UNIT-III

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Dimensional Analysis:

It is a method of dimensions. It is a mathematical technique used in research work for design and for conducting model tests.

It deals with the dimensions of the physical quantities involved. It uses three fixed dimensions in Fluid Mechanics viz Length L, mass M and time T.

Fixed dimensions are also known as fundamental dimensions.

Secondary or derived quantities:

These are those quantities which possess more than one fundamental dimension.

Determine the dimensions of the quantities below (i) Discharge (ii) kinematic viscosity (iii) Force (iv) specific weight (v) Dynamic viscosity.

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1 Discharge: $Q = a \times V$

$= L^2 \times \frac{m}{s}$

$= L^2 L T^{-1} = L^3 T^{-1}$

Kinematic viscosity: $\nu = \frac{\mu}{\rho}$

5 $\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{F}{A}}{\frac{L/T}{L}} = \frac{\frac{M \times \frac{L}{T^2}}{L^2}}{\frac{L}{T} \times \frac{1}{L}} = \frac{M \frac{L}{T^2} \times \frac{1}{L^2}}{\frac{1}{T}}$

$= \frac{ML}{T^2} \times \frac{1}{L} \times T = M L^{-1} T^{-1}$

$\rho = \frac{M}{V} = \frac{M}{L^3} = M L^{-3}$

$\nu = \frac{M L^{-1} T^{-1}}{M L^{-3}} = M L^{-1} T^{-1} M^{-1} L^3$

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iii) Force = Mass × Acceleration

Mass = M

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Acceleration = $\frac{\text{metre}}{\text{sec}^2} = \frac{L}{T^2}$

$\therefore$ Force $= M \times \frac{L}{T^2} = MLT^{-2}$ //

(iv) Specific Weight = $\frac{\text{Weight}}{\text{Volume}}$

Weight = Mass × Gravity (acceleration) ;

$= M \times \frac{m}{\sec^2} = MLT^{-2}$

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Volume $= L^3$

Team 1

Sp. Weight $= MLT^{-2}L^{-3}$

$= ML^{-2}T^{-2}$ //

(v) Dynamic viscosity $= \mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{F}{A}}{\frac{m/T}{L}} = \frac{\frac{M \times \frac{L}{T^2}}{L^2}}{\frac{m/T}{L}}$

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$= \frac{M \times \frac{L}{T^2} \times \frac{1}{L^2}}{\frac{L}{T} \times \frac{1}{L}} = \frac{MT^{-2}L^{-1}}{T^{-1}} = MT^{-2}L^{-1}T^{1}$

$= MT^{-1}L^{-1}$ //

Dimensional Homogenity :

It means the dimensions of each terms in an equation on both sides equal.

If the dimensions of each term on both sides of an equation are the same, the equation is known as dimensionally homogeneous equation. The powers of fundamental dimensions (i.e. L, M, T) on both sides of the equation will be identical for a dimensionally homogeneous equation

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Prove that $V = \sqrt{2gh}$ is a dimensionally homo. equation.

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$V = \sqrt{2gh}$

LHS, $V = \frac{L}{T} = LT^{-1}$,

RHS $\sqrt{2gh} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} = LT^{-1}$,

LHS = RHS, hence it is a dimensionally homogeneous equation.

Methods of dimensional Analysis:

Two methods of dimensional analysis

1. Rayleigh's method and
2. Bukingham's π - theorem.

Rayleigh's method:

This method is used for determining the expression for a variable which depends upon maximum three or four variables only.

Let X is a variable, which depends on X_1, X_2, X_3 variables.

Mathematically, $\boxed{X = f[X_1, X_2, X_3]}$

also

$$\boxed{X = K X_1^a \cdot X_2^b \cdot X_3^c}$$

K - Constant; a, b, c - arbitarily powers.

Find the expression for the Power P, developed by a pump depends upon the head H, the discharge Q and specific weight w of the fluid.

Power = Head 'H', Discharge 'Q', Specific weight 'w'

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$$\boxed{P = K H^a \cdot Q^b \cdot W^c} \leftarrow \text{I}$$

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~~$ML^3 T^3 = K L^a$~~

$$\text{Power} = \frac{\overset{MLT^{-2} \times L}{\text{Force} \times \text{distance}}}{\text{time } T} = \frac{\text{Workdone}}{\text{time}} = ML^2T^{-3}$$

$$\text{Head} = L$$

$$\text{Discharge} = \text{Area} \times \text{Velocity} = L^2 \times L \times T^{-1}$$
$$= L^3T^{-1}$$

$$\text{Specific weight} = \frac{\text{Weight}}{\text{Volume}} = \frac{\cancel{\text{Density}}\ \text{Mass} \times \text{Acceleration}}{\text{volume}}$$

$$= MLT^{-2}L^{-3}$$
$$= ML^{-2}T^{-2}$$

$$ML^2T^{-3} = KL^a \cdot (L^3T^{-1})^b \cdot (ML^{-2}T^{-2})^c$$

Power of M $\quad 1 = c$

Power of L $\quad 2 = a + 3b - 2c \rightarrow (1)$

Power of T $\quad -3 = -b - 2c \rightarrow (2)$

$c = 1$ in (2)

$b = 3 - 2c$
$= 3 - 2$
$\boxed{b = 1}$

$c = 1$ and $b = 1$ in (1)

$a + 3 - 2 = 2$
$a + 1 = 2$
$a = 2 - 1$
$\boxed{a = 1}$

Substituting all in (I)

$P = KH^1 \cdot Q^1 \cdot W^1$

$$\boxed{P = KHQW}$$

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The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.

