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$$= \sqrt{\frac{V^2}{\frac{\sigma}{\rho L}}} = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

$$\boxed{W_e = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}}$$

Mach's Number: (M)

Defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force

$$M = \sqrt{\frac{F_i}{F_e}} \qquad [K = \text{Elastic stress}]$$

$$= \sqrt{\frac{\rho A V^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K L^2}} = \sqrt{\frac{\rho V^2}{K}} = \sqrt{\frac{V^2}{\frac{K}{\rho}}}$$

$$M = \frac{V}{\sqrt{\frac{K}{\rho}}} = \frac{V}{C}$$

where $\sqrt{\frac{K}{\rho}} = C$ - Velocity of sound in fluid

$$\boxed{M = \frac{V}{C}} \rightarrow \boxed{C = \sqrt{\frac{K}{\rho}}}$$

MODEL LAWS (OR) SIMILARITY LAWS:

The laws on which the models are ~~designated~~ designed for dynamic similarity are called model laws or laws of similarity. Followings are the model laws:

1. Reynold's model law
2. Froude model law
3. Euler model law
4. Weber model law
5. Mach model law

Reynold's model law:

It is the law in which models are based on Reynold's number. Applications: (i) pipe flow (ii) Resistance experienced by sub-marines, airplanes, etc.

→

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It is given by:

$$[Re]_m = [Re]_p$$

$$\boxed{\frac{\rho_m L_m V_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}}$$

$$\frac{\rho_p V_p L_p}{\rho_m V_m L_m} \times \frac{\mu_m}{\mu_p} = 1$$

$$\boxed{\frac{\rho_r . V_r L_r}{\mu_r} = 1}$$

A pipe of diameter 1.5m is required to transport an oil of sp. gr. 0.90 and viscosity 3×10^{-2} poise at the rate of 3000 litres/s. Tests were conducted on a 15 cm diameter pipe using water at 20°c. Find the velocity and rate of flow in the model. Viscosity of water at 20°c = 0.01 poise.

Given data:

→* Distorted Model

→* Undistorted Model

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UNIT - III

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Model Analysis

* $\left[\dfrac{\rho V L}{\mu}\right]_m = \left[\dfrac{\rho V L}{\mu}\right]_p \longrightarrow [Re]_m = [Re]_p$

* $\left[\rho L^2 V^2\right]_m = \left[\rho L^2 V^2\right]_p \longrightarrow [F]_m = [F]_p \rightarrow$ Drag force
  $\hookrightarrow \frac{1}{2}\rho A U^2 C_D$

* $\left[\dfrac{V}{\sqrt{gL}}\right]_m = \left[\dfrac{V}{\sqrt{gL}}\right]_p$ — Froude's law

The ratio of lengths of a sub-marine and its model is 30:1. The speed of sub-marine is 10 m/s. The model is to be tested in a wind tunnel. Find the speed of air in wind tunnel. Also determine the ratio of the drag between the model and its prototype. Take the value of kinematic viscosities for sea water and air as 0.012 stokes and 0.016 stokes respectively. The density ~~and~~ for sea-water and air is given as 1030 kg/m³ and 1.24 kg/m³ respectively.

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Given data:

Model	Prototype:
$\dfrac{L_2}{L_1} = \dfrac{30}{1}$	$V_2 = 10$ m/s.
$\dfrac{F_2}{F_1} = ?\ ;\quad V_1 = ?$	$\nu_2 = 0.012$ stokes $= 0.012 \times 10^{-4}\ \dfrac{m^2}{s}$
$\nu_1 = 0.016 \times 10^{-4}\ \dfrac{m^2}{s}$	$\rho_2 = 1030$ kg/m³
$\rho_1 = 1.24$ kg/m³.	

Solution:

$$\frac{V_1 L_1}{\nu_1} = \frac{V_2 L_2}{\nu_2}$$

$$V_1 = \frac{V_2 L_2 \times \nu_1}{\nu_2 L_1}$$

$$= \frac{10 \times 30 \times 0.016 \times \cancel{10^{-4}}}{0.012 \times \cancel{10^{-4}}}$$

$$= 400\ m$$

Drag force $= \rho L^2 V^2 \left[\frac{1}{2}\rho A U^2 \times C_D\right]$

$$\frac{\frac{1}{2}\rho A U}{\frac{1}{2}\rho A U^2 \times C_D}$$

$$\frac{F_2}{F_1} = \frac{\rho_2 L_2^2 V_2^2}{\rho_1 L_1^2 V_1^2} = \frac{1030}{1.24} \times 30^2 \times \frac{10^2}{400^2}$$

$$= 467.22$$

A ship 300 m long moves in sea-water, whose density is 1030 kg/m³. A 1:100 model of this ship is to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30 m/s and the resistance of the model is 60N. Determine the velocity of ship in sea-water and also the resistance of the ship in sea-water. The density of air is given as 1.24 kg/m³. Take the kinematic viscosity of sea-water and air as 0.012 stokes and 0.018 stokes respectively.

