

SNS COLLEGE OF ENGINEERING Coimbatore – 641 107

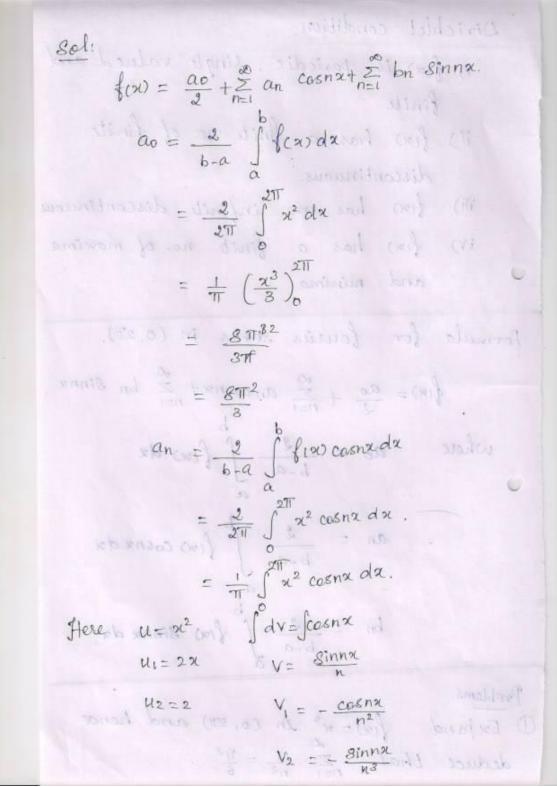


TOPIC : 2 – PROBLEMS BASED ON FULL RANGE SERIES

Dirichlet condition i) fix) is periodic, single valued and finite ii) fix) has a finite no. of finite discontinuous. iii) fix has no infinite discontinuous iv) fire has a finite no. of maxima and minima (1) + Formula for fourier series in (0,271). $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ where $a_0 = \frac{2}{b-a} \int \int \int \int \int dx$ $an = \frac{2}{b-a} \int_{a}^{b} f(x) \cos nx \, dx$ $bn = \frac{2}{b-a} \int f(\alpha) \sin n\alpha \, d\alpha$ Problems: () Expand fix) = x2 in (0, 27) and hence deduce that $\frac{2}{2} - \frac{1}{r^2} = \frac{\pi^2}{6}$.

