

Flow through Siphon:

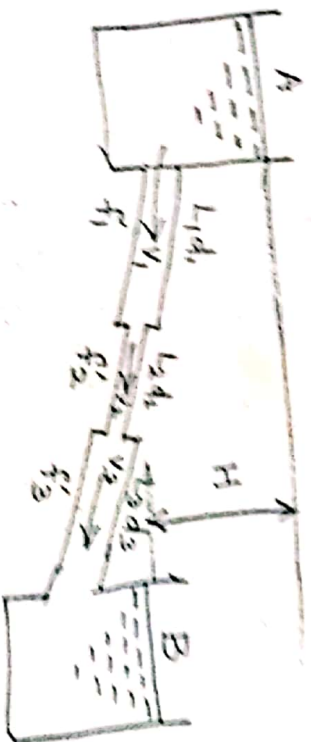
Siphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high land ground. As shown below:



Highest point of the siphon is called as summit. Siphon is used in the following cases:

1. To carry water from one reservoir to another reservoir separated by a hill or ridge (low) surface.
2. To take out the liquid from a tank which is not having any outlet.
3. To empty a channel not provided with any outlet like canals.

Flow through pipes in series or through compound pipes: Pipes in series or compound pipes is defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line.



By continuity equation,

$$Q = a_1 v_1 = a_2 v_2 = a_3 v_3$$

$H = \text{Sum of all losses}$

$$= 0.5 \frac{V_1^2}{2g} + \frac{4f_1' L_1 V_1^2}{2gd_1} + 0.5 \frac{V_2^2}{2g} + \frac{4f_2' L_2 V_2^2}{2gd_2} + \frac{(V_2 - V_3)^2}{2g} \\ + \frac{4f_3' L_3 V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

Neglecting all minor losses

$$H = \frac{4f_1' L_1 V_1^2}{2gd_1} + \frac{4f_2' L_2 V_2^2}{2gd_2} + \frac{4f_3' L_3 V_3^2}{2gd_3}$$

If $f_1' = f_2' = f_3'$, then

$$H = \frac{4f'}{2g} \left[\frac{L_1 V_1^2}{d_1} + \frac{L_2 V_2^2}{d_2} + \frac{L_3 V_3^2}{d_3} \right]$$

Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400 m, 200 m, and 300 m respectively. They are connected in series to make a compound pipe. The end of this compound pipe are connected with two tanks whose difference of water levels is 16 m. If coefficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first minor losses and then including them.

Given data:

$$d_1 = 400 \text{ mm} = 0.4 \text{ m}$$

$$d_2 = 200 \text{ mm} = 0.2 \text{ m}$$

$$d_3 = 300 \text{ mm} = 0.3 \text{ m}$$

$$L_1 = 400 \text{ m}$$

$$L_2 = 200 \text{ m}$$

$$L_3 = 300 \text{ m}$$

$$H = 16 \text{ m}$$

$$f' = 0.005$$

Q = (i) neglecting minor loss
(ii) Including minor loss

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Solution:

By continuity equation,

$$Q = a_1 v_1 = a_2 v_2 = a_3 v_3$$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{\frac{\pi}{4} d_1^2 \times v_1}{\frac{\pi}{4} d_2^2} = \frac{0.4^2}{0.2^2} v_1 = 4v_1$$

$$v_3 = \frac{a_1 v_1}{a_3} = \frac{0.4^2}{0.3^2} v_1 = 1.78 v_1$$

(i) Neglecting minor losses.

$$H = \frac{4f}{2g} \left[\frac{L_1 v_1^2}{d_1} + \frac{L_2 v_2^2}{d_2} + \frac{L_3 v_3^2}{d_3} \right]$$

$$16 = \frac{4 \times 0.005}{2 \times 9.81} \left[\frac{400}{0.4} v_1^2 + \frac{200}{0.2} (4v_1)^2 + \frac{300}{0.3} (1.78^2 v_1^2) \right]$$

$$= 1.02 \times 10^{-3} \left[1000 v_1^2 + 16000 v_1^2 + 3168.4 v_1^2 \right]$$

$$16 = 1.02 \times 10^{-3} \left[20168.4 v_1^2 \right]$$

$$v_1^2 = \sqrt{\frac{16}{1.02 \times 10^{-3} \times 20168.4}}$$

$$= 0.882 \frac{m}{s}$$

$$Q = a_1 v_1 = \frac{\pi}{4} 0.4^2 \times 0.882$$

$$= 0.1108 \frac{m^3}{s} = 110.82 \frac{\text{litres}}{s}$$

(ii) Including minor losses.

