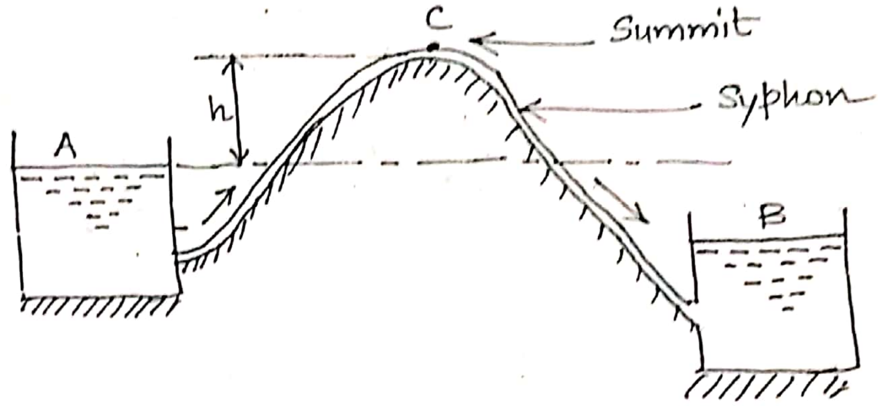
Flow Through Syphon:

Syphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high level ground as shown below.



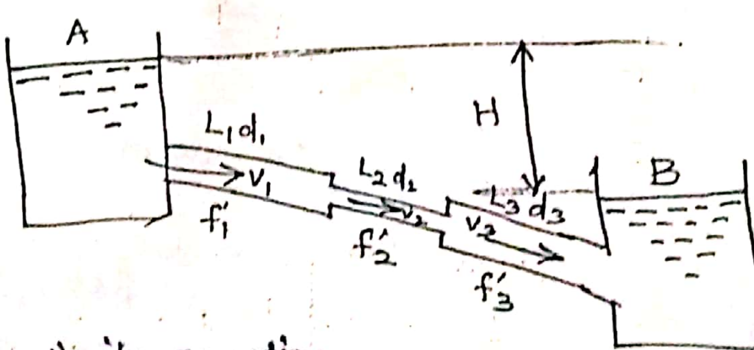
Highest point of the syphon is called as summit.

Syphon is used in the following cases:

1. To carry water from one reservoir to another reservoir separated by a hill, ~~or~~ ridge. (BuG) narrow elevation of land
2. To take out the liquid from a tank which is not having any outlet
3. To empty a channel not provided with any outlet sluice (channel)

Flow through pipes in series or Through Compound pipes:

Pipes in series or compound pipes is defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line



By continuity equation,

$$Q = a_1 V_1 = a_2 V_2 = a_3 V_3$$

H = Sum of all losses

$$= \frac{0.5V_1^2}{2g} + \frac{4f_1'L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2'L_2V_2^2}{2gd_2} + \frac{(V_2-V_3)^2}{2g}$$

$$+ \frac{4f_3'L_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

Neglecting all minor losses

$$H = \frac{4f_1'L_1V_1^2}{2gd_1} + \frac{4f_2'L_2V_2^2}{2gd_2} + \frac{4f_3'L_3V_3^2}{2gd_3}$$

If $f_1' = f_2' = f_3'$, then

$$\boxed{H = \frac{4f'}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]}$$

Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400 m, 200 m, and 300 m respectively. They are connected in series to make a compound pipe. The end of this compound pipe are connected with two tanks whose difference of water levels is 16 m. If co-efficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first minor losses and then including them.

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Given data:

$d_1 = 400$ mm $= 0.4$ m

$d_2 = 200$ mm $= 0.2$ m

$d_3 = 300$ mm $= 0.3$ m

$L_1 = 400$ m

$L_2 = 200$ m

$L_3 = 300$ m

$H = 16$ m

$f' = 0.005$

Q = (i) neglecting minor loss

(ii) Including minor loss

Solution:

By continuity equation,

$$Q = a_1 V_1 = a_2 V_2 = a_3 V_3$$

$$V_2 = \frac{a_1 V_1}{a_2} = \frac{\frac{\cancel{\pi}}{\cancel{4}} d_1^2 \times V_1}{\frac{\cancel{\pi}}{\cancel{4}} d_2^2} = \frac{0.4^2}{0.2^2} V_1 = 4 V_1$$

$$V_3 = \frac{a_1 V_1}{a_3} = \frac{0.4^2}{0.3^2} V_1 = 1.78 V_1$$

(i) Neglecting minor losses.