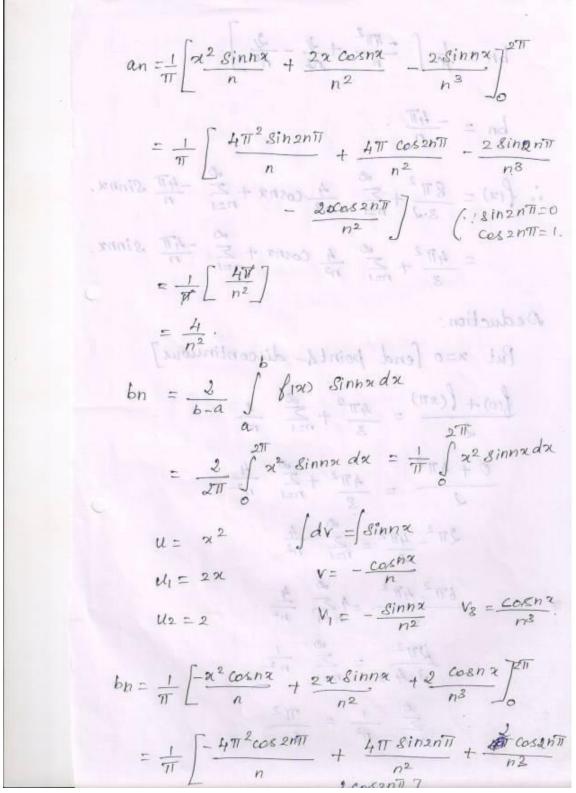




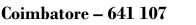














 $bn = \frac{1}{7} \left[-\frac{4\pi^2}{n} + \frac{2}{7^2} - \frac{2}{7^3} \right]$ bn = - 4/ . Trachie That - Trachie That -The Carl : $f(x) = \frac{8\pi^2}{3\cdot 2} + \frac{2}{n=1} \frac{4}{n^2} \cos nx + \frac{2}{n=1} - \frac{4\pi}{n} \sin nx$. $= \frac{4\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos n x + \sum_{n=1}^{\infty} \frac{-4\pi}{n} \sin n x.$ Deduction: Put = 0 [end point 4 discontinuous] $\frac{\int (0) + \int (2\pi)}{2} = \frac{4\pi^2}{2} + \frac{5}{n=1} \frac{4}{n^2}$ $\frac{0 + 4\pi^2}{1} = \frac{4\pi^2 + 2}{3} \frac{4}{n^2}$ $2\pi^2 - \frac{4\pi^2}{3} = \sum_{n=1}^{\infty} \frac{4}{n^2}$ $\frac{6\pi^2 - 4\pi^2}{3} = 4\sum_{n=1}^{\infty} \frac{4}{n^2}$ $\frac{1}{3.42} = \frac{5}{n^2}$ $\therefore \sum_{n=1}^{\infty} \frac{1}{p^2} = \frac{\pi^2}{6}.$





(2) Expand
$$\int (w) = (\pi - x)^2 in (c, 2\pi)$$
 and hence
deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi r^2}{b}$.
Sol:
 $\int (x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} an \cos x + \sum_{n=1}^{\infty} bn \sin x$.
 $a_0 = \frac{2}{b-a} \int_{0}^{b} f(w) dx$
 $= \frac{2}{2\pi r} \int_{0}^{2\pi} (\pi - x)^2 dx$
 $= \frac{1}{\pi r} \left[-\frac{(\pi - x)^2}{2} \right]_{0}^{2\pi}$
 $= \frac{1}{\pi r} \left[-\frac{(\pi - x)^2}{2} \right]_{0}^{2\pi}$
 $= \frac{1}{\pi r} \int_{0}^{2\pi} (\pi - x)^2 \cos x dx$
 $= \frac{2\pi r^2}{2}$.
 $a_n = \frac{2}{b-a} \int_{0}^{2\pi} f(x) \cos x dx$
 $= \frac{2}{\sqrt{\pi r}} \int_{0}^{2\pi} (\pi - x)^2 \cos x dx$
 $u = (\pi - x)^2 \int_{0}^{2\pi} dv = \int \cos x dx$
 $u_1 = -2(\pi - x)^2 \int_{0}^{2\pi} dv = \int \cos x dx$



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$$\begin{aligned} \alpha_{n} &= \frac{1}{n^{2}} \left[(\pi - x)^{2} \frac{\sin nx}{n} + 2 (\pi + x) \cos nx} - \frac{2 \sin x}{n^{2}} \int_{0}^{\pi + x} \right] \\ &= \frac{1}{n^{2}} \left[-\frac{2 (\pi - x\pi) \cos x}{n^{2}} + \frac{2\pi \cos x}{n^{2}} \right] \\ &= \frac{1}{\pi} \left[-\frac{2 (\pi - x\pi) \cos x}{n^{2}} + \frac{2\pi \cos x}{n^{2}} \right] \\ &= \frac{1}{n^{2}} \left(\frac{4\pi \cos x}{n^{2}} \right) = \frac{1}{n^{2}} \left(\frac{4\pi^{2}}{n^{2}} \right) \\ &= \frac{4}{n^{2}} \cdot \\ &= \frac{4}{n^{2}} \cdot \\ &= \frac{1}{n^{2}} \int_{0}^{2\pi} (\pi - x)^{2} \sin nx} \\ &= (\pi - x)^{2} \int_{0}^{2\pi} dx = \int_{0}^{2\pi \sin x} dx, \\ &= (\pi - x)^{2} \int_{0}^{2\pi} dx = \int_{0}^{2\pi \sin x} dx, \\ &= (\pi - x)^{2} \int_{0}^{2\pi} dx = \int_{0}^{2\pi \sin x} dx, \\ &= (\pi - x)^{2} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= (\pi - x)^{2} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx = \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi - x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^{2\pi \sin x} dx, \\ &= \int_{0}^{2\pi \sin x} \int_{0}^$$

