SNS COLLEGE OF ENGINEERING

Kurumbapalayam(Po), Coimbatore – 641 147

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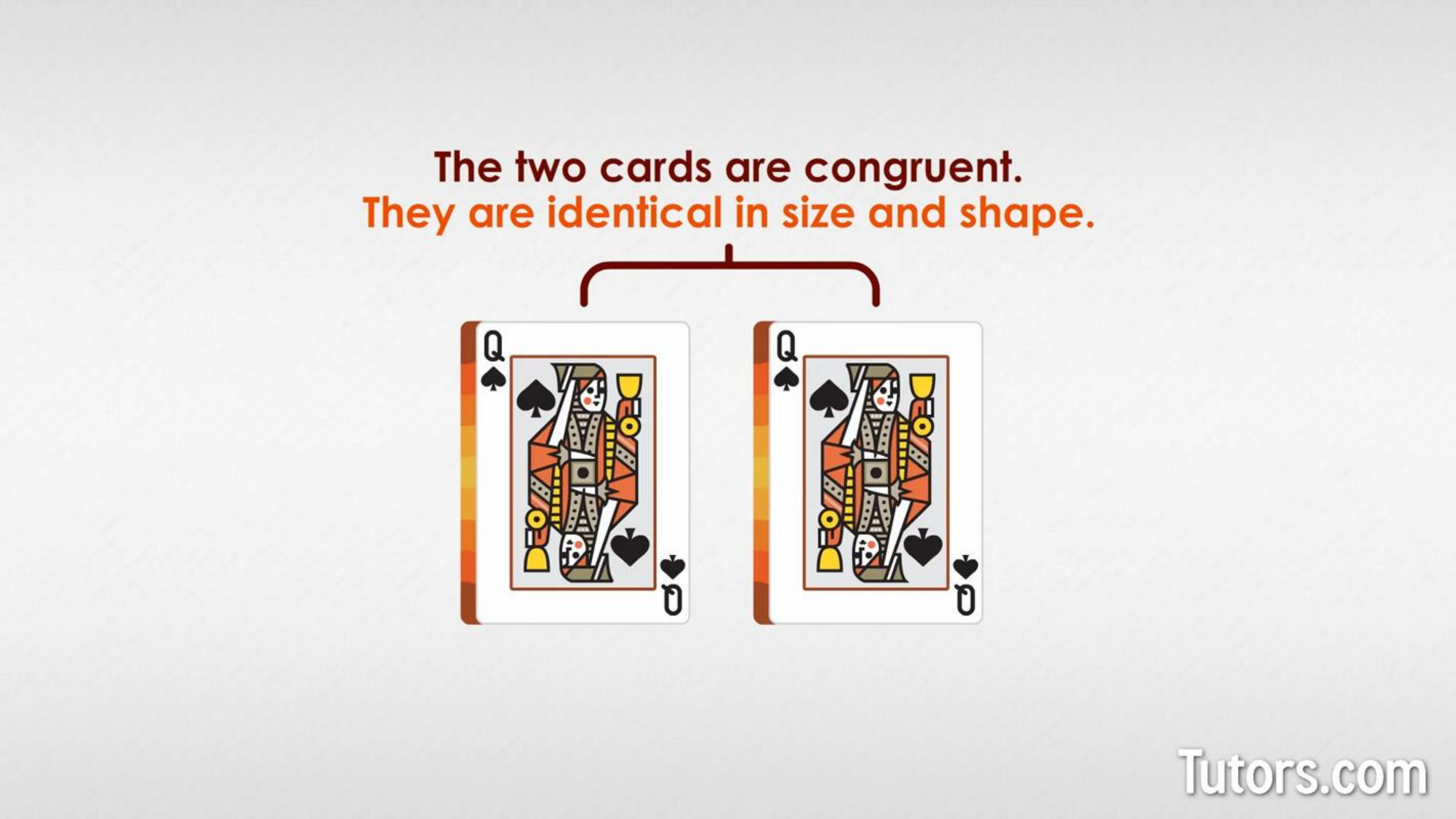
DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Course Code and Name : 19CS503 – CRYPTOGRAPHY AND NETWORK SECURITY

