TOPIC:7-Consistency of premised and Indirect method of proof

Consistency and Inconsistency of premises

A set of formulae $H_1, H_2, \cdots H_m$ is said to be inconsistent if their conjunction implies contradiction.

i.e) $H_1 \wedge H_2 \wedge \cdots \wedge H_m \Leftrightarrow F$

A set of formulae $H_1, H_2, \cdots H_m$ is said to be consistent if it is not inconsistent.

(1) Prove that $P \rightarrow Q$, $Q \rightarrow R$, $S \rightarrow \neg R$, $P \wedge S$ are inconsistent.

{1}	1) $P \to Q$	Rule P
{2}	2) $Q \to R$	Rule P
{1,2}	3) $P \to R$	Rule T $(P \to Q, Q \to R \Rightarrow P \to R)$
{4}	4) $S \to \neg R$	Rule P
{4}	5) $R \to \neg S$	Rule T $(P \to Q \Leftrightarrow \neg Q \to \neg P)$
{1,2,4}	6) $P \to \neg S$	Rule T $(P \to Q, Q \to R \Rightarrow P \to R)$
{1,2,4}	7) $\neg P \vee \neg S$	Rule T $(P \to Q \Leftrightarrow \neg P \vee Q)$
{1,2,4}	8) $\neg (P \wedge S)$	Rule T (Demorgan's)
{9}	9) $P \wedge S$	Rule P
{1,2,4,9}	10) $(P \wedge S) \wedge \neg (P \wedge S)$	Rule T $(P, Q \Rightarrow P \wedge Q)$

which is nothing but false value. Therefore given premises are inconsistent.

(2) Prove that $P\to Q$, $Q\to R$, $R\to S$, $S\to\neg R$ and $P\wedge S$ are inconsistent.

{1}	1) $P\to Q$	Rule P
{2}	2) $Q\to R$	Rule P
{1,2}	3) $P\to R$	Rule T ($P\to Q, Q\to R \Rightarrow P\to R$)
{4}	4) $R\to S$	Rule P
{1,2,4}	5) $P\to S$	Rule T ($P\to Q, Q\to R \Rightarrow P\to R$)
{6}	6) $S\to\neg R$	Rule P
{6}	7) $R\to\neg S$	Rule T ($P\to Q \Leftrightarrow \neg Q\to\neg P$)
{6}	8) $\neg R\vee\neg S$	Rule T ($P\to Q \Leftrightarrow \neg P\vee Q$)
{6}	9) $\neg S$	Rule T ($P\vee Q \Rightarrow Q$)
{1,2,4,6}	10) $\neg P$	Rule T ($P\to Q, \neg Q \Rightarrow \neg P$)
{1,2,4,6}	11) $\neg P\vee\neg S$	Rule T ($P, Q \Rightarrow P\vee Q$)
{12}	12) $P\wedge S$	Rule P
{1,2,4,6}	13) $\neg(P\wedge S)$	Rule T (Demorgan's)
{1,2,4,6,12}	14) $(P\wedge S)\wedge\neg(P\wedge S)$	Rule T ($P, Q \Rightarrow P\wedge Q$)

which is nothing but false value. Therefore given premises are inconsistent.

Indirect Method of Proof

In order to show that a conclusion C follows logically from the premises $H_1, H_2, \cdots H_m$, we assume C is FALSE and consider $\neg C$ as an additional premises. If $H_1 \wedge H_2 \wedge \cdots \wedge H_m \wedge \neg C$ is a contradiction, then C follows logically from $H_1, H_2, \cdots, H_m$.

1. Using indirect method of proof, derive $p \to \neg s$ from the premises $p \to (q \vee r)$, $q \to \neg p$, $s \to \neg r$ and p.

We consider $\neg(p \to \neg s)$ as an additional premises
$= \neg(\neg p \vee \neg s) = p \wedge s$.

{1}	1) $p \wedge s$	Assumed premises
{2}	2) $p \to (q \vee r)$	Rule P
{3}	3) p	Rule P
{2,3}	4) $q \vee r$	Rule T $(P, P \to Q \Rightarrow Q)$
{1}	5) s	Rule T $(P \wedge Q \Rightarrow Q)$
{6}	6) $s \to \neg r$	Rule P
{1,6}	7) $\neg r$	Rule T $(P, P \to Q \Rightarrow Q)$.

$\{2,3\}$	8) $\neg q \to r$	Rule T $(P \to Q \Leftrightarrow \neg P \vee Q)$
$\{2,3\}$	9) $\neg r \to q$	Rule T (contrapositive)
$\{1,2,3,6\}$	10) q	Rule T $(P, P \to Q \Rightarrow Q)$
$\{11\}$	11) $q \to \neg p$	Rule P
$\{1,2,3,6,11\}$	12) $\neg p$	Rule T $(P, P \to Q \Rightarrow Q)$
$\{1,2,3,6,11\}$	13) $p \wedge \neg p$	Rule T $(P, Q \Rightarrow P \wedge Q)$

which is nothing but false value.

. By method of contradiction, $p \to \neg s$

2) Show that the following argument is valid.

"My father praises me only if I can be proud of myself. Either I do well in sports or I cannot be proud of ~~himself~~ myself. If study hard, then I cannot do well in sports. Therefore, if father praises me, then I do not study well".

Let A : My father praises me

B : I can be proud of myself

C : I do well in sports

D : I study hard

Then, the premises are

$A \rightarrow B$, $C \vee \neg B$, $D \rightarrow \neg C$

Conclusion is $A \rightarrow \neg D$

{1}	1) A	Assumed premises
{2}	2) $A \to B$	Rule P
{1,2}	3) B	Rule T $(P, P \to Q \Rightarrow Q)$
{4}	4) $C \vee \neg B$	Rule P
{4}	5) $B \to C$	Rule T $(P \to Q \Leftrightarrow \neg P \vee Q)$
{1,2,4}	6) C	Rule T $(P, P \to Q \Rightarrow Q)$
{7}	7) $D \to \neg C$	Rule P
{7}	8) $C \to \neg D$	Rule T
{1,2,4,7}	9) $\neg D$	Rule T $(P, P \to Q \Rightarrow Q)$
	10) $A \to \neg D$	Rule CP