



① Demonstrate that R is a valid inference from the premises $P \rightarrow Q$, $Q \rightarrow R$ and P.

Here given premises are

- (1) $P \rightarrow Q$
- (2) $Q \rightarrow R$
- (3) P

$\{1\}$	1) $P \rightarrow Q$	Rule P
$\{2\}$	2) P	Rule P
$\{1,2\}$	3) Q	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
$\{4\}$	4) $Q \rightarrow R$	Rule P
$\{1,2,4\}$	5) R	Rule T ($P, P \rightarrow Q \Rightarrow Q$)



2. show that $(P \rightarrow Q) \wedge (R \rightarrow S), (Q \rightarrow M) \wedge (S \rightarrow N)$

$\neg(M \wedge N)$ and $(P \rightarrow R) \Rightarrow \neg P$.

Given premises are $(P \rightarrow Q) \wedge (R \rightarrow S),$
 $(Q \rightarrow M) \wedge (S \rightarrow N), \neg(M \wedge N)$ and $(P \rightarrow R)$

Conclusion is $\neg P$.

{1}	1) $(P \rightarrow Q) \wedge (R \rightarrow S)$	Rule P
{1}	2) $P \rightarrow Q$	Rule T ($P \wedge Q \Rightarrow P$)
{1}	3) $R \rightarrow S$	Rule T ($P \wedge Q \Rightarrow Q$)
{4}	4) $(Q \rightarrow M) \wedge (S \rightarrow N)$	Rule P
{4}	5) $Q \rightarrow M$	Rule T ($P \wedge Q \Rightarrow P$)
{4}	6) $S \rightarrow N$	Rule T ($P \wedge Q \Rightarrow Q$)
{1,4}	7) $P \rightarrow M$	Rule T ($P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$)
{1,4}	8) $R \rightarrow N$	Rule T ($P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$)
{9}	9) $P \rightarrow R$	Rule P



{1, 4, 9}	10) $P \rightarrow N$	Rule T ($P \rightarrow a, a \rightarrow R \Rightarrow P \rightarrow R$)
{1, 4, 9}	11) $\neg N \rightarrow \neg P$	Rule T (Taking \neg)
{1, 4}	12) $\neg M \rightarrow \neg P$	Rule T (Taking \neg)
{1, 4, 9}	13) $(\neg M \vee \neg N) \rightarrow \neg P$	Rule T ($a \rightarrow b, c \rightarrow b \Rightarrow (a \vee c) \rightarrow b$)
{1, 4, 9}	14) $\neg (M \wedge N) \rightarrow \neg P$	Rule T (Demorgan's law)
{15}	15) $\neg (M \wedge N)$	Rule P
{1, 4, 9, 15}	16) $\neg P$	Rule T ($P, P \rightarrow Q \Rightarrow Q$)

3.

Prove that the following argument is valid:

$$p \rightarrow \neg q, r \rightarrow q, r \Rightarrow \neg p$$

Given premises are $p \rightarrow \neg q, r \rightarrow q, r$

Conclusion is $\neg p$.



$\{1\}$	1) γ	Rule P
$\{2\}$	2) $\gamma \rightarrow q$	Rule P
$\{1,2\}$	3) q	Rule T ($P, P \rightarrow q \Rightarrow q$)
$\{4\}$	4) $p \rightarrow \neg q$	Rule P
$\{1,2,4\}$	5) $\neg p$	Rule T ($P \rightarrow \neg q, q \Rightarrow \neg P$)