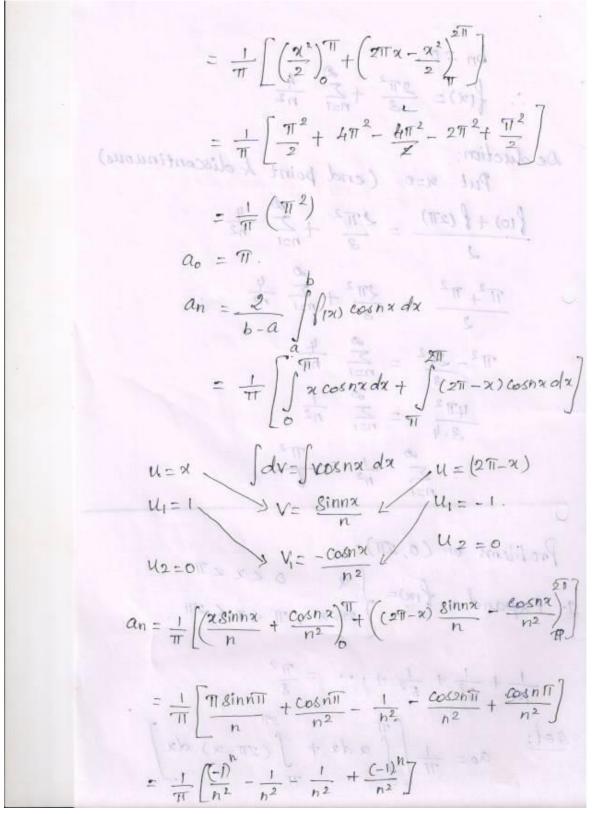




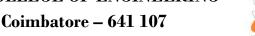
 $b_{n} = 0$ $\therefore \quad \int_{1}^{\infty} \left(\frac{2\pi^{2}}{3} + \sum_{n=1}^{\infty} \frac{4}{n^{2}} \right)$ De oluction: $Put \quad \pi=0 \quad (end \ point \ k \ oliscontinuous)$ $\frac{f(0) + f(2\pi)}{1} = \frac{2\pi^2}{3} + \frac{2}{n^2} + \frac{4}{n^2}$ $\frac{\pi^2 + \pi^2}{2} = \frac{2\pi^2}{3} + \frac{5}{n_{=1}} \frac{4}{n^2}$ $\pi^{2} - 2\pi^{2} = \sum_{n=1}^{\infty} \frac{4}{n^{2}},$ $\frac{4\pi^{2}}{2.4} = \sum_{n=1}^{\infty} \frac{1}{n^{2}},$ $(R-R) = \sum_{n=1}^{\infty} \frac{1}{h^2} = \frac{\pi^2}{6}$ n=1 Problems on (0,211): 1. Expand fix)= fx 0 < x < 11 2. Expand fix)= fx 0 < x < 211. $\frac{1}{1^{2}} + \frac{1}{3^{2}} + \frac{1}{5^{2}} + \cdots = \frac{71^{2}}{8}$ sol: $a_{0} = \frac{1}{77} \int \int x \, dx + \int (277 - x) \, dx \Big]$



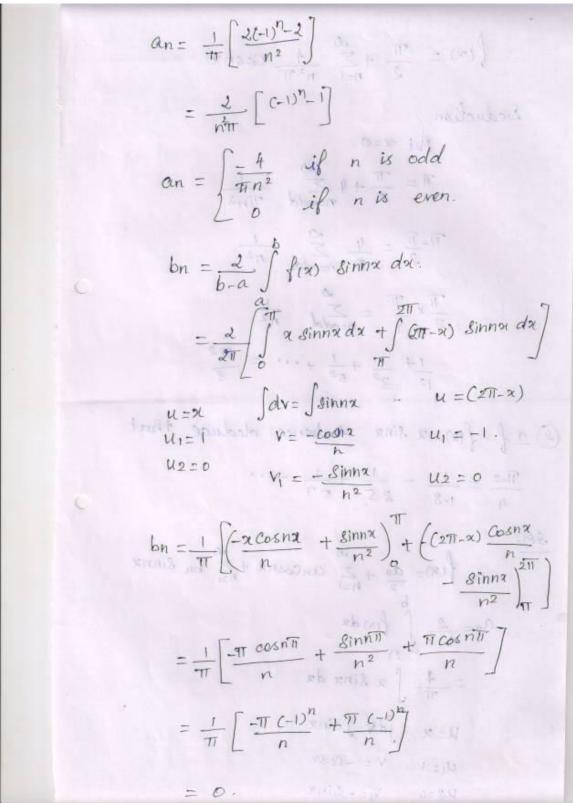




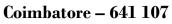














$$\int f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{4}{n^{2}\pi} \cos nx.$$

