





Fourier series are very useful in electrical engineering problems.

Deduction: Sum of the Fourier Series

Continuous	Discontinuous
Substitute the value directly	End point Middle point Average values at endpoints $\frac{L.H.S + R.H.S}{2}$

Problems:

① Sum of the Fourier series for

$$f(x) = \begin{cases} x^2, & -\pi \leq x \leq 0 \\ 0, & 0 \leq x \leq \pi \end{cases} \text{ at } x = \frac{\pi}{2}, \frac{3\pi}{2}$$

sol:

$x = \frac{\pi}{2}$  is a continuous point at  $(0, \pi)$

$$\therefore f(x) = 0$$

$x = -\frac{\pi}{2}$  is a continuous point at  $(-\pi, 0)$

$$f(x) = x^2$$

$$\Rightarrow f\left(-\frac{\pi}{2}\right) = \left(-\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4}$$

$$\therefore f(x) = \frac{\pi^2}{4}$$



② Sum the Fourier series for

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2, & 1 < x < 2 \end{cases} \text{ at } x=0.$$

Sol:  $x=0$  is a discontinuous and end point.

$$\begin{aligned} \text{Sum of the Fourier series} &= \frac{f(0) + f(2)}{2} \\ &= \frac{0 + 2}{2} = \frac{2}{2} \end{aligned}$$

Sum of the Fourier series } = 1.

③ Sum of the Fourier series for

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ x^2, & \pi < x < 2\pi \end{cases} \text{ at } x=\pi.$$

Sol:  $x=\pi$  is a discontinuous and middle point.

$$\begin{aligned} \text{Sum of the Fourier series} &= \frac{f(\pi^-) + f(\pi^+)}{2} \\ &= \frac{\pi + \pi^2}{2}. \end{aligned}$$





Dirichlet condition:

- i)  $f(x)$  is periodic, single valued and finite
- ii)  $f(x)$  has a finite no. of finite discontinuities.
- iii)  $f(x)$  has no infinite discontinuities.
- iv)  $f(x)$  has a finite no. of maxima and minima.

Formula for fourier series in  $(0, 2\pi)$ .

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where  $a_0 = \frac{2}{b-a} \int_a^b f(x) dx$

$$a_n = \frac{2}{b-a} \int_a^b f(x) \cos nx dx$$

$$b_n = \frac{2}{b-a} \int_a^b f(x) \sin nx dx$$

Problems:

- ① Expand  $f(x) = x^2$  in  $(0, 2\pi)$  and hence deduce that  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ .