



Unit - I  
Partial Differential Equations

Formation of Partial Differential equations  
by Elimination of arbitrary constants

Consider an equation  $f(x, y, z, a, b) = 0$  (1)  
where  $a$  &  $b$  denote arbitrary constants.

A p.d.e is formed by eliminating the arbitrary constants that occur in the functional relation between the variables.

Using  $\frac{\partial z}{\partial x} = p$ ,  $\frac{\partial z}{\partial y} = q$

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1. Form the p.d.e by eliminating the arbitrary constants  $a$  &  $b$  from  $z = ax + by$  (1)

Diff p.w.r.to  $x$  we get  
 $\frac{\partial z}{\partial x} = a \Rightarrow p = a$

Diff p.w.r.to  $y$  we get  
 $\frac{\partial z}{\partial y} = b \Rightarrow q = b$

$\therefore$  Eqn (1) becomes,  $z = px + qy$ .



2. Eliminate the arbitrary constants  
a & b from  $z = (x^2+a)(y^2+b)$ .

Sol:  
 $z = (x^2+a)(y^2+b)$

Diff p.w.r to x,

$$p = \frac{\partial z}{\partial x} = 2x(y^2+b) \Rightarrow \frac{p}{2x} = y^2+b$$

Diff p.w.r to y,

$$q = 2y(x^2+a) \Rightarrow \frac{q}{2y} = x^2+a$$

$\therefore$  Eqn ① becomes,

$$z = \frac{p}{2x} \cdot \frac{q}{2y}$$

$$4xyz = pq$$

3.  $z = a(x+y)+b$

Sol:  
 $z = a(x+y)+b$

Diff w.r to x,

$$p = a \rightarrow \text{①}$$

Diff w.r to y,

$$q = a \rightarrow \text{②}$$

From ① & ② we get  $pq = a$   $p = q$ .



4. Find the partial differential equation of all planes having equal intercepts on the  $x$  &  $y$  axis

Sol:

Intercept form of the plane equation is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

Given  $a=b$  (Equal intercepts on the  $x$  &  $y$  axis)

$$\frac{x}{a} + \frac{y}{a} + \frac{z}{c} = 1.$$

Diff. w.r to  $x$ , we get

$$\frac{1}{a} + \frac{1}{c} \frac{\partial z}{\partial x} = 0$$

$$\Rightarrow \frac{1}{a} = -\frac{1}{c} \frac{\partial z}{\partial x} = -\frac{1}{c} p \rightarrow \text{①}$$

Diff. w.r to  $y$ , we get

$$\frac{1}{a} + \frac{1}{c} \frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \frac{1}{a} = -\frac{1}{c} \frac{\partial z}{\partial y} = -\frac{1}{c} q \rightarrow \text{②}$$

From ① & ②,

$$-\frac{1}{c} p = -\frac{1}{c} q$$

$$p = q.$$



6.  $(x-a)^2 + (y-b)^2 + z^2 = 1.$

Sol:

Diff w.r. to  $x$ , we get

$$2(x-a) + 2z \frac{\partial z}{\partial x} = 0$$

$$2(x-a) = -2z p$$

$$x-a = -z p \rightarrow \textcircled{1}$$

Diff p.w.r. to  $y$ , we get

$$2(y-b) + 2z \frac{\partial z}{\partial y} = 0$$

$$2(y-b) = -2z q$$

$$y-b = -z q \rightarrow \textcircled{2}$$

Using  $\textcircled{1}$  &  $\textcircled{2}$ , we get

$$(-z p)^2 + (-z q)^2 + z^2 = 1$$

$$z^2 p^2 + z^2 q^2 + z^2 = 1$$

$$z^2 (p^2 + q^2 + 1) = 1.$$



(6)  $Z = (x+a)^3 + (y-b)^2$   
Sol:  
Diff p.w.r. to  $x$ , we get  
$$\frac{\partial Z}{\partial x} = 3(x+a)^2 \cdot 1$$
$$\Rightarrow \frac{p}{3} = (x+a)^2$$
  
Diff p.w.r. to  $y$ , we get  
$$\frac{\partial Z}{\partial y} = 2(y-b)$$
$$\frac{q}{2} = y-b$$
  
$$z = \left(\frac{p}{3}\right)^{3/2} + \left(\frac{q}{2}\right)^2$$
$$z = \left(\frac{p^{3/2}}{3^{3/2}}\right) + \left(\frac{q}{2}\right)^2$$

(7)  $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$   
Sol:  
$$(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$$
  
Diff w.r. to  $x$  we get  
$$2(x-a) = 2z p \cot^2 \alpha$$
$$x-a = z p \cot^2 \alpha$$
  
Diff w.r. to  $y$ , we get  
$$2(y-b) = 2z q \cot^2 \alpha$$
$$y-b = z q \cot^2 \alpha$$
  
$$z^2 p^2 \cot^4 \alpha + z^2 q^2 \cot^4 \alpha = z^2 \cot^2 \alpha$$
  
$$\div z^2 \cot^2 \alpha, \quad p^2 \cot^2 \alpha + q^2 \cot^2 \alpha = 1.$$