TOPIC : 10 - SOLUTIONS OF LINEAR EQUATIONS OF SECOND AND HIGHER ORDER WITH CONSTANT COEFFICIENTS

Higher order partial differential equations

$$a\frac{\partial^2 z}{\partial x^2} + b\frac{\partial^2 z}{\partial y \partial x} + c\frac{\partial^2 z}{\partial y^2} = F(x,y)$$

Here a, b, c are constants

$F(x,y)$ = Functions of x & y.

Take $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$

$(xD^2 + bDD' + cD'^2)z = F(x,y) \rightarrow (1)$

Solution of (1) is given by.

$z = C.F + P.I$

To find C.F

Replace D by m

D' by 1

$\therefore am^2 + bm + c = 0$

Solve we get two roots m_1, m_2.

case: (i)
If $m_1 \neq m_2$

$$C.F = f_1(y+m_1x) + f_2(y+m_2x).$$

case: (ii) If $m_1 = m_2$

$$C.F = f_1(y+m_1x) + xf_2(y+m_2x)$$

To find P.I:

$$P.I = \frac{1}{aD^2 + bDD' + cD'^2} F(x,y).$$

Type: (i) (Homogeneous Equations)
R.H.S = 0

(4) Solve: $(D^2 - 5DD' + 6D'^2)z = 0$

Sol:
The A.E is $m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$\therefore m = 2, 3.$

$$C.F = f_1(y+2x) + f_2(y+3x).$$

$$P.I = 0.$$

$$\therefore z = C.F + P.I$$

$$z = f_1(y+2x) + f_2(y+3x)$$

(5) Solve: $(D^3 - 3DD'^2 + 2D'^3)z = 0.$

Sol:
The A.E is $m^3 - 3m + 2 = 0.$

$$
\begin{array}{r|rrrr}
1 & 1 & 0 & -3 & 2 \\
  & \downarrow & 1 & 1 & -2 \\
\hline
1 & 1 & 1 & -2 & \boxed{0} \\
  & \downarrow & 1 & 2 & \\
\hline
-2 & 1 & 2 & \boxed{0} & \\
  & \downarrow & -2 & & \\
\hline
  & 1 & \boxed{0} & & \\
\end{array}
$$

$\therefore m = 1, 1, -2.$

C.F is $f_1(y+x) + f_2(y+x) + f_3(y-2x)$

$P.I = 0.$

$z = C.F + P.I$

$z = f_1(y+x) + f_2(y+x) + f_3(y-2x)$

Type: (ii)

$R.H.S = e^{ax+by}$

Working Rule:

① Replace D by a ; D' by b

② If $dr \neq 0$ then we get P.I

③ If $dr = 0$ put x in Nr & Diff dr w.r.to D and apply same method.

(1) Solve: $(D^2 + 2DD' + D'^2) = e^{2x+3y}$ (M)

Sol:

The A.E is $m^2+2m+1=0$

$(m+1)(m+1)=0$

$m=-1,-1$

$C.F = f_1(y-x) + x f_2(y-x)$

$$P.I = \frac{1}{D^2+2DD'+D'^2} e^{2x+3y}$$

D by 2

D' by 3

$$= \frac{1}{(2)^2+2\cdot2\cdot3+3^2} e^{2x+3y}$$

$$= \frac{1}{4+12+9} e^{2x+3y}$$

$$= \frac{1}{25} e^{2x+3y}$$

$z = C.F + P.I$

$$z = f_1(y-x) + x f_2(y-x) + \frac{1}{25} e^{2x+3y}.$$

2. Solve: $(D^2 - D'^2) z = e^{x+2y}$

Sol:

The A.E is $m^2-1=0$

$m^2=1$

$m=\pm 1.$

$C.F = f_1(y-x) + f_2(y+x)$

$$P.I = \frac{1}{(D^2-D'^2)} e^{x+2y}$$

$$P.I = -\frac{1}{3} e^{x+2y}$$

$$z = C.F + P.I$$

$$= f_1(y-x) + f_2(y+x) - \frac{1}{3} e^{x+2y}$$