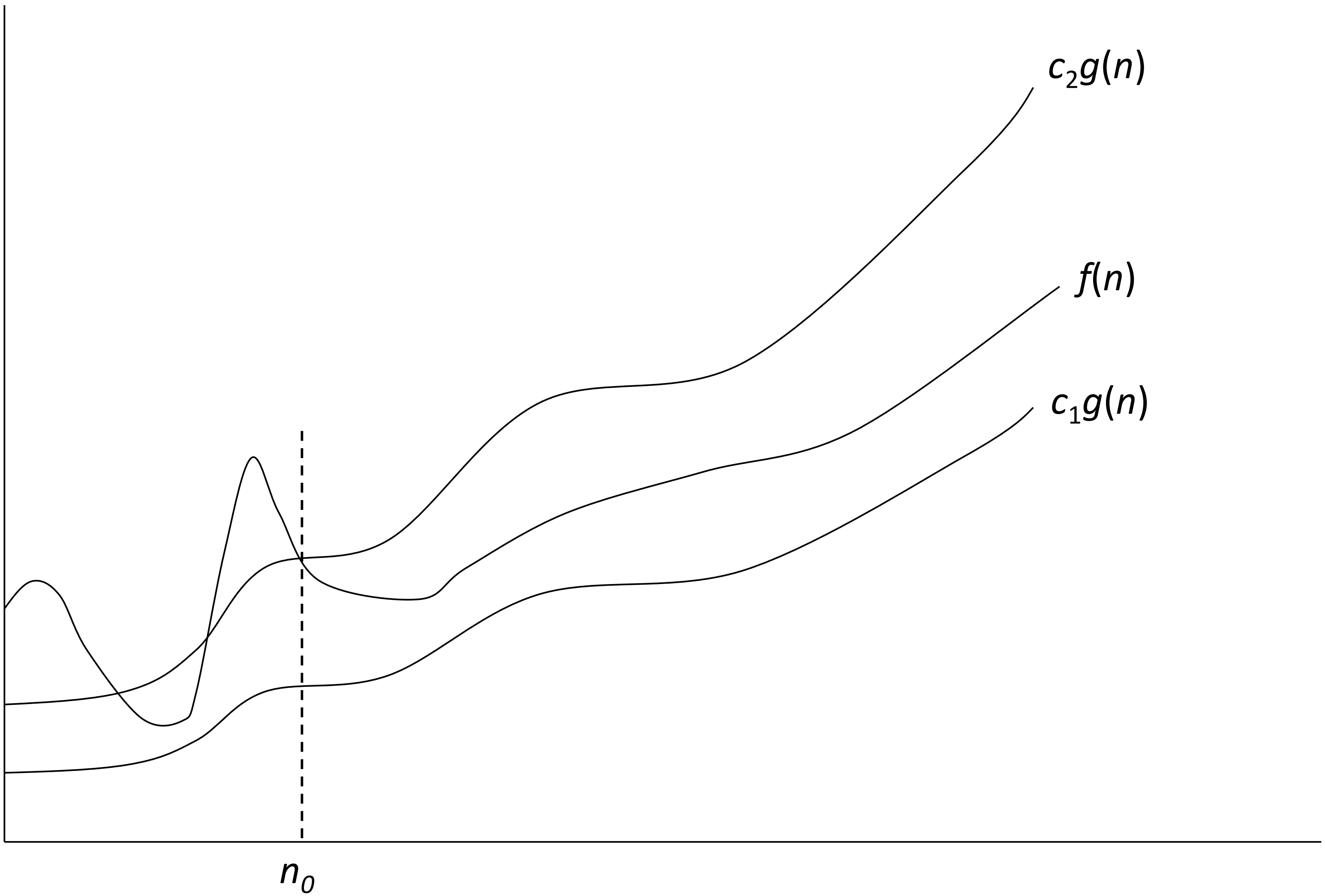
Big-*O* is not a tight upper bound.

