SNS COLLEGE OF ENGINEERING
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Accredited by NAAC-UGC with ‘A’ Grade
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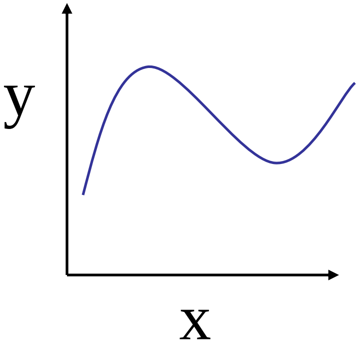
Parametric Form of Curve

Explicit Representation

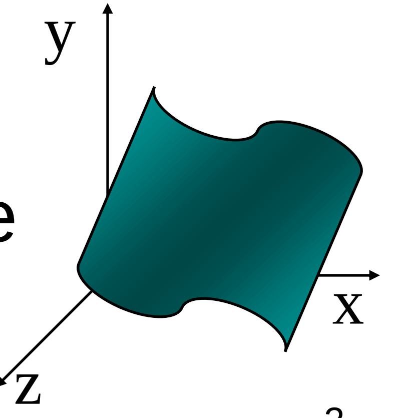
- Most familiar form of curve in 2D

$$y=f(x)$$

- Cannot represent all curves
  - Vertical lines
  - Circles



- Extension to 3D
  - $y=f(x)$, $z=g(x)$
  - The form $z = f(x,y)$ defines a surface



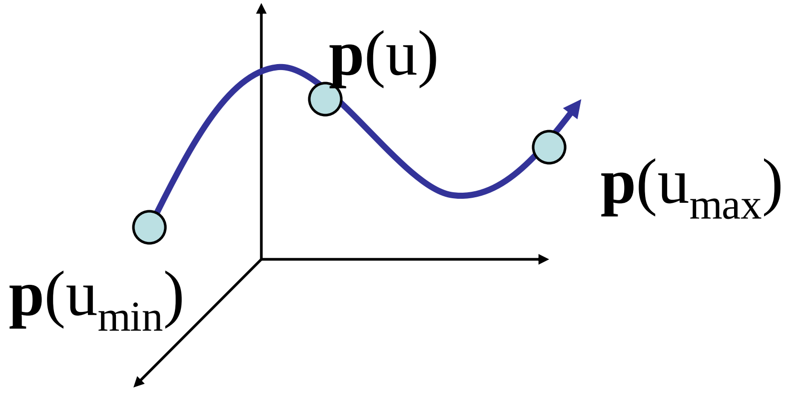
Parametric Curves

- Separate equation for each spatial variable

$$x=x(u)$$
$$y=y(u)$$
$$z=z(u)$$

$$\mathbf{p}(u)=[x(u), y(u), z(u)]^T$$

- For $u_{max} \geq u \geq u_{min}$ we trace out a curve in two or three dimensions

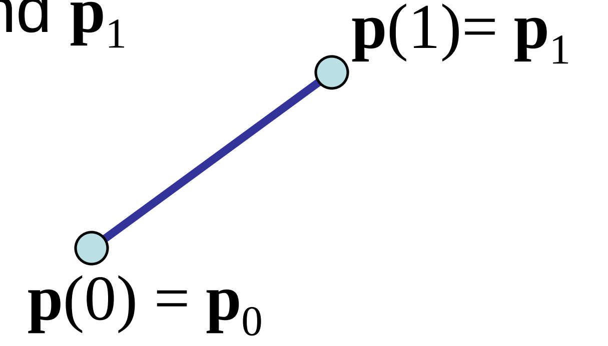


Parametric Lines

We can normalize u to be over the interval (0,1)

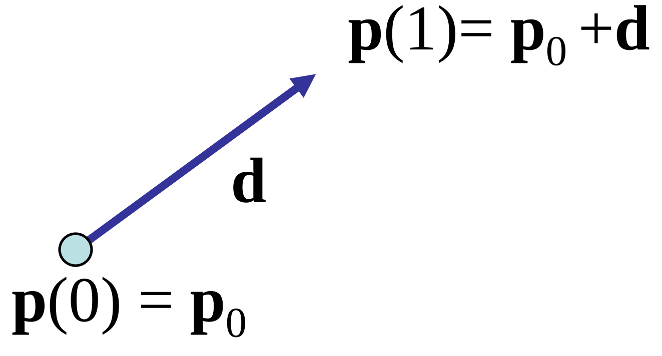
Line connecting two points $\mathbf{p}_0$ and $\mathbf{p}_1$

$$\mathbf{p}(u)=(1-u)\mathbf{p}_0+u\mathbf{p}_1$$



Ray from $\mathbf{p}_0$ in the direction $\mathbf{d}$

$$\mathbf{p}(u)=\mathbf{p}_0+u\mathbf{d}$$

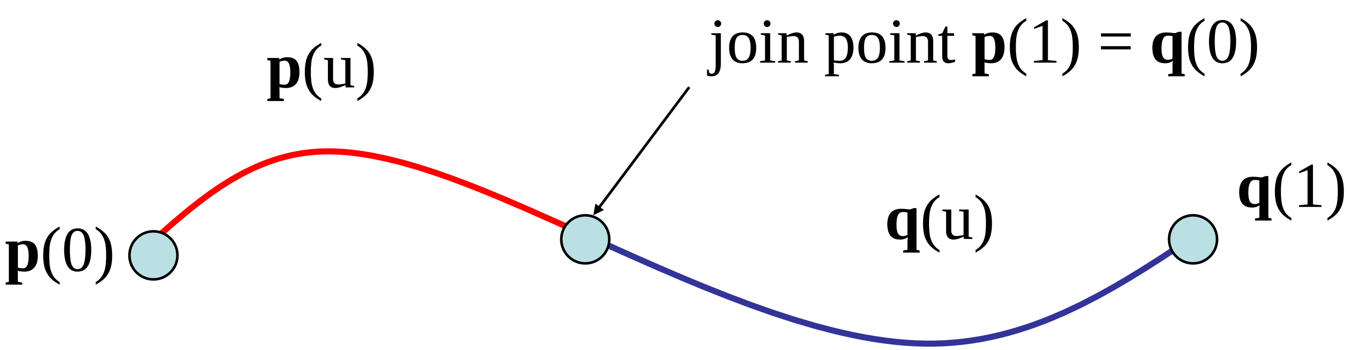


Curve Segments

- After normalizing u, each curve is written

  $\mathbf{p}(u)=[x(u), y(u), z(u)]^T, \quad 1 \geq u \geq 0$

- In classical numerical methods, we design a single global curve
- In computer graphics and CAD, it is better to design small connected curve *segments*



Parametric Polynomial Curve

$$x(u) = \sum_{i=0}^{N} c_{xi} u^{i} \quad y(u) = \sum_{j=0}^{M} c_{yj} u^{j} \quad z(u) = \sum_{k=0}^{L} c_{zk} u^{k}$$

- If N=M=K, we need to determine 3(N+1) coefficients

- Equivalently we need 3(N+1) independent conditions

- Noting that the curves for x, y and z are independent, we can define each independently in an identical manner

- We will use the form $\mathrm{p}(u) = \sum_{k=0}^{L} c_k u^{k}$ where p can be any of x, y, z

Why Polynomials

- Easy to evaluate
- Continuous and differentiable everywhere
  - Must worry about continuity at join points including continuity of derivatives