Deduction:

$$\int \mu t = \pi = 0,$$

$$\pi = \frac{\pi}{2} + 4 \sum_{n=0}^{\infty} \frac{1}{\pi^{n}}$$

$$\int \frac{\pi}{2} = \frac{\pi}{2} + 4 \sum_{n=0}^{\infty} \frac{1}{\pi^{n}}$$

$$\int \frac{\pi}{2} = \frac{\pi}{2} + \frac{5}{2} + \frac{5}{\pi^{n}}$$

$$\int \frac{\pi}{2} - \frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{1}{n^{2}}$$

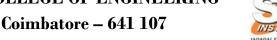
$$\int \frac{\pi}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots = \frac{\pi}{8}$$

$$\int \frac{\pi}{2} + \frac{1}{1,8} - \frac{1}{2,5} + \frac{1}{5,7} - \frac{1}{5,7}$$

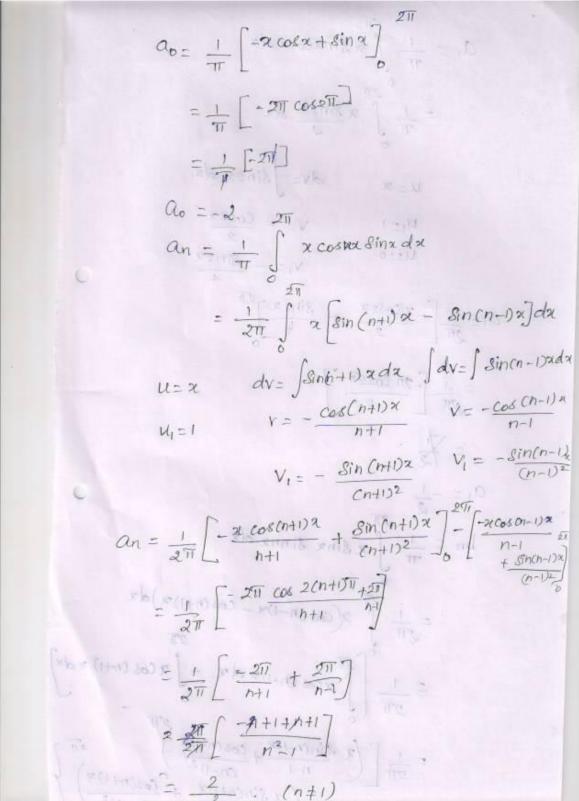
$$\int \frac{\pi}{4} + \frac{1}{1,8} - \frac{1}{2,5} + \frac{1}{5,7} - \frac{1}{5,7}$$

$$\int \frac{\pi}{4} + \frac{\pi}{4} + \frac{1}{5} + \frac{\pi}{5,7} + \frac{\pi}{5,$$

