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University, Chennai**

Department of Artificial Intelligence and Data Science Course Name - Introduction to Artificial Intelligence

II Year / III Semester

Unit 2 Rule based deduction system



Rule-Based Deduction Systems

The way in which a piece of knowledge is expressed by a human expert carries important information,

example: if the person has fever and feels tummy-pain then she may have an infection.

In logic it can be expressed as follows:

$$\forall x. (\text{has_fever}(x) \ \& \ \text{tummy_pain}(x) \ \rightarrow \ \text{has_an_infection}(x))$$

If we convert this formula to **clausal form** we lose the content as then we may have equivalent formulas like:

(i) $\text{has_fever}(x) \ \& \ \sim\text{has_an_infection}(x) \ \rightarrow \ \sim\text{tummy_pain}(x)$

(ii) $\sim\text{has_an_infection}(x) \ \& \ \text{tummy_pain}(x) \ \rightarrow \ \sim\text{has_fever}(x)$

Notice that:

- (i) and (ii) are **logically equivalent** to the original sentence
- they have **lost the main information** contained in its formulation.



Forward production systems

- The main idea behind the forward/backward production systems is:
 - to take **advantage of the implicational form** in which production rules are stated by the expert
 - and use that information to help achieving the goal.
- In the present systems the formulas have two forms:
 - rules
 - and facts



Forward production systems

- Rules are the **productions** stated in implication form.
 - Rules express specific knowledge about the problem.
 - Facts are assertions not expressed as implications.
 - The task of the system will be to prove a **goal formula** with these facts and rules.
 - In a **forward production system** the rules are expressed as **F-rules**
 - F-rules operate on the **global database** of facts until the termination condition is achieved.
 - This sort of proving system is a **direct system** rather than a **refutation system**.

- Facts
 - Facts are expressed in **AND/OR form**.
 - An expression in AND/OR form consists on sub-expressions of literals connected by & and V symbols.
 - An expression in AND/OR form **is not in clausal form**.

Forward production systems

Steps to **transform** facts **into AND/OR form** for forward system:

1. *Eliminate* (temporarily) implication symbols.
2. Reverse quantification of variables in first disjunct by *moving negation symbol*.
3. *Skolemize* existential variables.
4. Move all **universal quantifiers** to the front and drop.
5. **Rename variables** so the same variable does not occur in different main conjuncts
 - Main conjuncts are small AND/OR trees, not necessarily sum of literal clauses as in Prolog.

EXAMPLE

Original formula: $\exists u. \forall v. \{q(v, u) \& \sim[[r(v) \vee p(v)] \& s(u, v)]\}$

converted formula: $q(w, a) \& \{[\sim r(v) \& \sim p(v)] \vee \sim s(a, v)\}$

Conjunction of two main conjuncts →

Various variables in conjuncts

All variables appearing on the final expressions are assumed to be **universally quantified**.



