



Mathematical Induction

Principle of Mathematical Induction

Let $P(n)$ be a statement or proposition involving the natural number 'n', then we complete two steps:

Basic step we must prove that $P(1)$ is true.

Inductive step By assuming $P(k)$ is true, we must prove that $P(k+1)$ is also true.

① Use mathematical induction to show that $n! \geq 2^{n+1}$, $n = 5, 6, \dots$

Let $P(n) = n! \geq 2^{n+1}$, $n = 5, 6, \dots$

(i) $P(5) : 5! \geq 2^6 = 120 \geq 64$ is true

Assume $P(k) : k! \geq 2^{k+1}$ is true \rightarrow ①

(ii) claim : $P(k+1)$ is true

By ①, $k! \geq 2^{k+1}$ Multiply both sides by 2

$$2k! \geq 2^{k+2}$$



$$\Rightarrow (k+1)k! \geq 2^{k+2}$$

$$\Rightarrow (k+1)! \geq 2^{k+2}$$

$\therefore P(k+1)$ is true.

Hence, by the principle of mathematical induction,

$$n! \geq 2^{n+1}, \text{ for } n = 5, 6, \dots$$

② Use Mathematical induction, show that

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$\text{Let } P(n) : 1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$(i) P(1) : 1 = \frac{1(2)}{2} \text{ is true.}$$

$$(ii) \text{ Assume } P(k) : 1+2+3+\dots+k = \frac{k(k+1)}{2} \text{ is true.} \\ \rightarrow \textcircled{1}$$

(iii) claim : $P(k+1)$ is true.

$$P(k+1) : 1+2+3+\dots+k+k+1$$

$$= \frac{k(k+1)}{2} + k+1 \quad (\text{by } \textcircled{1})$$



$$= \frac{k(k+1) + 2k + 2}{2} = \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} \quad \text{which is true.}$$

By the principle of mathematical induction,

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

③ Prove that by the principle of mathematical induction, for 'n' a positive integer,

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\text{Let } P(n) = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(i) \quad P(1) : 1^2 = \frac{1(1+1)(2+1)}{6} \text{ is true.}$$

$$(ii) \quad \text{Assume } P(k) : 1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6} \text{ is true}$$

→ ①

(iii) claim : $P(k+1)$ is true.

$$P(k+1) : 1^2 + 2^2 + \dots + k^2 + (k+1)^2$$



$$\begin{aligned} &= \frac{K(K+1)(2K+1)}{6} + (K+1)^2 \quad \text{by (1)} \\ &= \frac{K(K+1)(2K+1) + 6(K+1)^2}{6} \\ &= \frac{(K+1)(K(2K+1) + 6(K+1))}{6} \\ &= \frac{(K+1)(K+2)(2K+3)}{6} \\ &= \frac{(K+1)((K+1)+1)(2(K+1)+1)}{6} \end{aligned}$$

$\therefore P(K+1)$ is true.

\therefore By the principle of mathematical induction,

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \forall n$$



SNS COLLEGE OF ENGINEERING
Coimbatore – 641 107

