



Higher order partial differential equations

$$a \frac{\partial^2 z}{\partial x^2} + b \frac{\partial^2 z}{\partial y \partial x} + c \frac{\partial^2 z}{\partial y^2} = f(x, y)$$

Here a, b, c are constants
 $F(x, y) =$ Functions of x & y .

Take $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$

$$(xD^2 + bDD' + cD'^2)z = F(x, y) \rightarrow \textcircled{1}$$

Solution of $\textcircled{1}$ is given by,

$$z = C.F + P.I$$

To find C.F
Replace D by m
 D' by 1

$$\therefore am^2 + bm + c = 0$$

So we get two roots m_1, m_2 .



Case: ci)

If $m_1 \neq m_2$

$$C.F = f_1(y+m_1x) + f_2(y+m_2x)$$

Case: cii) If $m_1 = m_2$

$$C.F = f_1(y+m_1x) + xf_2(y+m_2x)$$

To find P.I:

$$P.I = \frac{1}{aD^2 + bD + c} F(x, y)$$

Type: ci) (Homogeneous Equations)
R.H.S = 0.

$$\textcircled{1} \text{ solve: } (D^2 - 5D + 6)z = 0$$

Sol:

The A.E is $m^2 - 5m + 6 = 0$
 $(m-2)(m-3) = 0$
 $\therefore m = 2, 3$

$$C.F = f_1(y+2x) + f_2(y+3x)$$

$$P.I = 0$$

$$\therefore z = C.F + P.I$$

$$z = f_1(y+2x) + f_2(y+3x)$$

$$\textcircled{2} \text{ solve: } (D^3 - 3D^2 + 2D)z = 0$$

Sol:

The A.E is $m^3 - 3m^2 + 2 = 0$

$$m^3 + 0.m^2 - 3m + 2 = 0$$



$$\begin{array}{c|ccc} 1 & 1 & 0 & -3 \\ \hline & \downarrow & 1 & 1 \\ \hline 1 & 1 & 1 & -2 \\ \hline & \downarrow & 1 & 2 \\ \hline -2 & 1 & 2 & 0 \\ \hline & \downarrow & -2 & \\ \hline & 1 & 0 & \end{array}$$

$\therefore m = 1, 1, -2$

C.F is $f_1(y+x) + f_2(y+0x) + f_3(y-2x)$

$P.I = 0$

$Z = C.F + P.I$

$Z = f_1(y+x) + f_2(y+0x) + f_3(y-2x)$

Type: (ii) $R.H.S = e^{ax+by}$

Working Rule:

- ① Replace D by a ; D' by b
- ② If $dr \neq 0$ then we get P.I
- ③ If $dr = 0$ put x in nr & diff dr w.r.to D and apply same method.



① Solve: $(D^2 + 2D + 1)z = e^{2x+3y}$ (14)

Sol:
The A.E is $m^2 + 2m + 1 = 0$
 $(m+1)(m+1) = 0$
 $m = -1, -1$

C.F = $f_1(y-x) + x f_2(y-x)$

P.I = $\frac{1}{D^2 + 2D + 1} e^{2x+3y}$

$= \frac{1}{(2)^2 + 2 \cdot 2 \cdot 3 + 3^2} e^{2x+3y}$ D by 2
D' by 3

$= \frac{1}{4 + 12 + 9} e^{2x+3y}$

$= \frac{1}{26} e^{2x+3y}$

$z = C.F + P.I$

$z = f_1(y-x) + x f_2(y-x) + \frac{1}{26} e^{2x+3y}$

2. Solve: $(D^2 - D^1)z = e^{x+2y}$

Sol:
The A.E is $m^2 - 1 = 0$
 $m^2 = 1$
 $m = \pm 1$

C.F = $f_1(y-x) + f_2(y+x)$

P.I = $\frac{1}{(D^2 - D^1)} e^{x+2y}$

$= \frac{1}{1^2 - 0^2} e^{x+2y}$ D → 1
D' → 2



$$P \cdot Q = -\frac{1}{3} e^{x+2y}$$
$$Z = C.F + P \cdot Q$$
$$= f_1(y-x) + f_2(y+x) - \frac{1}{3} e^{x+2y}$$