Based Deduction Systems: forward production system

rules

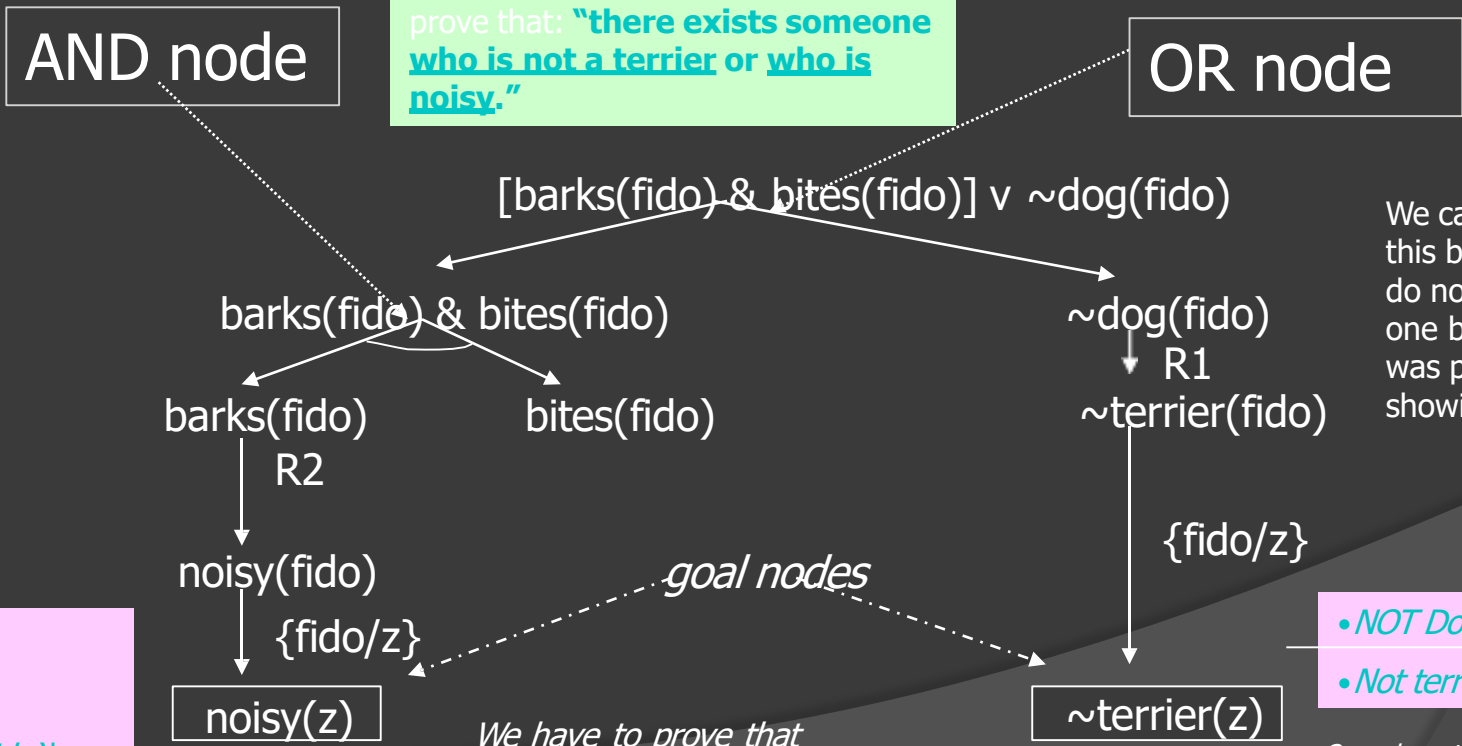
Rules in a **forward production system** will be applied to the AND/OR graph to produce new transformed graph structures.

We assume that rules in a forward production system are of the form:

$$L \implies W,$$

where L is a literal and **W is a formula in AND/OR form.**

- Recall that a rule of the form $(L1 \vee L2) \implies W$ is equivalent to the pair of rules: $L1 \implies W \vee L2 \implies W$.



We cannot prove this branch but we do not have to since one branch of OR was proven by showing Fido

- $Dog(Fido)$
- $barks(Fido)$
- $Not\ terrier(Fido)$
- $Noisy(Fido)$

- $NOT\ Dog(Fido)$
- $Not\ terrier(Fido)$

We have to prove that there is X that is noisy. $X=Fido$

Or we have to prove that there is X that X is not a terrier



forward production systems



Steps to transform the rules into a free-quantifier form:

1. Eliminate (temporarily) implication symbols.
2. Reverse quantification of variables in first disjunct by moving negation symbol.
3. Skolemize existential variables.
4. Move all universal quantifiers to the front and drop.
5. Restore implication.

All variables appearing on the final expressions are assumed to be universally quantified.

E.g. Original formula: $\forall x. (\exists y. \forall z. (p(x, y, z)) \rightarrow \forall u. q(x, u))$
Converted formula: $p(x, y, f(x, y)) \rightarrow q(x, u).$

Skolem
function

Restored
implication



forward production systems



A full example:

- Fact: Fido barks and bites, or Fido is not a dog.
- (R1) All terriers are dogs.
- (R2) Anyone who barks is noisy.