In other words $n = O(n^2)$

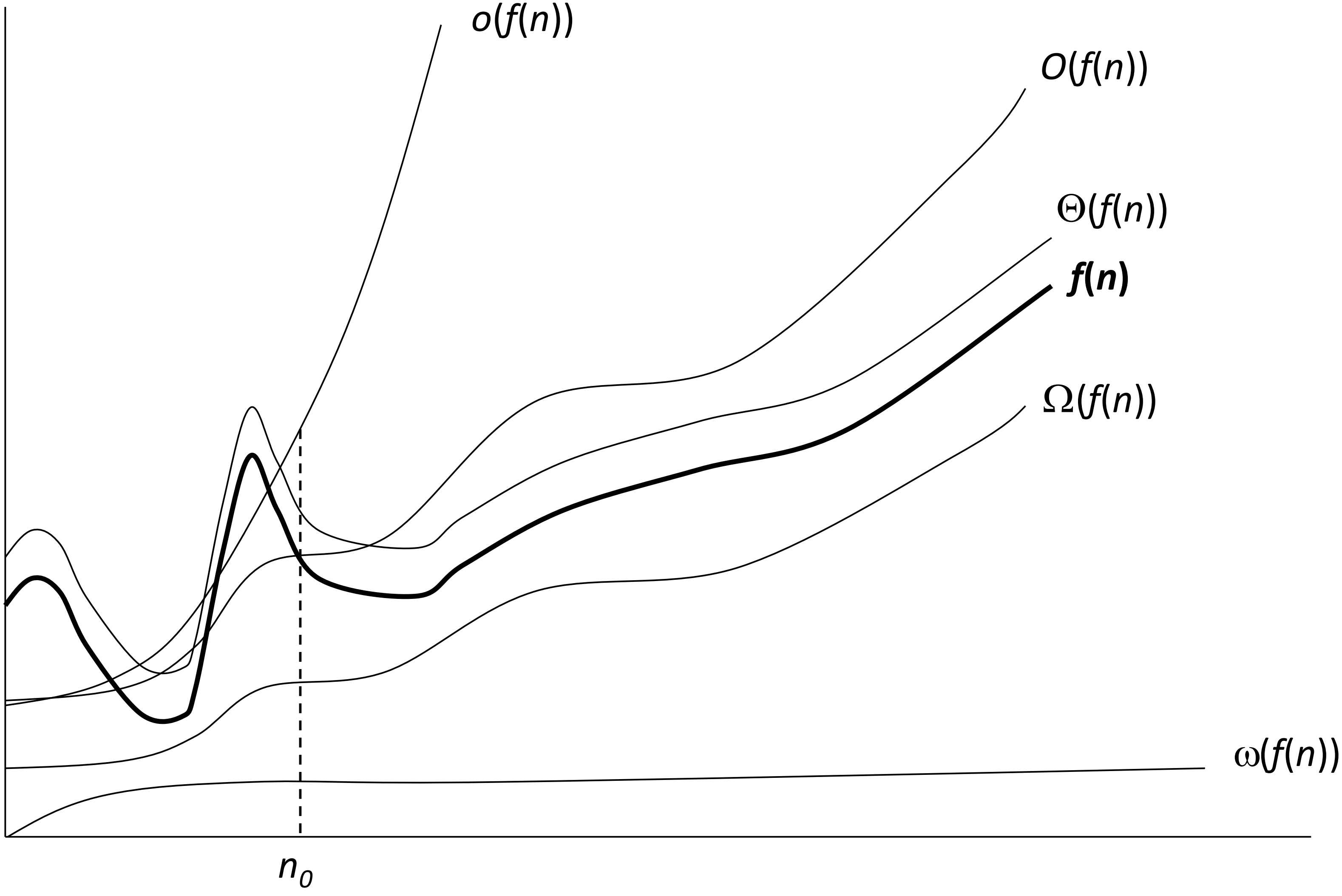
Θ provides a tight bound

$$f(n) = \Theta(g(n)): \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that}$$
$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0$$