$$H = \frac{4f'}{2g}\left[\frac{L_1 V_1^2}{d_1} + \frac{L_2 V_2^2}{d_2} + \frac{L_3 V_3^2}{d_3}\right]$$

$$16 = \frac{4 \times 0.005}{2 \times 9.81}\left[\frac{400 \times V_1^2}{0.4} + \frac{200 \; 4^2 V_1^2}{0.2} + \frac{300 \; 1.78^2 V_1^2}{0.3}\right]$$

$$= 1.02 \times 10^{-3}\left[1000 V_1^2 + 16000 V_1^2 + 3168.4 V_1^2\right]$$

$$16 = 1.02 \times 10^{-3}\left[20168.4 V_1^2\right]$$

$$V_1 = \sqrt{\frac{16}{1.02 \times 10^{-3} \times 20168.4}}$$

$$= 0.882 \; \frac{m}{s}$$

$$Q = a_1 V_1 = \frac{\pi}{4} 0.4^2 \times 0.882$$

$$= 0.1108 \; \frac{m^3}{s} = 110.82 \; \frac{\text{litres}}{s}.$$

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(ii) Including minor loss.

Major losses are $= h_i + hf_1 + h_c + hf_2 + h_e + hf_3 + h_o$

$$h_i = \frac{0.5 V_1^2}{2g} = \cancel{\frac{0.5 \times 0.882^2}{2 \times 9.81} = 0.0198 \, m}$$

$$h_{f1} = \frac{4 f_1' L_1 V_1^2}{2gd} = \frac{4 \times 0.005 \times 400}{0.4} \frac{V_1^2}{2g} = 20 \frac{V_1^2}{2g}$$

$$h_c = \frac{0.5 V_2^2}{2g} = 0.5 \frac{(4V_1)^2}{2g} = 8 \frac{V_1^2}{2g}$$

$$h_{f_2} = \frac{4 f' L_2 V_2^2}{2gd} = \frac{4 \times 0.005 \times 200 \times 4^2}{0.2} \frac{V_1^2}{2g} = 320 \frac{V_1^2}{2g}$$

$$h_e = \frac{(V_2 - V_3)^2}{2g} = \frac{(4V_1 - 1.78V_1)^2}{2g} = \frac{(2.22V_1)^2}{2g} = 4.93 \frac{V_1^2}{2g}$$

$$h_{f_3} = \frac{4 f' L_3 V_3^2}{2gd} = \frac{4 \times 0.005 \times 300 \times 1.78^2}{0.3} \frac{V_1^2}{2g} = 63.368 \frac{V_1^2}{2g}$$

$$h_o = \frac{V_3^2}{2g} = \frac{1.78^2 V_1^2}{2g} = 3.168 \frac{V_1^2}{2g}$$

$\therefore$ Total loss $= 0.5 \frac{V_1^2}{2g} + 20 \frac{V_1^2}{2g} + 8 \frac{V_1^2}{2g} + 320 \frac{V_1^2}{2g} + 4.93 \frac{V_1^2}{2g}$
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$$+ 63.3668 \frac{V_1^2}{2g} + 3.168 \frac{V_1^2}{2g}$$

$$= \frac{V_1^2}{2g} \left[0.5 + 20 + 8 + 320 + 4.93 + 63.368 + 3.168 \right]$$

$$16 = 419.97 \frac{V_1^2}{2g}$$

$$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.97}} = 0.87 \frac{m}{s}$$

$$\therefore Q = a_1 V_1 = \frac{\pi}{4} \times 0.4^2 \times 0.87$$

$$= 0.1087 \frac{m^3}{s}$$

$$Q = 108.65 \frac{\text{litres}}{s}$$

The difference in water surface levels in two tanks, which are connected by three pipe in series of lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively (i) neglecting minor loss (ii) including minor losses.

Given data:

$L_1 = 300 m$; $d_1 = 300$ mm $= 0.3m$; $H = 12 m$

$L_2 = 170m$; $d_2 = 200$ mm $= 0.2m$; $f_1' = 0.005$

$L_3 = 210m$; $d_3 = 400$ mm $= 0.4m$; $f_2' = 0.0052$

$f_3' = 0.0048$

$Q = ?$

Solution:

By continuity equation, $Q = a_1V_1 = a_2V_2 = a_3V_3$

$$V_2 = \frac{a_1V_1}{a_2} = \frac{\frac{\pi}{4}\,0.3^2 \times V_1}{\frac{\pi}{4}(0.2)^2} = 2.25V_1$$

$$V_3 = \frac{a_1V_1}{a_3} = \frac{\frac{\pi}{4}\times 0.3^2 \times V_1}{\frac{\pi}{4}\times 0.4^2} = 0.5625V_1$$

