DISCRETE MATHEMATICS

PART-A

1. **Express the statement, "The crop will be destroyed if there is a flood," in symbolic form.**
Solution:
Let C: The crop will be destroyed.
F: There is a flood
Symbolic form: $F \rightarrow C$

2. **State the truth table of "If tigers have wings then the earth travels round the sun."**
Solution:
Let P: Tigers have wings. "F"
Q: The earth travels round the sun. "F"
Therefore, given statement is $P \rightarrow Q$, has the truth value "T".

P	Q	$P \rightarrow Q$
F	F	T

3. **Give the converse and contrapositive of the implication " If it is raining, then I get wet".**
Solution:
P: It is raining.
Q: I get wet.
Converse: $(Q \rightarrow P)$ If I get wet, then it is raining.
Contrapositive: $(\neg Q \rightarrow \neg P)$ If I do not get wet, then it is not raining.

4. **What are the contrapositive, the converse and the inverse of the conditional statement. "If you work hard then you will be rewarded."**
Solution:
P:You work hard ⌐P: You will not work hard
Q: You will be rewarded ⌐Q: You will not be rewarded.
Converse: $(Q \rightarrow P)$ You will be rewarded only if you work hard.
Contrapositive: $(\neg Q \rightarrow \neg P)$ If you will not be rewarded then you will not work hard.
Inverse: $(\neg P \rightarrow \neg Q)$ If you will not work hard then you will not be rewarded.

5. **Define Tautology with an example.**
Solution:
A statement that is true for all possible values of propositional variables is called a tautology or universally valid formula or a logical truth.
Example: $P \vee \neg P$ is a tautology.
6. **Using truth table, show that the proposition $p \vee \neg(p \wedge q)$ is a tautology.**
Solution:

p	q	$p\wedge q$	$\urcorner(p\wedge q)$	$p\vee\urcorner(p\wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Therefore, $p\vee\urcorner(p\wedge q)$ is a tautology.

7. Show that the proposition $P\rightarrow Q$ and $\urcorner P\vee Q$ are logically equivalent.

Solution:

We should prove that $(P\rightarrow Q) \Leftrightarrow \urcorner P\vee Q$.

i.e., To prove $(P\rightarrow Q) \leftrightarrow \urcorner P\vee Q$ is tautology.

P	Q	$P\rightarrow Q$	$\urcorner P$	$\urcorner P\vee Q$	$(P\rightarrow Q) \leftrightarrow \urcorner P\vee Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$(P\rightarrow Q) \Leftrightarrow \urcorner P\vee Q$.

8. Define functionally complete set of connectives and give an example.

Solution:

A set of connectives is said to be functionally complete if any formula can be written as an equivalent formula containing only these connectives.

Eg: The set of connectives $\{\wedge, \urcorner\}$ & $\{\vee, \urcorner\}$ are functionally complete.

9. Show that $(\wedge,\vee)$ is not functionally complete.

Solution:

$\urcorner P$ cannot be expressed using the connectives $\{\vee, \wedge\}$. Since no such contribution of statement exist with $\{\vee, \wedge\}$ as input is T and the output is F.

10. State the rules of inference theory.

Solution:

Rule P: A premise may be introduced at any point in the derivation.

Rule T: A formula S may be introduced in a derivation of s is tautologically implied by any one or more of the preceding formulas in the derivation.

Rule CP: If we can derive S from R and a set of premises, then we can derive $R\rightarrow S$ from the set of premises alone.

11. Determine whether the conculsion C follows logically from the premises H_1 and H_2 or not.

H_1: $P\rightarrow Q$, H_2:P, C:Q.

Solution:

P	Q	$P\to Q$	$(P\to Q)\wedge P$	$[(P\to Q)\wedge P]\to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Therefore conclusion is valid.

12. Write the negation of the statement $(\exists x)\ (\forall y)\ p(x,y)$.

Solution:

Given : $(\exists x)\ (\forall y)\ p(x,y)$

Negation : $(\forall x)\ (\exists y)\ \overline{p(x,y)}$.

13. Define bound and free variables.

Solution:

A variable is bounded by a quantifier is called bound variable.

A variable which is not bounded by any quantifier is called free variable.

14. Write the scope of the quantifiers in the formula $(x)\ (P(x)\to(\exists y)R(x, y))$

Solution:

The scope of (x) is $P(x)\to(\exists y)R(x,y)$.

The scope of $(\exists y)$ is $R(x,y)$.

15. "Every parrot is ugly"-Express using quantifiers.

Solution:

$P(x)$: x is a parrot;

$Q(x)$: x is ugly.

Symbolic form: $(x)(P(x)\to Q(x))$.

16. Symbolize the statement "Some men are genius"

Solution:

Let $M(x)$: x is a man

$G(x)$: x is genius.

Symbolic form: $(\exists x)(M(x)\wedge G(x))$.