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Given data:

Model

$L_1 = 3\,m$

$V_1 = 30\,\frac{m}{s}$

$F_1 = 60\,N$

$\rho_1 = 1.24\,\frac{kg}{m^3}$

$\nu_1 = 0.018 \times 10^{-4}\,\frac{m^2}{s}$

Prototype

$L_2 = 300\,m$

$\rho_2 = 1030\,kg/m^3$

$\nu_2 = 0.012$ stokes $= 0.012 \times 10^{-4}\,\frac{m^2}{s}$

$V_2 = ?$

$F_2 = ?$

Solution:

$$\frac{\rho_1 V_1 L_1}{\mu_1} = \frac{\rho_2 V_2 L_2}{\mu_2}$$

$$\frac{V_1 L_1}{\nu_1} = \frac{V_2 L_2}{\nu_2}$$

$$V_2 = \frac{V_1 L_1 \times \nu_2}{\nu_1 \times L_2}$$

$$= \frac{30 \times 3 \times 0.012}{0.018 \times 300}$$

$$= 0.2\,\frac{m}{s}$$

$$\frac{F_1}{F_2} = \frac{\rho_1 L_1^2 V_1^2}{\rho_2 L_2^2 V_2^2}$$

$$F_2 = \frac{F_1 \times \rho_2 L_2^2 V_2^2}{\ldots}$$

$$F_2 = \frac{60 \times 1030 \times 300^2 \times 0.2^2}{1.24 \times 3^2 \times 30^2}$$

$$= 22.15\,KN$$

1:15 model of a flying boat is towed through water. The prototype is moving in sea-water density 1024 kg/m3 at a velocity of 20 m/s. Find the corresponding speed of the model. Also determine the resistance due to waves on model if the resistance due to waves of prototype is 600 N.

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Given data:

Model

$\frac{L_1}{L_2} = \frac{1}{15}$

$V_1 = ?$

$F_1 = ?$

Prototype

$\rho_2 = 1024 \ kg/m^3$

$V_2 = 20 \ \frac{m}{s}$

$F_2 = 600 \ N$

Solution:

From Froude law,

$$\frac{V_1}{\sqrt{L_1 g}} = \frac{V_2}{\sqrt{L_2 g}}$$

$$V_1 = \frac{V_2 \times \sqrt{L_1}}{\sqrt{L_2}} = 20 \times \sqrt{\frac{1}{15}} = 5.164 \ \frac{m}{s}.$$

We know that,

$$\frac{F_1}{F_2} = \frac{\cancel{\frac{1}{2}}\ \rho_1 A_1 U_1^2 \times C_{D_1}}{\cancel{\frac{1}{2}}\ \rho_2 A_2 U_2^2 \times C_{D_2}} = \frac{\rho_1 \times L_1^2 \times V_1^2}{\rho_2 \times L_2^2 \times V_2^2} =$$

$$F_1 = \frac{\rho_1 \times L_1^2 \times V_1^2}{\rho_2 \times L_2^2 \times V_2^2} \times F_2 = \frac{1000 \times 1 \times 5.164^2 \times 600}{1024 \times 15^2 \times 20^2}$$

$$= 0.1736 \ N$$

Types of models:

1. Undistorted models
2. Distorted models.

Undistorted model:

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These are those models which are geometrically similar to their prototypes. The same scale ratio will be maintained.

Behaviour can be easily predicted.

Distorted model:

A model is said to be distorted if it is not geometrically similar to its prototype.

E.g. rivers, reservoirs, etc.

