



TOPIC:3- Normal Forms

Normal Form
If we write the given statement in a particular form (interms of \wedge , \vee and \neg), then it is called Normal form.

Elementary Product
A product of the statement variables and their negations in a formula is called Elementary products.

For example, let P and Q be any two atomic variables. Then possible elementary products are
 $P, Q, \neg P, \neg Q, \neg P \wedge Q, \neg Q \wedge P, P \wedge \neg P, Q \wedge \neg Q, P \wedge \neg P \wedge Q$.

Elementary Sum
A sum of the two statement variables and their negation is called Elementary sum.

Let P and Q be any two atomic variables. Then $P, Q, P \vee Q, \neg P \vee Q, P \vee \neg Q, P \vee \neg P \vee Q$ are some examples of elementary sum.



Disjunctive Normal Form (DNF)

A statement formula which is equivalent to a given formula and which consists of a sum of elementary products is called a Disjunctive Normal Form of the given formula.

Conjunctive Normal Form (CNF)

A statement formula which is equivalent to a given formula and which consists of a product of elementary sum is called a conjunctive Normal Form of the given formula.

Principal Normal Forms

Let P and Q be two statement variable then the minterms are

$$P \wedge Q, P \wedge \neg Q, \neg P \wedge Q, \neg P \wedge \neg Q$$

The maxterms are

$$P \vee Q, P \vee \neg Q, \neg P \vee Q, \neg P \vee \neg Q$$



Principal Normal Forms

Let P and Q be two statement variables
then the minterms are

$$P \wedge Q, P \wedge \neg Q, \neg P \wedge Q, \neg P \wedge \neg Q$$

The maxterms are

$$P \vee Q, P \vee \neg Q, \neg P \vee Q, \neg P \vee \neg Q$$

Principal Disjunctive Normal Forms (PDNF)

For a given statement formula, an equivalent formula consisting of disjunction of minterms is called a Principal Disjunctive Normal Forms

Principal Conjunctive Normal Forms (PCNF)

For a given statement formula, an equivalent formula consisting of conjunction of maxterms only is known as its Principal conjunction normal form (PCNF).



Q) Obtain the PCNF and PDNF for

$$(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$$

Let S:

$$(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$$

$$\Leftrightarrow (\neg \neg P \vee R) \wedge [(Q \rightarrow P) \wedge (P \rightarrow Q)]$$

$$\Leftrightarrow (P \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)]$$

$$\Leftrightarrow (P \vee R \vee F) \wedge [(\neg Q \vee P \vee F) \wedge (\neg P \vee Q \vee F)]$$

$$\Leftrightarrow (P \vee R \vee (Q \wedge \neg Q)) \wedge [(\neg Q \vee P \vee (R \wedge \neg R)) \wedge (\neg P \vee Q \vee (R \wedge \neg R))]$$

$$\Leftrightarrow [(P \vee R \vee Q) \wedge (P \vee R \vee \neg Q)] \wedge [(\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \neg R)]$$

$$\wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$$

$$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R)$$

$$\wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$$

$$S \Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \quad (\text{PCNF})$$

$$\neg S : (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$$

$$\neg(\neg S) : (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$$

$$S : (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R) \quad (\text{PDNF})$$



2. Obtain the principal disjunctive and conjunctive

normal forms $(P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$

$$\begin{aligned} \text{Let } S &\Leftrightarrow (P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R)) \\ &\Leftrightarrow (\neg P \vee (Q \wedge R)) \wedge (\neg \neg P \vee (\neg Q \wedge \neg R)) \\ &\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee R) \wedge (P \vee \neg Q) \wedge (P \vee \neg R) \\ &\Leftrightarrow (\neg P \vee Q \vee F) \wedge (\neg P \vee R \vee F) \wedge (P \vee \neg Q \vee F) \wedge (P \vee \neg R \vee F) \\ &\Leftrightarrow (\neg P \vee Q \vee (R \wedge \neg R)) \wedge (\neg P \vee R \vee (Q \wedge \neg Q)) \\ &\quad \wedge (P \vee \neg Q \vee (R \wedge \neg R)) \wedge (P \vee \neg R \vee (Q \wedge \neg Q)) \\ &\Leftrightarrow (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee R \vee Q) \wedge (\neg P \vee R \vee \neg Q) \\ &\quad \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee \neg R \vee Q) \wedge (P \vee \neg R \vee \neg Q) \\ S &\Leftrightarrow (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \\ &\quad \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R) \end{aligned}$$

This is required PCNF

$$\begin{aligned} S &\Leftrightarrow (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee R) \\ &\quad \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \quad (\text{PCNF}) \end{aligned}$$

$$\neg S \Leftrightarrow (\neg P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee R)$$

$$\neg(\neg S) \Leftrightarrow (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$$

$$S \Leftrightarrow (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R) \quad (\text{PDNF})$$



Obtain the PDNF of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.

3.

P	Q	R	$P \wedge Q$	$\neg P$	$\neg P \wedge R$	$Q \wedge R$	$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$	Min term
T	T	T	T	F	F	T	⊙	$P \wedge Q \wedge R$
T	T	F	T	F	F	F	⊙	$P \wedge Q \wedge \neg R$
T	F	T	F	F	F	F	F	
T	F	F	F	F	F	F	F	
F	T	T	F	T	T	T	⊙	$\neg P \wedge Q \wedge R$
F	T	F	F	T	F	F	F	
F	F	T	F	T	T	F	⊙	$\neg P \wedge \neg Q \wedge R$
F	F	F	F	T	F	F	F	