Based on these facts, prove that: “there exists someone who is not a terrier or who is noisy.”

Logic representation:

$(\text{barks}(\text{fido}) \ \& \ \text{bites}(\text{fido})) \ \vee \ \sim\text{dog}(\text{fido})$

R1: $\text{terrier}(x) \rightarrow \text{dog}(x)$

R2: $\text{barks}(y) \rightarrow \text{noisy}(y)$

goal: $\exists w. (\sim\text{terrier}(w) \vee \text{noisy}(w))$

goal



From facts to goal

AND node

OR node

AND/OR Graph for the 'terrier' problem:

$$[\text{barks}(\text{fido}) \ \& \ \text{bites}(\text{fido})] \vee \sim\text{dog}(\text{fido})$$

barks(fido) & bites(fido)

~dog(fido)

barks(fido)

bites(fido)

↓ **R1** applied in reverse

~terrier(fido)

↓ **R2** applied forward

noisy(fido)

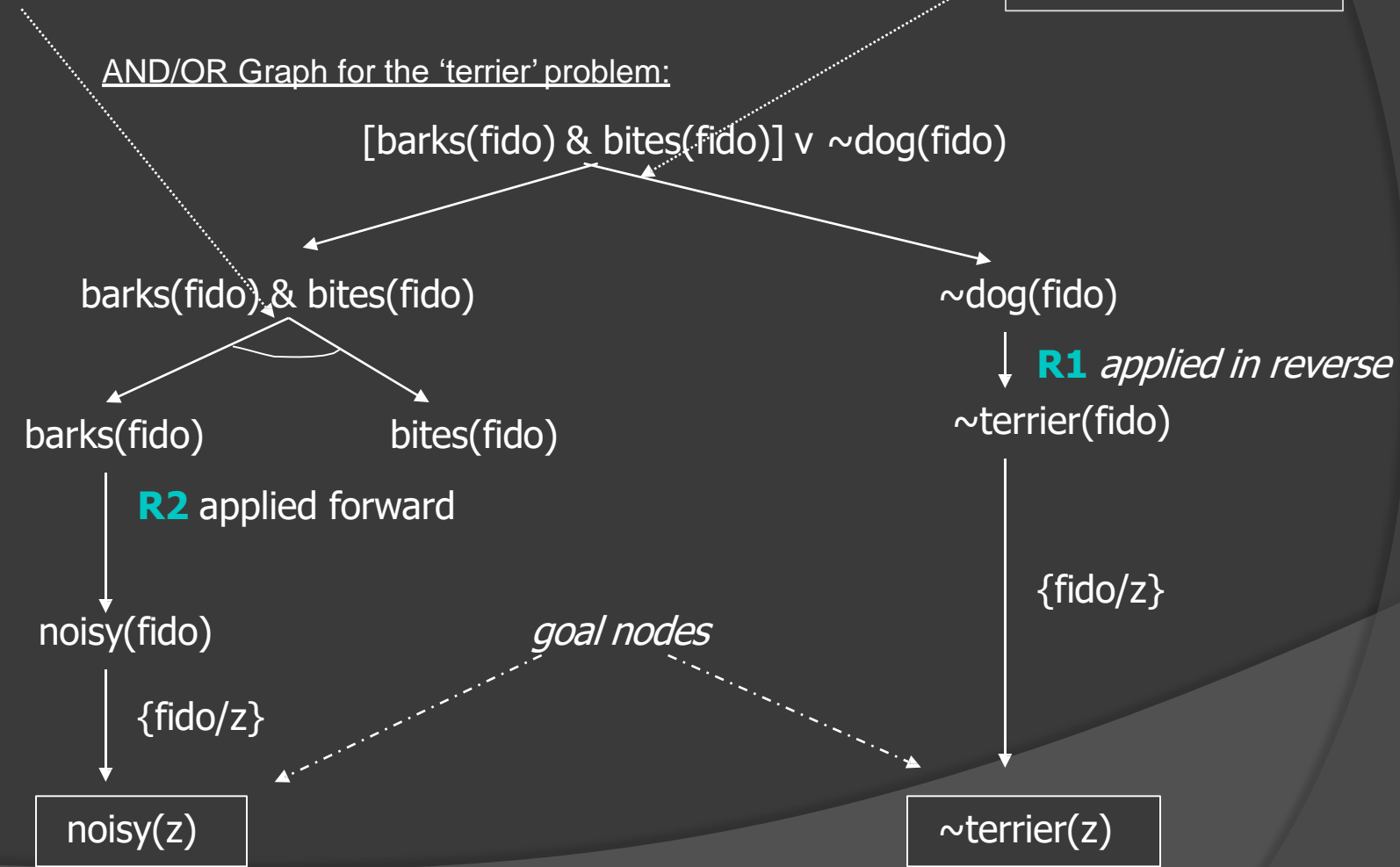
{fido/z}

{fido/z}

goal nodes

noisy(z)

~terrier(z)





Backward production system

B-Rules

We restrict B-rules to expressions of the form: $W \implies L$,
where W is an expression in AND/OR form and L is a literal,
and the scope of quantification of any variables in the implication **is the entire implication.**