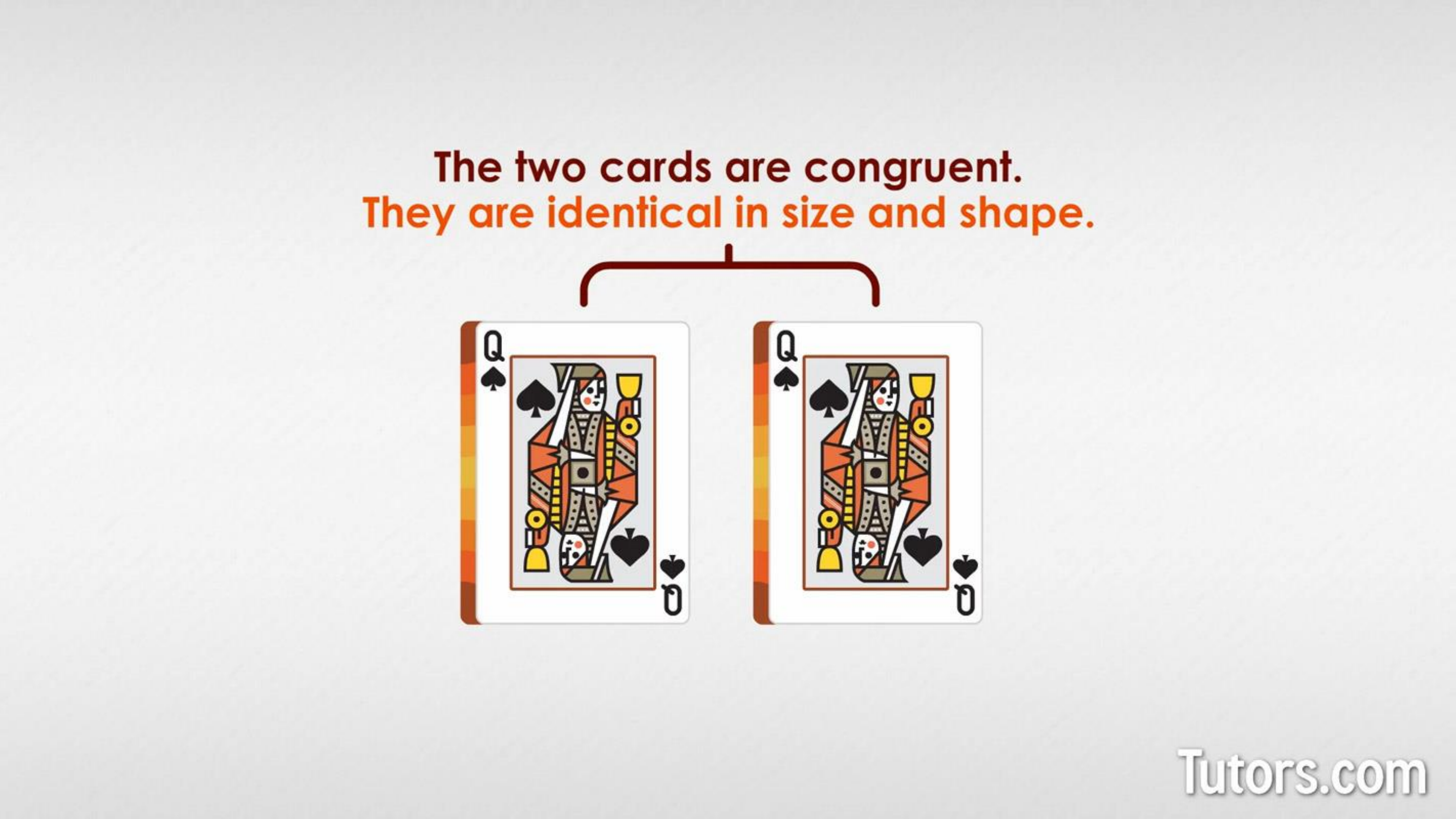
Unit 2: Symmetric Cryptography

Topic : Congruence and matrices

Congruence and matrices



Congruence and matrices



MATRICES

A matrix of size $l \times m$

$$\text{Matrix } \mathbf{A}: \begin{array}{cc} & m \text{ columns} \\ l \text{ rows} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{l1} & a_{l2} & \cdots & a_{lm} \end{bmatrix} \end{array}$$