Major losses are = $h_i + h_{f1} + h_c + h_{f2} + h_e + h_{f3}$
+ h_o

$$h_i = \frac{0.5 v_1^2}{2g} = \frac{0.5 \times 0.882^2}{2 \times 9.81} = 0.0198 m$$

$$h_{f1} = \frac{4f_1 L_1 v_1^2}{2gd} = \frac{4 \times 0.005 \times 400}{0.4} \frac{v_1^2}{2g} = \frac{20 v_1^2}{2g}$$

$$h_c = \frac{0.5 V_2^2}{2g} = 0.5 \frac{(4V_1)^2}{2g} = 8 \frac{V_1^2}{2g}$$

$$hf_2 = \frac{4 f' L_2 V_2^2}{2gd} = \frac{4 \times 0.05 \times 200 \times 4^2 V_1^2}{0.2 \times 2g} = 320 \frac{V_1^2}{2g}$$

$$h_e = \frac{(V_2 - V_3)^2}{2g} = \frac{(4V_1 - 1.78V_1)^2}{2g} = \frac{(2.22V_1)^2}{2g} = 4.93 \frac{V_1^2}{2g}$$

$$hf_3 = \frac{4 f' L_3 V_3^2}{2gd} = \frac{4 \times 0.005 \times 300 \times 1.78^2 V_1^2}{0.3 \times 2g} = 63.368 \frac{V_1^2}{2g}$$

$$h_o = \frac{V_3^2}{2g} = \frac{1.78^2 V_1^2}{2g} = 3.168 \frac{V_1^2}{2g}$$

$$\therefore \text{Total loss} = \frac{0.5 V_1^2}{2g} + 20 \frac{V_1^2}{2g} + 8 \frac{V_1^2}{2g} + 320 \frac{V_1^2}{2g} + 4.93 \frac{V_1^2}{2g} + 63.368 \frac{V_1^2}{2g} + 3.168 \frac{V_1^2}{2g}$$

$$= \frac{V_1^2}{2g} [0.5 + 20 + 8 + 320 + 4.93 + 63.368 + 3.168]$$

$$16 = 419.97 \frac{V_1^2}{2g}$$

$$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.97}} = 0.87 \frac{m}{s}$$

$$\therefore Q = a_1 V_1 = \frac{\pi \times 0.4^2}{4} \times 0.87 = 0.1087 \frac{m^3}{s}$$