Recall that $W \implies (L1 \ \& \ L2)$ is equivalent to the two rules: $W \implies L1$ and $W \implies L2$.

An important property of logic is the **duality** between assertions and goals in theorem-proving systems.

Duality between assertions and goals allows the **goal expression** to be treated as **if it were an assertion.**

Conversion of the goal expression into AND/OR form:

1. Elimination of implication symbols.
2. Move negation symbols in.
3. Skolemize existential variables.
4. Drop existential quantifiers. Variables remaining in the AND/OR form are considered to be existentially quantified.

Goal clauses are **conjunctions of literals** and the **disjunction of these clauses** is the **clause form** of the goal well-formed formula.



Example 1 of formulation of Rule-Based Deduction System



1. Facts:
 dog(fido)
 ~barks(fido)
 wags-tail(fido)
 meows(myrtle)

R2

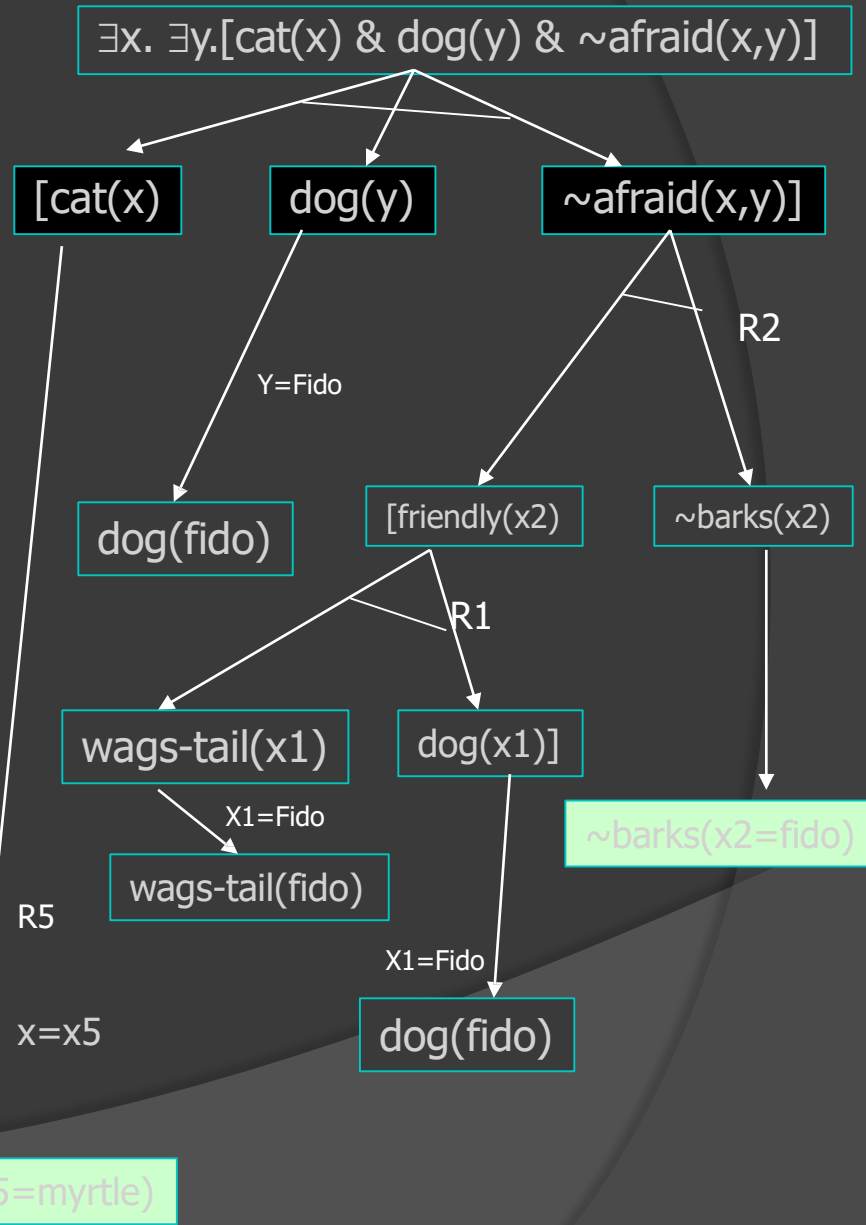
Rules:
 R1: [wags-tail(x1) & dog(x1)] → friendly(x1)
 R2: [friendly(x2) & ~barks(x2)] → ~afraid(y2,x2)
 R3: dog(x3) → animal(x3)
 R4: cat(x4) → animal(x4)
 R5: meows(x5) → cat(x5)