(i) Neglecting minor losses:

$$H = \frac{4}{2g}\left[\frac{f_1'L_1V_1^2}{d_1} + \frac{f_2'L_2V_2^2}{d_2} + \frac{f_3'L_3V_3^2}{d_3}\right]$$

$$12 = \frac{4}{2\times 9.81}\left[\frac{0.005\times 300\times V_1^2}{0.3} + \frac{0.0052\times 170\times 2.25^2 V_1^2}{0.2} + \frac{0.0048\times 210\times 0.5625^2}{0.4}\right]$$

$$12 = 0.204\left[5V_1^2 + 22.38V_1^2 + 0.797V_1^2\right]$$

$$\frac{12}{0.204} = 28.174\,V_1^2$$

$$V_1 = \sqrt{\frac{12}{0.204\times 28.174}} = 1.445\ \frac{m}{s}$$

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$$Q = a_1 V_1 = \frac{\pi}{4} \times 0.3^2 \times 1.445$$

$$= 0.1022 \ \frac{m^3}{s} = 102.18 \ \frac{\text{litres}}{s.}$$

ii) Including minor losses,

$$H = \frac{4}{2g} \left[0.5 V_1^2 + \frac{f_1' L_1 V_1^2}{d_1} + 0.5 V_2^2 + \frac{f_2' \times L_2 V_2^2}{d_2} + (V_2 - V_3)^2 + \frac{f_3' L_3 \times V_3^2}{d_3} + V_3^2 \right]$$

X (wrong)

$$12 = 0.204 \left[0.5 V_1^2 + 5 V_1^2 + 0.5 (2.25)^2 V_1^2 + 22.38 V_1^2 + (2.25 - 0.5625)^2 + 0.797 V_1^2 + 0.5625 V_1^2 \right.$$

$$\frac{12}{0.204} = 0.5 V_1^2 + 5 V_1^2 + 2.53125 V_1^2 + 22.38 V_1^2 + 2.85 V_1^2 + 0.797 V_1^2 + 0.5625 V_1^2$$

$$58.86 = 34.615 V_1^2$$

$$V_1 = \cancel{1.30} \ \frac{m}{s} \quad 1.41 \ \frac{m}{s} \quad 1.407 \ m/s.$$

$$Q = a_1 V_1 = \frac{\pi}{4} \times 0.3^2 \times 1.30$$

$$= \cancel{0.0923 \ \frac{m^3}{s}} = 92.17 \ \frac{\cancel{\text{litres}}}{\text{sec.}} = 99.45 \ \text{lps.}$$

Equivalent Pipe:

It is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe, which consists of several pipes of different lengths and diameters.

The uniform diameter of the equivalent pipe is called as equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

In compound pipe, loss of head, neglecting minor losses,

$$H = \frac{4f_1'L_1V_1^2}{2gd_1} + \frac{4f_2'L_2V_2^2}{2gd_2} + \frac{4f_3'L_3V_3^2}{2gd_3} \longrightarrow (1)$$

Assuming $f_1' = f_2' = f_3'$ and by continuity equation

$$Q = a_1V_1 = a_2V_2 = a_3V_3$$

$$Q = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2 = \frac{\pi}{4}d_3^2V_3$$

$$\boxed{V_1 = \frac{4Q}{\pi d_1^2}\ ;\ V_2 = \frac{4Q}{\pi d_2^2}\ ;\ V_3 = \frac{4Q}{\pi d_3^2}}$$

Substituting above in (1)

$$H = \frac{4f_1'L_1\left(\frac{4Q}{\pi d_1^2}\right)^2}{2gd_1} + \frac{4f_2'L_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{2gd_2} + \frac{4f_3'L_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{2gd_3}$$

$$= \frac{4\times16f'Q^2}{\pi^2\times2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

In equivalent pipe,

$$H = \frac{4f'LV^2}{2gd} \quad \text{and} \quad Q = aV\ ;\ V = \frac{Q}{a} = \frac{Q}{\frac{\pi}{4}d^2}$$

$$= \frac{4f'L\left(\frac{Q}{\frac{\pi}{4}d^2}\right)^2}{2gd} = \frac{4\times16f'Q^2}{\pi^2\times2g}\left(\frac{L}{d^5}\right)$$

Now, loss of head in compound pipe = loss of head in equivalent pipe

$$\cancel{\frac{4\times16fQ^2}{\pi^2\times2g}}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] = \cancel{\frac{4\times16fQ^2}{\pi^2\times2g}}\left[\frac{L}{d^5}\right]$$

$$\boxed{\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}} \longrightarrow \text{Dupuit's equation.}$$