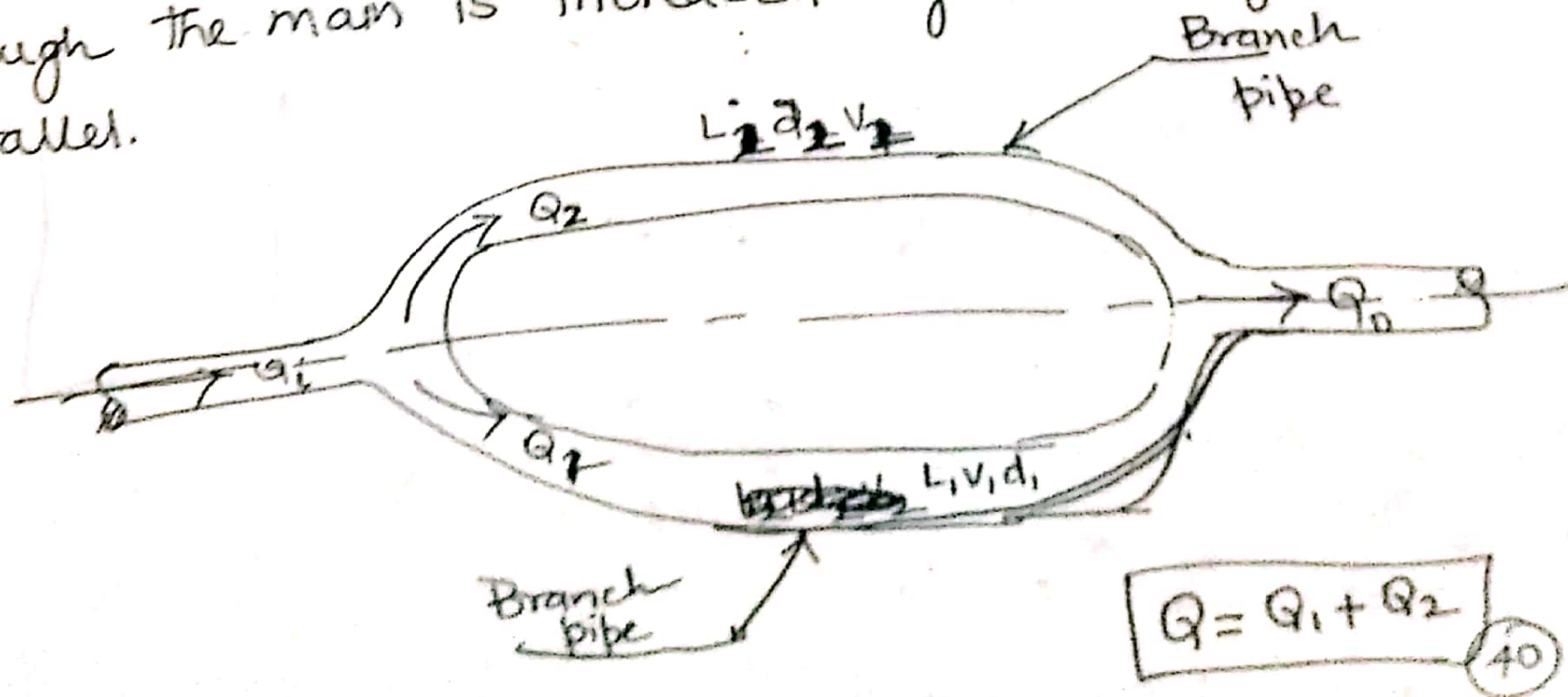
$$Q = 108.65 \frac{\text{litres}}{s}$$



$$d = 371.88 \text{ mm}$$

Flow Through Parallel Pipes

A main pipe which divides into two or more branches as shown in fig. and again join together downstream, to form a single pipe, then the branch pipes are said to be connected in parallel. The discharge through the main is increased by connecting pipes in parallel.



A main pipe divides into two parallel pipes which again forms one pipe. The length and diameter for the first parallel pipe are 2000m and 1.0m respectively, while the length and diameter of 2nd parallel pipe are 2000m and 0.8m. Find the rate of flow in each parallel pipe, if total flow in the main is $3.0 \frac{m^3}{s}$. The coefficient of friction for each parallel pipe is same and equal to 0.005.

Given Data:

$$L_1 = 2000m \quad L_2 = 2000m \quad Q_0 = 3 \frac{m^3}{s}$$

$$d_1 = 1m \quad d_2 = 0.8m \quad f = 0.005$$

$$Q_1 = ? \quad Q_2 = ?$$

Solution:

$$Q_0 = Q_1 + Q_2$$

We know that,

$$\frac{f L_1 K_1 V_1^2}{R g d_1} = \frac{f L_2 K_2 V_2^2}{R g d_2}$$

$$Q_1 = a_1 v_1$$

$$Q_2 = a_2 v_2$$

$$\frac{V_1^2}{1} = \frac{V_2^2}{0.8}$$

$$V_1 = \sqrt{\frac{V_2^2}{0.8}} = \frac{V_2}{\sqrt{0.8}} = 1.12 V_2$$

$$\Rightarrow Q_0 = Q_1 + Q_2$$

$$3 = \frac{\pi}{4} d_1^2 V_1 + \frac{\pi}{4} d_2^2 V_2$$

$$3 = \frac{\pi}{4} 1^2 \cdot 1.12 V_2 + \frac{\pi}{4} \times 0.8^2 V_2$$

$$3 = 0.8796 V_2 + 0.5026 V_2$$

$$3 = 1.382 V_2$$

$$V_2 = 2.17 \frac{m}{s}$$

$$\therefore V_1 = 1.12 V_2 = 2.43 \frac{m}{s}$$

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$$Q_1 = a_1 v_1 = \frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} 1^2 \times 2.43 = 1.906 \frac{\text{m}^3}{\text{s}}$$

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$$Q_2 = Q_0 - Q_1 = 1.094 \frac{\text{m}^3}{\text{s}}$$

A pipe line of 0.6m diameter is 1.5 km long. To increase the discharge, another line of the same diameter is introduced parallel to the first in the second half of the length. Neglecting minor losses, find the increase in discharge if $f' = 0.01$. The head at inlet is 300 mm.