Different scale ratios for horizontal & vertical dimensions. Advantages:

* Cost of the model is reduced
* Vertical dimension can be ~~reduced~~ measured accurately

40, 45, 54, 55, 57, 62, 71

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UNIT - III

* Rayleigh's method:

$$\boxed{\begin{aligned} X &= f[x_1, x_2, x_3] \\ X &= K\, x_1^a \cdot x_2^b \cdot x_3^c \end{aligned}}$$

* Buckingham's π theorem:

$$\boxed{X_1 = f(X_2, X_3 \ldots\ldots X_n)}$$

$$\boxed{f(X_1, X_2, X_3, \ldots\ldots X_n) = 0}$$

$$\boxed{f_1(\pi_1, \pi_2, \ldots\ldots \pi_{n-m}) = 0}$$

$$\left.\begin{aligned} \pi_1 &= X_2^{a_1} \cdot X_3^{b_1} \cdot X_4^{c_1} \cdot X_1 \\ \pi_2 &= X_2^{a_2} \cdot X_3^{b_2} \cdot X_4^{c_2} \cdot X_5 \\ \pi_3 &= X_2^{a_{n-m}} \cdot X_3^{b_{n-m}} \cdot X_4^{c_{n-m}} \cdot X_n \end{aligned}\right\} \text{Equations}$$

$$\boxed{\begin{aligned} \pi_1 &= \phi[\pi_2, \pi_3, \ldots \pi_{n-m}] \\ \pi_2 &= \phi[\pi_1, \pi_3, \ldots \pi_{n-m}] \end{aligned}}$$

$$\boxed{\text{No. of dimensionless } \pi\text{-terms} = n-m}$$

$$\boxed{\text{Each } \pi\text{-term} = m+1}$$

MODEL ANALYSIS

* Geometric Similarity:

$$\boxed{\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r}$$

$$\boxed{\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2}$$

$$\boxed{\frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 = L_r^3}$$

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* Kinematic Similarity:

$$\boxed{\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}} = V_r}$$ ← velocity ratio

$$\boxed{\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r}$$ ← acceleration ratio

* Dynamic Similarity:

$$\boxed{\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = \frac{(F_p)_p}{(F_p)_m} = \frac{(F_s)_p}{(F_s)_m} = \frac{(F_e)_p}{(F_e)_m} = F_r}$$

(F_i) - Inertia force
(F_v) – Viscous force
(F_g) – gravity force
F_p – Pressure force
F_s – Surface tension force
F_e – Elastic force

* Reynold's number: (Re)

$$\boxed{Re = \frac{V \times L}{\nu} = \frac{V \times D}{\nu} = \frac{\rho V L}{\mu} = \frac{\rho V D}{\mu}}$$

* Froude's number: (Fe)

$$\boxed{Fe = \frac{V}{\sqrt{Lg}}}$$

* Euler's number: (Eu)

$$\boxed{Eu = \frac{V}{\sqrt{\frac{P}{\rho}}}}$$

* Weber's number: (We)

$$\boxed{We = \frac{V^2}{\sqrt{\frac{\sigma}{\rho L}}}}$$

* Mach's number: (M)

$$\boxed{M = \frac{V}{c}} \qquad C = \sqrt{\frac{K}{\rho}}$$

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* Reynold's model law:

$$[Re]_m = [Re]_p \qquad \left[\frac{VD}{\nu}\right]_m = \left[\frac{VD}{\nu}\right]_p$$

$$\left[\frac{VL}{\nu}\right]_m = \left[\frac{VL}{\nu}\right]_p \qquad \left[\frac{\rho VL}{\mu}\right]_m = \left[\frac{\rho VD}{\mu}\right]_p$$

* Froude's model law:

$$[Fe]_m = [Fe]_p$$

$$\left[\frac{V}{\sqrt{Lg}}\right]_m = \left[\frac{V}{\sqrt{Lg}}\right]_m$$

* Euler's model law:

$$[Eu]_m = [Eu]_p$$

$$\left[\frac{V}{\sqrt{\frac{P}{\rho}}}\right]_m = \left[\frac{V}{\sqrt{\frac{P}{\rho}}}\right]_m$$

* Weber's model law:

$$[We]_m = [We]_p$$

$$\left[\frac{V^2}{\sqrt{\frac{\sigma}{\rho L}}}\right]_m = \left[\frac{V^2}{\sqrt{\frac{\sigma}{\rho L}}}\right]_p$$

* Mach's model law:

$$[M]_m = [M]_p$$

$$\left[\frac{V}{C}\right]_m = \left[\frac{V}{C}\right]_p$$