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Solution:

Time period (t) = Length (L), Acceleration (g)

$$\boxed{t = KL^{a}.\, g^{b}} \leftarrow \text{I}$$

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Time period = T

Length = L

Acceleration = LT^{-2}

$$T = KL^{a}.\,(LT^{-2})^{b}$$

Power of T $\quad 1 = -2b \rightarrow (1)$

Power of L $\quad 0 = a+b \rightarrow (2)$

Taking (1)

$$\boxed{b = -\frac{1}{2}}$$

Substituiting $\boxed{b = -\frac{1}{2}}$ in (2)

$$0 = a - \frac{1}{2}$$

$$\boxed{a = \frac{1}{2}}$$

Substituting $a = \frac{1}{2}$ and $b = -\frac{1}{2}$ in (I)

$$t = KL^{\frac{1}{2}}.\, g^{-\frac{1}{2}}$$

$$\boxed{t = K\sqrt{\frac{L}{g}}}$$

From experiment $K = 2\pi$

$$\therefore \boxed{t = 2\pi\sqrt{\frac{L}{g}}}$$

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Find an expression for the drag force on smooth sphere of diameter D, moving with a uniform velocity V in a fluid of density ρ and dynamic viscosity μ.

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Given data:

Drag Force = Sphere Diameter D, Uniform velocity V, Density ρ, Dynamic viscosity μ

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Solution:

$$\boxed{F = K\, D^a \cdot V^b \cdot \rho^c \cdot \mu^d} \leftarrow \text{I}$$

Force = MLT^{-2}
Diameter = L
Velocity = LT^{-1}
Density = ML^{-3}
Viscosity = $ML^{-1}T^{-1}$

$$MLT^{-2} = K\, L^a \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (ML^{-1}T^{-1})^d$$

Power of M $\quad 1 = c + d \rightarrow (1)$

Power of L $\quad 1 = a + b - 3c - d \rightarrow (2)$

Power of T $\quad -2 = -b - d \rightarrow (3)$

[4 Variable ≠ 3 equation not possible to solve]

$\boxed{c = 1-d} \rightarrow$ from (1)

$\boxed{b = 2-d} \rightarrow$ from (3)

$a = 1 - b + 3c + d$

$= 1 - 2 + d + 3(1-d) + d$

$= 1 - 2 + d + 3 - 3d + d$

$\boxed{a = 2 - d}$

$$F = K\, D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d} \cdot \mu^d$$

$$= K D^2 \cdot V^2 \cdot \rho^1 \left(D^{-d} \cdot V^{-d} \cdot \rho^{-d} \cdot \mu^d\right)$$

$$\boxed{F = K \rho D^2 V^2 \left(\frac{\mu}{DV\rho}\right)^d}$$

Buckingham's π-theorem.

It states, "If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contains m fundamentals dimensions (M, L, T), then the variables are arranged Into $(n-m)$ dimensionless terms. Each term is called π-term".

Let X_1 = dependent variable

$X_2, X_3 \ldots X_n$ = independent variables, then

$$\boxed{X_1 = f(X_2, X_3 \ldots X_n)}$$

Can also, be written as

$$\boxed{f(X_1, X_2, X_3, \ldots X_n) = 0.}$$

According to Buckingham's π theorem it can be written as

$$\boxed{f_1(\pi_1, \pi_2, \ldots \pi_{n-m}) = 0} \quad (1)$$

∴ Each π term contains $m+1$ variables, where m is the number of fundamental dimensions and also called as repeating variables.

$$\left.\begin{aligned} \pi_1 &= X_2^{a_1} \cdot X_3^{b_1} \cdot X_4^{c_1} \cdot X_1 \\ \pi_2 &= X_2^{a_2} \cdot X_3^{b_2} \cdot X_4^{c_2} \cdot X_5 \\ \pi_{n-m} &= X_2^{a_{n-m}} X_3^{b_{n-m}} \cdot X_4^{c_{n-m}} \cdot X_n \end{aligned}\right\} \text{Equations}$$

Each equation is solved by the principle of dimensional homogenity and values of a_1, b_1, c_1, etc are obtained. Then these values are substituted in equation and values of π's $\pi_1, \pi_2, \pi_3 \ldots \pi_{n-m}$ are obtained. These values of π's are then substituted in equation (1). Final equation is as follows

$$\pi_1 = \phi\,[\pi_2, \pi_3, \ldots \pi_{n-m}]$$

$$\pi_2 = \phi_1\,[\pi_1, \pi_3, \ldots \pi_{n-m}]$$