Given data:

$$d = 0.6 \text{ m} = d_1 = d_2$$

$$L = 1500 \text{ m} = L$$

$$L_1 = 750 \text{ m}$$

$$L_2 = 750 \text{ m}$$

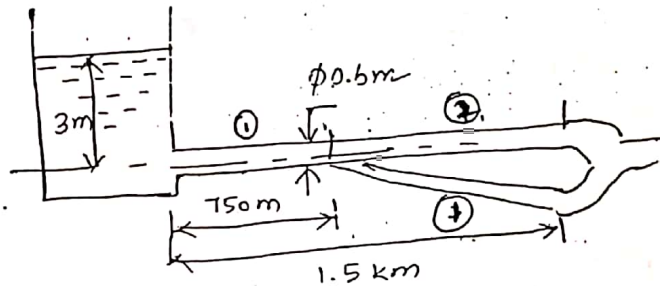
$$f' = 0.01 = f'_1 = f'_2$$

$$H_i = 300 \text{ m} = 0.3 \text{ m}$$

$$H_o = 0$$

$$h_f = H_i - H_o = 0.3 \text{ m}$$

$$Q_0 - Q_i = ?$$



Solution:

$$Q^* = a \times v^*$$

$$h_f = \frac{4 f' L v^{*2}}{2gd}$$

$$v^* = \sqrt{\frac{h_f \times 2gd}{4 \times f' \times L}} = \sqrt{\frac{0.3 \times 2 \times 9.81 \times 0.6}{4 \times 0.01 \times 1500}} = 0.243 \frac{\text{m}}{\text{s}}$$

$$\therefore Q^* = \frac{\pi}{4} d^2 v^* = \frac{\pi}{4} \times 0.6^2 \times 0.243 = 0.0685 \frac{\text{m}^3}{\text{s}}$$

Now, $Q_1 = Q_2 + Q_3$
 $Q_1 = Q_4 + Q_5$ (Being dia & lengths are same)
 $Q_1 = 2Q_2$
 $Q_2 = \frac{Q_1}{2}$

$$h_f = h_{f1} + h_{f2}$$

$$h_{f1} = \frac{4 f' L_1 v_1^2}{2 g d_1} = \frac{4 f' L_1 \left(\frac{Q_1}{a_1}\right)^2}{2 g d_1} = \frac{4 f' L_1 \left(\frac{Q_1}{\frac{\pi}{4} d_1^2}\right)^2}{2 g d_1}$$

$$h_{f1} = \frac{4 \times 0.01 \times 750 \times \frac{Q_1^2}{\left(\frac{\pi}{4} \times 0.6^2\right)^2}}{2 \times 9.81 \times 0.6} = 31.88 Q_1^2$$

$$h_{f2} = \frac{4 f' L_2 v_2^2}{2 g d_2} = \frac{4 f' L_2 \left(\frac{Q_2}{a_2}\right)^2}{2 g d_2} = \frac{4 f' L_2 \left(\frac{\frac{Q_2}{2}}{\frac{\pi}{4} d_2^2}\right)^2}{2 g d_2}$$

$$h_{f2} = \frac{4 \times 0.01 \times 750 \times \frac{Q_2^2}{\left(\frac{\pi}{4} \times 0.6^2\right)^2}}{2 \times 9.81 \times 0.6} = 7.97 Q_2^2$$

$$h_f = h_{f1} + h_{f2}$$

$$0.3 = 31.88 Q_1^2 + 7.97 Q_2^2$$

$$0.3 = 39.847 Q_1^2$$

$$Q_1 = 0.0868 \frac{\text{m}^3}{\text{s}}$$

$$\therefore \text{Increase in flow rate} = Q_1 - Q_1^*$$

$$= 0.0868 - 0.0685$$

$$= 0.0183 \frac{\text{m}^3}{\text{s}}$$

$$= 18.27 \frac{\text{litres}}{\text{c}}$$

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 10 sharp ended pipes of diameters 50 mm and 100 mm respectively, each of length 100 m are connected in parallel between two reservoirs which have difference of level of 10 m. If the co-efficient of friction for each pipe is 0.08, calculate the rate of flow for each pipe and also the diameter of a single pipe 100 m long which would give the same discharge, if it were substituted for the original two pipes.

