TOPIC : 1 - Proposition & Connectives

Proposition

A proposition (statement) is a declarative sentence which is either true or false but not both.

Example

(a) New Delhi is the capital of India. (True)

(b) chennai is in England (False)

(c) $10+6 = 16$ (True)

(d) For any x, $x+2 = 2+x$ (True)

(e) The sum of two and four is seven (False)

The following sentences are not Propositions

(a) $x+4 = 2$ (neither true nor false)

(b) what a wonderful joke this is (Exclamatory)

(c) obey my orders. (command)

(d) what is the height of Himalaya? (Interogative

Atomic statements (Primary Statements)

A declarative sentence which cannot be further split up into simple sentences are called primary statements.

Example Ram is a boy.

Connectives

Connective is an operation which is used to connect two or more than two statemen

S. No.	Connectives	Name	Symbols	Type of operator
1.	Not	Negation	$\neg$ (or) $\sim$	Unary
2.	And	Conjunction	$\wedge$	Binary
3.	Or	Disjunction	$\vee$	Binary
4.	If ... then	Conditional	$\rightarrow$	Binary
5.	If and only if	Biconditional	$\leftrightarrow$	Binary

Molecular Statement (Compound Statement)

New statements can be formed from atomic statements through the use of connectives such as 'and', 'or', 'but' etc. The resulting statements are called molecular or compound statements.

Example

Niranjan is a boy $\boxed{\text{and}}$ Nirmala is a girl.

Truth table

The truth value of a proposition is either true (T) or False (F).

A truth table is a table that shows the truth values of a compound proposition for all possible cases.

Negation ($\neg$ or $\sim$) (Not)

The negation of a statement is generally formed by introducing the word 'not' at a proper place in the statement.

If 'P' has the truth value 'T', then $\neg P$ has the truth value 'F'.

III^{ly} If 'P' has the truth value 'F', then $\neg P$ has the truth value 'T'.

Truth Table (for Negation)	
P	$\neg P$
T	F
F	T

Conjuction ($\wedge$) (and)

The conjunction of two statements P and Q is the statement $P \wedge Q$ which is read as P and Q.

Truth Table for $P \wedge Q$		
P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F

Disjunction ($\vee$) (or)

The disjunction of two statements P and Q is the statement $P \vee Q$ which is read as P or Q.

Truth Table for $P \vee Q$		
P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional statement ($\longrightarrow$) (If, ... then)

If P and Q be any two statements, then the statement $P \rightarrow Q$ which is read as "if P then Q" is called a conditional statement.

Truth Table for $P \rightarrow Q$		
P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional statement ($\longleftrightarrow$) (if and only if)

If P and Q are any two statements, then the statement $P \longleftrightarrow Q$ which is read as "P if and only if Q" is called a biconditional statement.

Truth Table for $P \longleftrightarrow Q$		
P	Q	$P \longleftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Converse and contrapositive

If $P \longrightarrow Q$ is an implication, then the converse of $P \rightarrow Q$ is the implication $Q \rightarrow P$, and the contrapositive of $P \longrightarrow Q$ is the implication $\neg Q \longrightarrow \neg P$.

(1) Give the converse and contrapositive of the implication "If it is raining, then I get wet"

Let P : It is raining
Q : I get wet

Converse ($Q \rightarrow P$) : If I get wet, then it is raining

Contrapositive ($\neg Q \rightarrow \neg P$)

If I do not get wet, then it is not raining

(5) Construct a truth table for the compound proposition $(p \to q) \to (q \to p)$.

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(7) Construct the truth table for the compound proposition $(p \to q) \leftrightarrow (\neg p \to \neg q)$.

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$(p \to q) \leftrightarrow (\neg p \to \neg q)$
T	T	T	F	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	F
F	F	T	T	T	T	T