

TOPIC : 11 - SOLUTIONS OF LINEAR EQUATIONS OF SECOND AND HIGHER ORDER WITH  
CONSTANT COEFFICIENTS

Type: (iii)

$$\text{R.H.S} = \sin(ax+by) \text{ or } \cos(ax+by)$$

$$\text{Replace } D^2 \rightarrow -a^2$$

$$D D' \rightarrow -ab$$

$$D'^2 \rightarrow -b^2$$

$$\text{Solve: } (D^2 + 2DD' + D'^2)z = \sin(2x-y)$$

Sol:

$$\text{The A.E is } m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$\Rightarrow m = -1, -1.$$

$$\text{C.F} = f_1(y-x) + x f_2(y-x)$$

$$\text{P.S} = \frac{1}{D^2 + 2DD' + D'^2} \sin(2x-y)$$

$$= \frac{1}{-4 + 2 \cdot 2 \cdot -1} \sin(2x-y) \quad \begin{array}{l} D^2 \rightarrow -4 \\ DD' \rightarrow 2 \\ D'^2 \rightarrow -1 \end{array}$$

$$= \frac{1}{-1} \sin(2x-y)$$

$$= -\sin(2x-y)$$

$$z = \text{C.F} + \text{P.S}$$

$$z = f_1(y-x) + x f_2(y-x) - \sin(2x-y)$$



2. Solve:  $(D^2 - 4D')z = \cos 2x \cos 3y$

Sol.

$$(D^2 - 4D')z = \frac{1}{2} [\cos(2x+3y) + \cos(2x-3y)]$$

The A.E is  $m^2 - 4 = 0$

$$m^2 = 4$$

$$m = \pm 2$$

$$C.F = f_1(y+2x) + f_2(y-2x)$$

$$P.I_1 = \frac{1}{D^2 - 4D'} \cdot \frac{1}{2} \cos(2x+3y)$$

$$= \frac{1}{-4 - 4(-9)} \cdot \frac{1}{2} \cos(2x+3y)$$

$$= \frac{1}{-4+36} \cdot \frac{1}{2} \cos(2x+3y)$$

$$= \frac{1}{32} \cdot \frac{1}{2} \cos(2x+3y)$$

$$= \frac{1}{64} \cos(2x+3y)$$

$$P.I_2 = \frac{1}{D^2 - 4D'} \cdot \frac{1}{2} \cos(2x-3y)$$

$$= \frac{1}{-4 + 36} \cdot \frac{1}{2} \cos(2x-3y)$$

$$= \frac{1}{64} \cos(2x-3y)$$

$$C.F = f_1(y+2x) + f_2(y-2x) + \frac{1}{64} \cos(2x+3y)$$

$$+ \frac{1}{64} \cos(2x-3y)$$



Type: (iv)

$$\text{R.H.S} = x \text{ \& } y$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

1. Solve:  $(D^2 + 3DD' + 2D'^2)z = x+y$ .

Sol:

The A.E is  $m^2 + 3m + 2 = 0$

$$(m+1)(m+2) = 0$$

$$\therefore m = -1, -2.$$

$$\text{C.F} = f_1(y-x) + f_2(y-2x)$$

$$\text{P.I} = \frac{1}{D^2 + 3DD' + 2D'^2} (x+y)$$

$$= \frac{1}{D^2 \left[ 1 + \left( \frac{3DD' + 2D'^2}{D^2} \right) \right]} (x+y)$$

$$= \frac{1}{D^2} \left[ 1 + \left( \frac{3DD' + 2D'^2}{D^2} \right) \right]^{-1} (x+y)$$

$$= \frac{1}{D^2} \left[ 1 - \left( \frac{3D'}{D} + \frac{2D'}{D^2} \right) + \dots \right] (x+y)$$

$$= \frac{1}{D^2} \left[ x+y - \frac{3D'}{D} (x+y) \right]$$

$$= \frac{1}{D^2} \left[ x+y - \frac{3}{D} (1) \right]$$



$$\begin{aligned} P.I &= \frac{1}{D^2} [x+y-3x] = R.A \\ &= \frac{1}{D^2} [y-2x] \\ &= \left( \frac{1}{D} \left[ yx - \frac{2x^2}{2} \right] \right) \end{aligned}$$

$$P.I = \frac{yx^2}{2} - \frac{x^3}{3}$$

$$Z = f_1(y-2x) + f_2(y-2x) + \frac{yx^2}{2} - \frac{x^3}{3}$$

2. Solve:  $(D^2 + DD' - 6D'^2)Z = x^2y$

Sol:

The A.E is  $m^2 + m - 6 = 0$   
 $(m+3)(m-2) = 0$   
 $m = -3, 2.$

$$C.F = f_1(y-3x) + f_2(y+2x)$$

$$P.I = \frac{1}{D^2 + DD' - 6D'^2} (x^2y)$$

$$= \frac{1}{D^2 \left[ 1 + \left( \frac{DD' - 6D'^2}{D^2} \right) \right]} (x^2y)$$

$$= \frac{1}{D^2} \left[ 1 + \left( \frac{D'}{D} - \frac{6D'^2}{D^2} \right) \right]^{-1} x^2y$$

$$= \frac{1}{D^2} \left[ 1 - \left( \frac{D'}{D} - \frac{6D'^2}{D^2} \right) \right] x^2y$$

$$= \frac{1}{D^2} \left[ x^2y - \frac{D'}{D} (x^2y) \right]$$



$$\begin{aligned} P.I &= \frac{1}{D^2} \left( x^2 y - \frac{x^2}{D} \right) = R_1 \\ &= \frac{1}{D^2} \left( x^2 y - \frac{x^3}{3} \right) = \\ &= \frac{1}{D} \left( \frac{x^3}{3} y - \frac{x^4}{12} \right) = \\ &= \frac{x^4}{12} y - \frac{x^5}{60} = R_2 \\ z &= f_1 (y - 3x) + f_2 (y + 2x) \\ &\quad + \frac{x^4}{12} y - \frac{x^5}{60} \end{aligned}$$