Suppose we want to ask if there are a cat and a dog such that the cat is unafraid of the dog.

The goal expression is:

$\exists x. \exists y.[cat(x) \& dog(y) \& \sim afraid(x,y)]$

We treat the goal expression as an assertion





network: formulation of Rule-Based Deduction

2. The blocks-world situation is described by the following set of wffs:

on_table(a)	clear(e)
on_table(c)	clear(d)
on(d,c)	heavy(d)
on(b,a)	wooden(b)
heavy(b)	on(e,b)

The following statements provide general knowledge about this blocks world:

Every big, blue block is on a green block.

Each heavy, wooden block is big.

All blocks with clear tops are blue.

All wooden blocks are blue.

Represent these statements by a set of implications having single-literal consequents.

Draw a consistent AND/OR solution tree (using B-rules) that solves the problem: “Which block is on a green block?”



HOMEWORK Problem 2. Transformation rules and goal:



Facts:

- f1: on_table(a)
- f2: on_table(c)
- f3: on(d,c)
- f4: on(b,a)
- f5: heavy(b)
- f6: clear(e)
- f7: clear(d)
- f8: heavy(d)
- f9: wooden(b)
- f10: on(e,b)

Rules:

- R1: $big(y1) \wedge blue(y1) \rightarrow green(g(y1))$ Every big, blue block is on a green block.
- R2: $big(y0) \wedge blue(y0) \rightarrow on(y0,g(y0))$ “ “ “ “ “ “ “ “ “
- R3: $heavy(z) \wedge wooden(z) \rightarrow big(z)$ Each heavy, wooden block is big.
- R4: $clear(x) \rightarrow blue(x)$ All blocks with clear tops are blue.
- R5: $wooden(w) \rightarrow blue(w)$ All wooden blocks are blue.

Goal:

$green(u) \wedge on(v,u)$ Which block is on a green block?



HOMEWORK PROBLEM 3. Information Retrieval System



- We have a set of facts containing personnel data for a business organization
- and we want an automatic system to answer various questions about personal matters.

□ Facts

John Jones is the manager of the Purchasing Department.

manager(p-d, john-jones)

works_in(p-d, joe-smith)

works_in(p-d, sally-jones)

works_in(p-d, pete-swanson)

Harry Turner is the manager of the Sales Department.

manager(s-d, harry-turner)

works_in(s-d, mary-jones)

works_in(s-d, bill-white)

married(john-jones, mary-jones)