Visualization of $\Theta(g(n))$



Visualization of Asymptotic Growth



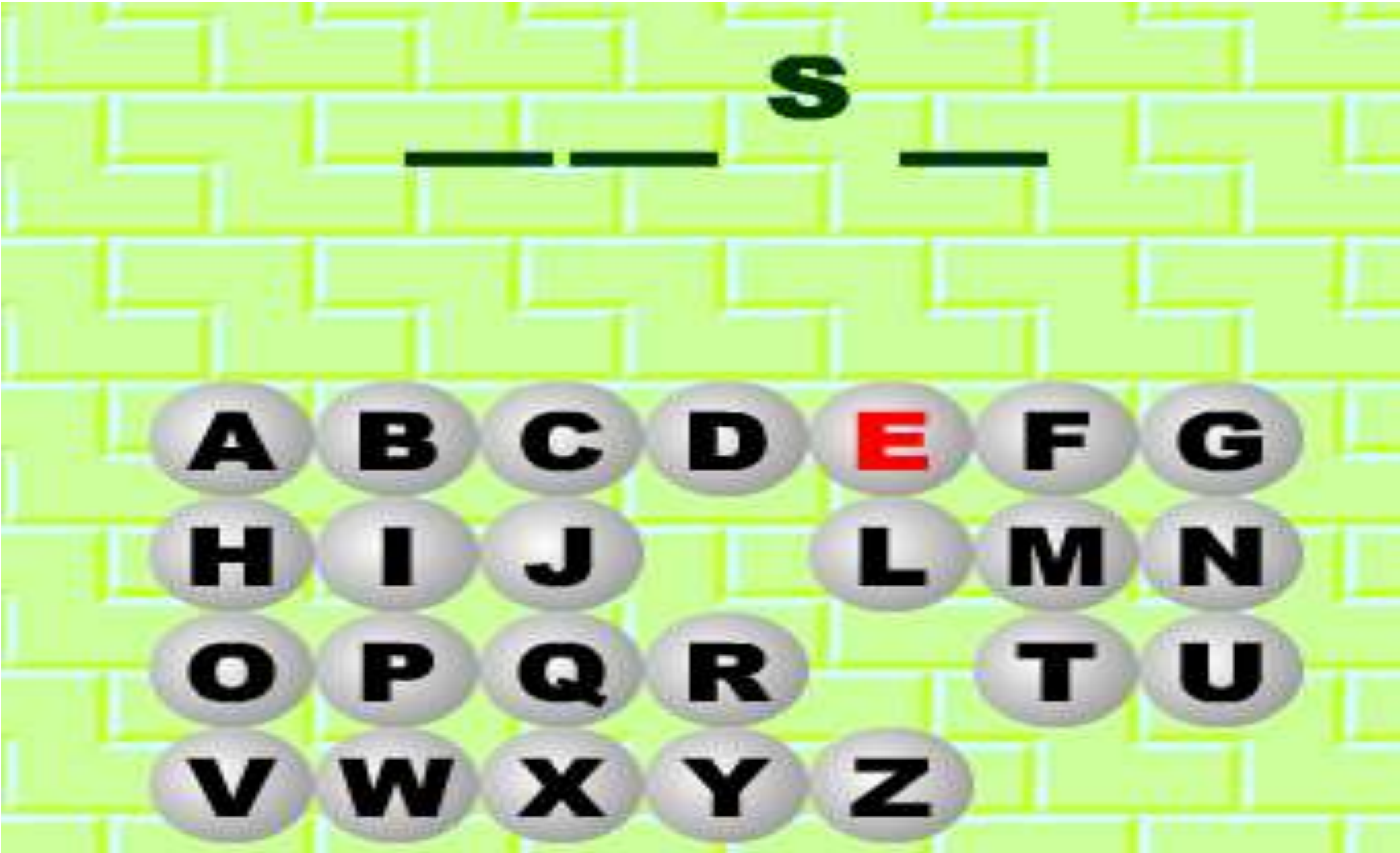
Analogy to Arithmetic Operators

$$f(n) = O(g(n)) \qquad \approx \qquad a \leq b$$

$$f(n) = \Omega(g(n)) \qquad \approx \qquad a \geq b$$

$$f(n) = \Theta(g(n)) \qquad \approx \qquad a = b$$

BREAK



Answer:

2 3 1