Vectors

Two points $P = (1,3)$ & $Q = (4,1)$

Displacement - Vector is often drawn as an arrow of a certain length pointing in certain direction.

P to Q' is a vector v having components $(3,-2)$

(·) Calculated by subtracting the coordinates of the points individually.

$P(1,3)$ $Q(4,1)$

$$Q - P = (4-1, 1-3) = (3,-2)$$

(·) Difference between 2 points is a vector: $v = Q - P$

(·) Sum of a point and a vector is a point: $P + v = Q$

2D Vectors

$r = (3.4, -7.78)$

3D Vectors

$t = (33, 142.7, 89.1)$

Operations with vectors

$a = (2,5,6)$ & $b = (-2,7,1)$

$$a + b = (2-2, 5+7, 6+1)$$
$$= (0, 12, 7)$$

and

$$6a = (2,5,6)$$
$$= 6(2,5,6) = (12,30,36)$$

Linear Combinations of Vectors

v & w

New vector $= av + bw$

$$w = a_1 v_1 + a_2 v_2 + \dots + a_m v_m$$

Affine Combinations of Vectors

$$\boxed{a_1 + a_2 + \dots + a_m = 1}$$

Magnitude of a Vector

$$\boxed{|w| = \sqrt{w_1^2 + w_2^2 + \dots + w_n^2}}$$

(eg) $w = (4, -2)$

$$w = \sqrt{(4)^2 + (-2)^2}$$
$$= \sqrt{16+4} = \sqrt{20}$$

Dot Product

Properties

1. Symmetry: $a.b = b.a$
2. Linearity: $(a+c) \cdot b = a.b + c.b$
3. Homogeneity $(sa).b = s(a.b)$
4. $|b|^2 = b.b$

Angle between two vectors

$$\boxed{\begin{array}{c} b = (|b| \cos \phi_b, |b| \sin \phi_b) \\ \& \\ c = (|c| \cos \phi_c, |c| \sin \phi_c) \end{array}}$$

$$b.c = |b|\,|c| \cos(\theta)$$