TOPIC:3- Normal Forms

Normal form

If we write the given statement in a particular form (interms of $\wedge$, $\vee$ and $\neg$), then it is called Normal form.

Elementry Product

A product of the statement variables and their negations in a formula is called Elementry products.

For example, let P and Q be any two atomic variables. Then possible elementry products are

$P, Q, \neg P, \neg Q, \neg P \wedge Q, \neg Q \wedge P, P \wedge \neg P, Q \wedge \neg Q$

$P \wedge \neg P \wedge Q$.

Elementry sum

A sum of the two statement variables and their negation is called Elementry sum.

Let P and Q be any two atomic variables.

Then $P, Q, P \vee Q, \neg P \vee Q, P \vee \neg Q, P \vee \neg P \vee Q$ are some examples of elementry sum.

Disjunctive Normal Form (DNF)

A statement formula which is equivalent to a given formula and which consists of a sum of elementry products is called a Disjunctive Normal Form of the given formula.

Conjunctive Normal Form (CNF)

A statement formula which is equivalent to a given formula and which consists of a product of elementry sum is called a conjunctive Normal Form of the given formula.

Principal Normal Forms

Let P and Q be two statement variable then the minterms are

$P \wedge Q$, $P \wedge \neg Q$, $\neg P \wedge Q$, $\neg P \wedge \neg Q$

The maxterms are

$P \vee Q$, $P \vee \neg Q$, $\neg P \vee Q$, $\neg P \vee \neg Q$

Principal Normal Forms

Let P and Q be two statement variables then the minterms are

$P \wedge Q$, $P \wedge \neg Q$, $\neg P \wedge Q$, $\neg P \wedge \neg Q$

The maxterms are

$P \vee Q$, $P \vee \neg Q$, $\neg P \vee Q$, $\neg P \vee \neg Q$

Principal Disjunctive Normal Forms (PDNF)

For a given statement formula, an equivalent formula consisting of disjunction of minterms is called a Principal Disjunctive Normal Forms

Principal Conjunctive Normal Forms (PCNF)

For a given statement formula, an equivalent formula consisting of conjunction of maxterms only is known as its Principal conjuction Normal form (PCNF).

1) Obtain the PCNF and PDNF for $(\neg P \to R) \wedge (Q \leftrightarrow P)$.

Let S: $(\neg P \to R) \wedge (Q \leftrightarrow P)$

$\Leftrightarrow (\neg\neg P \vee R) \wedge [(Q \to P) \wedge (P \to Q)]$

$\Leftrightarrow (P \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)]$

$\Leftrightarrow (P \vee R \vee F) \wedge [(\neg Q \vee P \vee F) \wedge (\neg P \vee Q \vee F)]$

$\Leftrightarrow (P \vee R \vee (Q \wedge \neg Q)) \wedge [(\neg Q \vee P \vee (R \wedge \neg R)) \wedge (\neg P \vee Q \vee (R \wedge \neg R))]$

$\Leftrightarrow [(P \vee R \vee Q) \wedge (P \vee R \vee \neg Q)] \wedge [(\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)]$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

$S \Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$ (PCNF)

$\neg S$: $(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee$

$\neg(\neg S)$: $(\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$

S : $(\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$ (PDNF)

2. Obtain the principal disjunctive and conjunctive normal forms $(P \to (Q \land R)) \land (\neg P \to (\neg Q \land \neg R))$.

Let $S \Leftrightarrow (P \to (Q \land R)) \land (\neg P \to (\neg Q \land \neg R))$

$\Leftrightarrow (\neg P \lor (Q \land R)) \land (\neg\neg P \lor (\neg Q \land \neg R))$

$\Leftrightarrow (\neg P \lor Q) \land (\neg P \lor R) \land (P \lor \neg Q) \land (P \lor \neg R)$

$\Leftrightarrow (\neg P \lor Q \lor F) \land (\neg P \lor R \lor F) \land (P \lor \neg Q \lor F) \land (P \lor \neg R \lor F)$

$\Leftrightarrow (\neg P \lor Q \lor (R \land \neg R)) \land (\neg P \lor R \lor (Q \land \neg Q))$
$\land (P \lor \neg Q \lor (R \land \neg R)) \land (P \lor \neg R \lor (Q \land \neg Q))$

$\Leftrightarrow (\neg P \lor Q \lor R) \land (\neg P \lor Q \lor \neg R) \land (\neg P \lor R \lor Q) \land (\neg P \lor R \lor \neg Q)$
$\land (P \lor \neg Q \lor R) \land (P \lor \neg Q \lor \neg R) \land (P \lor \neg R \lor Q) \land (P \lor \neg R \lor \neg Q)$

$S \Leftrightarrow (\neg P \lor Q \lor R) \land (\neg P \lor Q \lor \neg R) \land (\neg P \lor Q \lor R) \land (\neg P \lor \neg Q \lor R)$
$\land (P \lor \neg Q \lor R) \land (P \lor \neg Q \lor \neg R) \land (P \lor Q \lor \neg R) \land (P \lor \neg Q \lor \neg R)$

This is required PCNF

$S \Leftrightarrow (\neg P \lor Q \lor R) \land (\neg P \lor Q \lor \neg R) \land (\neg P \lor \neg Q \lor R) \land (P \lor \neg Q \lor R)$
$\land (P \lor \neg Q \lor \neg R) \land (P \lor Q \lor \neg R)$ (PCNF)

$\neg S \Leftrightarrow (\neg P \lor \neg Q \lor \neg R) \land (P \lor Q \lor R)$

$\neg(\neg S) \Leftrightarrow (P \land Q \land R) \lor (\neg P \land \neg Q \land \neg R)$

$S \Leftrightarrow (P \land Q \land R) \lor (\neg P \land \neg Q \land \neg R)$ (PDNF)

3. Obtain the PDNF of $(P\wedge Q)\vee(\neg P\wedge R)\vee(Q\wedge R)$.

P	Q	R	$P\wedge Q$	$\neg P$	$\neg P\wedge R$	$Q\wedge R$	$(P\wedge Q)\vee(\neg P\wedge R)\vee(Q\wedge R)$	Min term
T	T	T	T	F	F	T	T	$P\wedge Q\wedge R$
T	T	F	T	F	F	F	T	$P\wedge Q\wedge\neg R$
T	F	T	F	F	F	F	F	
T	F	F	F	F	F	F	F	
F	T	T	F	T	T	T	T	$\neg P\wedge Q\wedge R$
F	T	F	F	T	F	F	F	
F	F	T	F	T	T	F	T	$\neg P\wedge\neg Q\wedge R$
F	F	F	F	T	F	F	F	