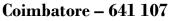




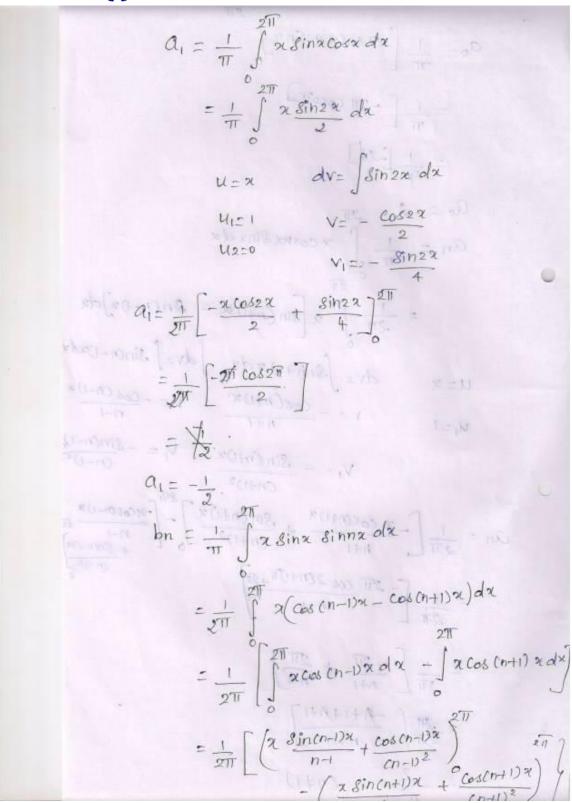










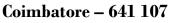






$$\begin{split} h_{n} &= \frac{1}{2\pi} \left[\frac{1}{(n+1)^{2}} + \frac{1}{(n+1)^{2}} - \frac{1}{(n+1)^{2}} + \frac{1}{(n+1)^{2}} \right] \\ h_{n} &= 0 \\ h_{n} &= 0 \\ h_{n} &= 0 \\ h_{n} &= 0 \\ h_{n} &= \frac{1}{2\pi} \int_{0}^{2\pi} x (\sin x \, dx) \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \cos 2x) \, dx \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \cos 2x) \, dx \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \cos 2x) \, dx \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \cos 2x) \, dx \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \cos 2x) \, dx \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \cos 2x) \, dx \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \sin 2x) - (\frac{x^{2}}{2} + \frac{\cos 2x}{4}) \int_{0}^{2\pi} x \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \cos 2x) \, dx \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x (i - \sin 2x) - (\frac{x^{2}}{2} + \frac{\cos 2x}{4}) \int_{0}^{2\pi} x \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} (2\pi) - \frac{4\pi^{2}}{2} - \frac{4}{2} + \frac{4\pi}{4} \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} (2\pi) - \frac{4\pi^{2}}{2} - \frac{4}{2} + \frac{4\pi}{4} \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} (2\pi) - \frac{4\pi^{2}}{2} - \frac{4\pi^{2}}{4} + \frac{4\pi}{4} \\ &= \frac{1}{2\pi} \int_{0}^{2\pi} x + \pi x \sin x + \frac{1}{2\pi} \int_{0}^{2\pi} x \\ &= -1 - \frac{1}{2} \cos x + \pi x \sin x + \frac{1}{2\pi} \int_{0}^{2\pi} \frac{1}{\pi^{2}} \int_{0}^{2\pi} \cos x \\ &= -1 - \frac{1}{2} \cos x + \pi x \sin x + \frac{1}{2\pi} \int_{0}^{2\pi} \frac{1}{\pi^{2}} \int_{0}^{2\pi} \cos x \\ &= \frac{\pi}{2} \int_{0}^{2\pi} \int_{0}^{2\pi} x + \pi x + \pi \int_{0}^{2\pi} x \int_{0}^{2\pi} \frac{1}{\pi^{2}} \int_{0}^{2\pi} x \\ &= \frac{\pi}{2} \int_{0}^{2\pi} \int_{0}^{2\pi} \frac{1}{\pi^{2}} \int_{0}^{2\pi} \frac{1}{\pi$$







 $\left(1-\frac{\pi}{2}\right)\cdot\frac{1}{2}=\sum_{n=2}^{\infty}\frac{1}{(n+1)(n-1)}\cos\frac{n\pi}{2}$ $\frac{1}{2} - \frac{11}{4} = \frac{1}{1 \cdot 3} \cos \pi + \frac{1}{4 \cdot 2} \cos \pi + \frac{1}{4 \cdot 2} \cos \pi + \frac{1}{2}$ $+\frac{1}{5\cdot 3}\cos\{\frac{3}{11}+\cdots,\frac{1}{5\cdot 7}+\frac{1}{5\cdot 7}+\cdots,\frac{1}{5\cdot 7}+\cdots,\frac{1}{5\cdot$ $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots = \frac{\pi - 2}{4}.$ $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{2\pi}{2} \frac{1}{2} \frac{1$ $\frac{1}{2\pi}\left[\frac{1}{2\pi}-\frac{1}{2\pi}\right]=\left[\frac{1}{2\pi}\left[\frac{1}{2\pi}\right]$ $b_1 = n.$ $b_2 = \frac{1}{2} + \frac{1}{2}$ $= -1 - \frac{1}{2} \cos \alpha + \pi \sin \alpha + \frac{1}{12} \cos \alpha + \frac{1}{2}$ fion $x = \frac{\pi}{2}$ このキャー 二世の語道