Given data:

$$d_1 = 50 \text{ mm} = 0.05 \text{ m}$$

$$d_2 = 100 \text{ mm} = 0.1 \text{ m}$$

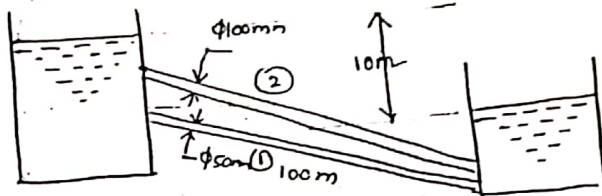
$$L_1 = 100 \text{ m} = L_2$$

$$H = 10 \text{ m} = h_f$$

$$f' = 0.08$$

$$Q_1, Q_2 = ?$$

$$d = ?$$



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Solution:

We know that,
$$h_f = \frac{4 f' L v^2}{2 g d}$$

For first pipe

$$h_f = \frac{4 f' L_1 v_1^2}{2 g d_1} = \frac{4 f' L_1 \left(\frac{Q_1}{a_1}\right)^2}{2 g d_1} = \frac{4 f' L_1 \left(\frac{Q_1^2}{\frac{\pi}{4} d_1^2}\right)^2}{2 g d_1}$$

$$10 = \frac{4 \times 0.08 \times 100 \times \frac{Q_1^2}{\left(\frac{\pi}{4} \times (0.05)^2\right)^2}}{2 \times 9.81 \times 0.05} = 8.625 \times 10^6 Q_1^2$$

$$Q_1 = \sqrt{\frac{10}{8.625 \times 10^6}} = 0.00108 \frac{\text{m}^3}{\text{s}}$$

Similarly, for second pipe,

$$h_f = \frac{4 f' L_2 v_2^2}{2 g d_2} = \frac{4 f' L_2 \left(\frac{Q_2}{a_2}\right)^2}{2 g d_2} = \frac{4 f' L_2 \left(\frac{Q_2^2}{\frac{\pi}{4} d_2^2}\right)^2}{2 g d_2}$$

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$$10 = \frac{4 \times 0.08 \times 100 \text{ m} \times \frac{Q_2^2}{\left(\frac{\pi \times (0.1)^2}{4}\right)^2}}{2 \times 9.81 \times 0.1} = 264.406 \times 10^3 Q_2^2$$

$$Q_2 = \sqrt{\frac{10}{264.406 \times 10^3}} = 0.00615 \frac{\text{m}^3}{\text{s}}$$

Then, discharge through single pipe

$$Q = Q_1 + Q_2$$

$$= 0.00108 + 0.00615$$

$$Q = 0.00723 \frac{\text{m}^3}{\text{s}}$$

Loss of head through single pipe

$$h_f = \frac{4 f' L v^2}{2 g d} = \frac{4 \times 0.08 \times 100 \times \left(\frac{Q}{\frac{\pi d^2}{4}}\right)^2}{2 g d} = \frac{4 \times 0.08 \times 100 \times \frac{16 Q^2}{\pi^2 d^4}}{2 \times 9.81 \times d}$$

$$= \frac{4 \times f' \times L \times 4^2 \times Q^2}{\pi^2 d^4 \times 2 \times 9.81 \times d} = \frac{4 \times f' \times L \times 4^2 \times Q^2}{2 \times 9.81 \times \pi^2 \times d^5}$$

$$d^5 = \frac{4 \times 0.08 \times 100 \times 4^2 \times (0.00723)^2}{2 \times 9.81 \times \pi^2 \times 10}$$

$$d^5 = 1.086 \times 10^{-5} \quad 1.382 \times 10^{-5}$$

$$d = 0.10165 \text{ m} \quad 0.1067 \text{ m}$$

$$\boxed{d = 101.65 \text{ mm}} \quad 106.687 \text{ mm}$$