TOPIC:5 –Pigeon HolePrinciple

Pigeon Hole principle

If $(n+1)$ pigeon occupies 'n' holes then atleast one hole has more than 1 pigeon.

Generalized Pigeon Hole Principle

If 'm' pigeon occupies 'n' holes $(m>n)$, then atleast one hole has more than $\left[\frac{m-1}{n}\right]+1$ pigeon.

(1) Show that, among 100 people, atleast 9 of them were born in the same month.

Here, No. of pigeon = m = No. of people
= 100

No. of Holes = n = No. of month
= 12

Then by generalized pigeon hole principle,

$$\left[\frac{m-1}{n}\right]+1 \Rightarrow \left[\frac{100-1}{12}\right]+1 = 8+1 = 9$$

∴ 9 were born in the same month.

(2) show that, among if seven colours are used to paint 50 bicycles, atleast 8 bicycles will be the same colour.

Here, No. of pigeon = m = No. of bicycle

$= 50$

No. of Holes $= n = 7$ = No. of colours.

Then by generalized pigeon hole principle,

$$\left[\frac{m-1}{n}\right]+1 \Rightarrow \left[\frac{50-1}{7}\right]+1 \Rightarrow 7+1$$

$$\Rightarrow 8$$

$\therefore$ atleast 8 bicycles will have the same colour.

(3) Prove that in any group of six people, there must be atleast 3 mutual friends or atleast 3 mutual enemies.

Let those six people will be A, B, C, D, E and F. Fix A. The remaining 5 peoples can be accommodate into 2 groups namely (1) Friends of A and (2) Enemies of A.

Now, by generalized pigeon hole principle, atleast one of the group must contain

$\left\lceil \frac{m-1}{n} \right\rfloor + 1 \Rightarrow \left\lfloor \frac{5-1}{2} \right\rfloor + 1 \Rightarrow 3$

Let the group friend of A contain 3 people

case (i)

If any two of these 3 people (B, C, D) are friends, then these two together with A form mutual friends.

case (ii)

If no two of these 3 people are friends, then these 3 people (B, C, D) are mutual enemies.

In either case, we get the required conclusion.

If the group of enemises of A contain 3 people, by the above similar argument we get the required conclusion.

(4) What is the maximum number of students required in a discrete mathematics class to be sure that atleast six will receive the same grade if there are five possible grades A, B, C, D and E?

No. of pigeon hole = No. of grades

$= n = 5$

Let K be number of students (pigeon) in discrete mathematics class.

$\therefore K+1 = 6$

$\Rightarrow \boxed{K = 5}$

$\therefore$ The total number of students $= Kn+1$

$= 25+1 = 26$

$\therefore$ Minimum number of students $= 26$