MATRICES

$$\begin{bmatrix} 2 & 1 & 5 & 11 \end{bmatrix}$$

Row matrix

$$\begin{bmatrix} 2 \\ 4 \\ 12 \end{bmatrix}$$

Column matrix

$$\begin{bmatrix} 23 & 14 & 56 \\ 12 & 21 & 18 \\ 10 & 8 & 31 \end{bmatrix}$$

Square matrix

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

0

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

I

Operations and Relations

$$\begin{bmatrix} 12 & 4 & 4 \\ 11 & 12 & 30 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 2 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 2 & 3 \\ 8 & 10 & 20 \end{bmatrix}$$

C = A + B

$$\begin{bmatrix} -2 & 0 & -2 \\ -5 & -8 & 10 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 2 & 10 \end{bmatrix} - \begin{bmatrix} 7 & 2 & 3 \\ 8 & 10 & 20 \end{bmatrix}$$

D = A − B

Operations and Relations

the product of a row matrix (1 × 3) by a column matrix (3 × 1). The result is a matrix of size 1 × 1.

$$\begin{array}{ccccc} C & & A & & B \\ \begin{bmatrix} 53 \end{bmatrix} & = & \begin{bmatrix} 5 & 2 & 1 \end{bmatrix} & \times & \begin{bmatrix} 7 \\ 8 \\ 2 \end{bmatrix} \end{array}$$

In which: $\boxed{53 = 5 \times 7 + 2 \times 8 + 1 \times 2}$

Residue Matrices

Cryptography uses residue matrices: matrices where all elements are in Zn. A residue matrix has a multiplicative inverse if gcd (det(A), n) = 1.

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & 7 & 2 \\ 1 & 4 & 7 & 2 \\ 6 & 3 & 9 & 17 \\ 13 & 5 & 4 & 16 \end{bmatrix} \qquad \mathbf{A}^{-1} = \begin{bmatrix} 15 & 21 & 0 & 15 \\ 23 & 9 & 0 & 22 \\ 15 & 16 & 18 & 3 \\ 24 & 7 & 15 & 3 \end{bmatrix}$$

$$\det(\mathbf{A}) = 21 \qquad \det(\mathbf{A}^{-1}) = 5$$

LINEAR CONGRUENCE

Cryptography often involves solving an equation or a set of equations of one or more variables with coefficient in Zn. This section shows how to solve equations when the power of each variable is 1 (linear equation).

Single-Variable Linear Equations

Equations of the form $ax \equiv b \pmod{n}$ might have no solution or a limited number of solutions.

$\boxed{\text{Assume that the gcd } (a, n) = d.}$

If $d \nmid b$, there is no solution.

If $d \mid b$, there are d solutions.

Example 2.35

Solve the equation $10x \equiv 2 \pmod{15}$.

Solution

First we find the gcd (10 and 15) = 5. Since 5 does not divide 2, we have no solution.

Example 2.36

Solve the equation $14x \equiv 12 \pmod{18}$.

Solution

$$14x \equiv 12 \pmod{18} \to \quad 7x \equiv 6 \pmod{9} \quad \to x \equiv 6\,(7^{-1}) \pmod{9}$$
$$x_0 = (6 \times 7^{-1}) \bmod 9 = (6 \times 4) \pmod{9} = 6$$
$$x_1 = x_0 + 1 \times (18/2) = 15$$

LINEAR CONGRUENCE

Solve the equation $3x + 4 \equiv 6 \pmod{13}$.

Solution

First we change the equation to the form $ax \equiv b \pmod{n}$. We add -4 (the additive inverse of 4) to both sides, which give $3x \equiv 2 \pmod{13}$. Because gcd (3, 13) = 1, the equation has only one solution, which is $x_0 = (2 \times 3^{-1}) \bmod 13 = 18 \bmod 13 = 5$. We can see that the answer satisfies the original equation: $3 \times 5 + 4 \equiv 6 \pmod{13}$.

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ASSESSMENT SOLUTION

Solve the set of following three equations:

$$3x + 5y + 7z \equiv 3 \pmod{16}$$
$$x + 4y + 13z \equiv 5 \pmod{16}$$
$$2x + 7y + 3z \equiv 4 \pmod{16}$$

Solution

The result is $x \equiv 15 \pmod{16}$, $y \equiv 4 \pmod{16}$, and $z \equiv 14 \pmod{16}$. We can check the answer by inserting these values into the equations.

REFERENCES

William Stallings, Cryptography and Network Security: Principles and Practice, PHI 3rd Edition, 2006.

THANK YOU