17. Symbolize the statement "All men are giants."

Solution:

Case(i): Using $G(x)$: x is a giant.

$M(x)$: x is a man.

The given statement can be symbolized as $(\forall x)[(M(x)\to G(x)]$

Case(ii): However, if we restrict the variable x to the universe which is the class of men, then the statement is $(\forall x)G(x)$.

18. Symbolize the statement "x is the father of the mother of y".

Solution:

P(x): x is a person

F(x,y): x is father of y.

M(x,y): x is mother of y.

Symbolic form: $(\exists z)[P(z) \wedge F(x,z) \wedge M(z,x)]$.

19. Explain the two types of quantifiers through example.

Solution:

(i)Universal Quantifiers: The quantifier "all" is called the universal Quantifier and we shall denote it by $\forall x$.

Eg: For all x, x is an integer is written as $(\forall x)\ I(x)$.

(ii)Existential Quantifiers: The quantifier "some" is called the existential quantifier and we shall denote it by $\exists x$.

Eg: There exists an x such that x is a man is written as $(\exists x)\ M(x)$.

20. Define a rule of universal specification.

Solution:

From (x) A(x) one can conclude A(y).

If a statement of the from (x) A(x) is assumed to be true, then the universal quantifier can be dropped to obtain A(y) is true for any arbitrary object 'y' in the universe.

21. Given an Indirect proof of the theorem "If 3n+2 is odd, then n is odd".

Solution:

P: 3n+2 is odd

Q: n is odd

Hypothesis: Assume that $P \to Q$ is false.

i.e., Assume that p is true and Q is false. i.e., n is not odd $\Rightarrow$ n is even.

Analysis: If n is even then n=2k for some integer k. $3n+2=3(2k)+2=6k+2=2(3k+1)$

Conclusion: We observe that the R.H.S value of 3n+2 is divisible by 2. This means that 3n+2 is even. This contradicts the assumption P is true. In view of this contradiction, we infer that the given conditional $P \to Q$ is true.

PART-B

Problems based on Logical Equivalence:

1. **Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.**
2. **Without using the truth table, prove that $\neg p \rightarrow (q \rightarrow r) \Leftrightarrow (q \rightarrow (p \vee r)$.**

Problems based on Normal forms:

3. **Without using truth table find the PCNF and PDNF of $P \rightarrow (Q \wedge P) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$.**
   [Ans: PCNF: $(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R)$
   PDNF: $(P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$].
4. **Obtain the product of sum canonical form of the formula $(\neg P \rightarrow R) \rightarrow (Q \leftrightarrow P)$**

Problems based on Inference theory of Statement Calculus:

5. **Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$ and $(A \wedge \neg B) \rightarrow (R \vee S)$.**
6. **Show that $(P \rightarrow Q) \wedge (R \rightarrow S)$, $(Q \wedge M) \wedge (S \rightarrow N)$, $\neg (M \wedge N)$ and $(P \rightarrow R) \Rightarrow \neg P$**
7. **Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$, $\neg M$.**
8. **Prove that the premises $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent.**
9. Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ & Q
10. Using conditional proof prove that $\neg P \vee Q$, $\neg Q \vee R$, $R \rightarrow S \Rightarrow P \rightarrow S$
11. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R)$ and $(Q \rightarrow S)$.
12. **Show that the following set of premises are inconsistent**
    (i) If jack misses many classes through illness, then he fails high school
    (ii) If Jack fails high school, then he is uneducated
    (iii) If Jack reada a lot of books, then he is not uneducated.
    (iv) Jack misses many classes through illness and reads a lot of books.
13. **Show that the following implication by using indirect method. $(R \rightarrow \neg Q)$, $R \vee S$, $S \rightarrow \neg Q$, $P \rightarrow Q \Rightarrow \neg P$.**
14. **Using Indirect method of proof , derive $p \rightarrow \neg s$ from the premises $p \rightarrow (q \vee r)$, $q \rightarrow \neg p$, $s \rightarrow \neg r$ and p.**

Problems based on Predicate Calculus:

15. **Prove that $(\forall x)(P(x) \rightarrow Q(x))$, $(\forall x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (\forall x)(R(x) \rightarrow \neg P(x))$.**
16. **Show that the conclusion $(\forall x)(P(x) \rightarrow \neg Q(x))$ follows from the premises $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$ and $(\exists y)(M(y) \wedge \neg W(y))$.**
17. **Use Indirect method to prove that the conclusion $\exists z\, Q(z)$ follows from the premises $(\forall x)(P(x) \rightarrow Q(x))$ and $\exists y\, p(y)$.**
18. **Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)(P(x) \vee (\exists x)\, Q(x))$ by indirect method of proof.**
19. Show that $(x)(P(x) \rightarrow Q(x) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x)$

Problems based on Proof methods and strategy

20. **Prove that $\sqrt{2}$ is irrational by giving a proof using contradiction.**