UNIT-II

FLOW THROUGH CIRCULAR CONDUITS

Circular conduits is nothing, but, a pipe which is used to carry liquids under pressure.

When the pipe is running ~~fu~~ full, the flow is under pressure. This kind of flow is known as pipe flow.

If, when the pipe is not running full, the flow is not under pressure; only atmospheric pressure exists inside the pipe. This kind of flow is termed as channel flow.

Types of flow in pipes:

The flow in pipes are classified as,

1. Viscous flow — controlled by viscosity (viscous force)
2. Non-viscous flow

Classification of viscous flow:

1. Laminar flow — Viscous force > Inertia force
2. Turbulant flow — Viscous force < Inertia force

Viscous flow (or) Laminar flow:

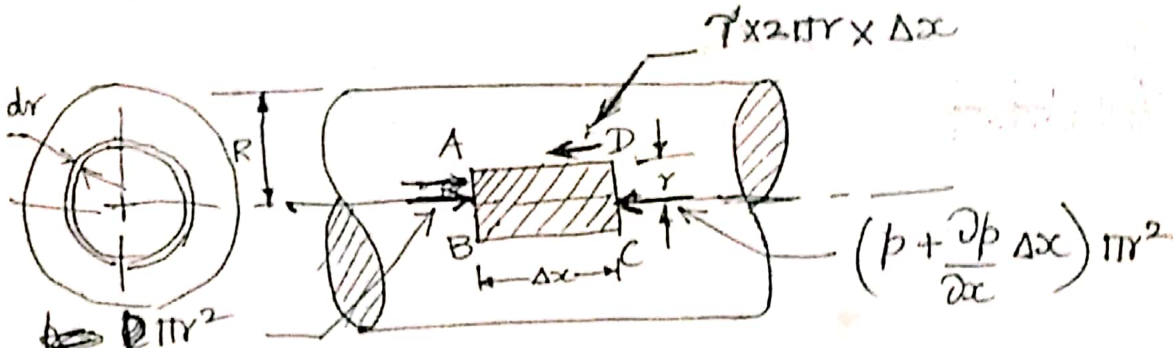
~~E~~ For this flow, following are to be determined,

2 * Velocity distribution across a section.
3 * ratio of maximum velocity to average velocity.
1 * shear stress distribution
4 * drop of pressure for a given length.

The flow through the circular pipe will be viscous or laminar, if the Reynolds number (Re) is less than 2000.

$$Re = \frac{\rho V D}{\mu}$$

ρ - Density
V - velocity
D - Diameter
μ - viscosity



Following forces are acting on the fluid,

1. Pressure force on face AB $= p\pi r^2$
2. Pressure force on face CD $= \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$
3. Shear force, $= \tau \times 2\pi r \Delta x$ on the surface

Since, there is no acceleration, the summation of all forces in the direction of flow should be zero

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

$$\cancel{p\pi r^2} - \cancel{p\pi r^2} - \frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau 2\pi r \Delta x = 0$$

$\frac{\partial p}{\partial x} \leftarrow$ pressure gradient

$$\Rightarrow \Delta x \pi r \left[-\frac{\partial p}{\partial x} r - 2\tau\right] = 0$$

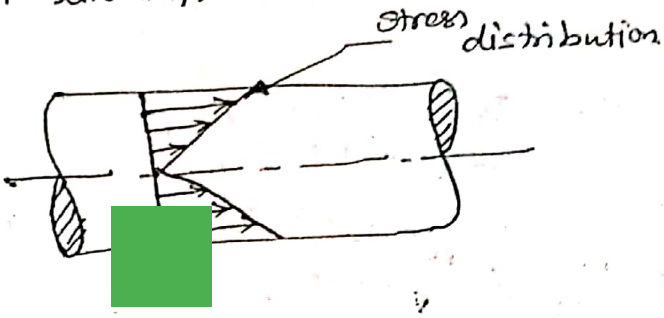
$$\Rightarrow -\frac{\partial p}{\partial x} r - 2\tau = 0$$

$$-2\tau = \frac{\partial p}{\partial x} r$$

$$\boxed{\tau = -\frac{\partial p}{\partial x}\frac{r}{2}} \quad \text{—— (1)}$$

Shear stress distribution

From the above equation, it can be seen that 'τ' varies with 'r' linearly, hence



Velocity distribution:

Velocity distribution across the section is obtained by substituting $\boxed{\tau = \mu \frac{du}{dy}}$ in equation (1)

Y is measured from the pipe wall,

$$y = R - r$$

then $dy = -dr$

$$\therefore \tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting the above in (1)

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu} \frac{\partial p}{\partial x} r$$

Integrating this above equation with respect to 'r'

$$\boxed{u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 + C} \qquad (2)$$

'C' in above equation is the constant of integration and the value is obtained from the boundary condition that $r = R$, $u = 0$

$$\therefore 0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + C$$

$$C = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

Substituting value of 'C' in (2)

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

$$\boxed{u = -\frac{1}{4\mu} \frac{\partial p}{\partial x} \left[R^2 - r^2\right]} \qquad (3)$$

(3)

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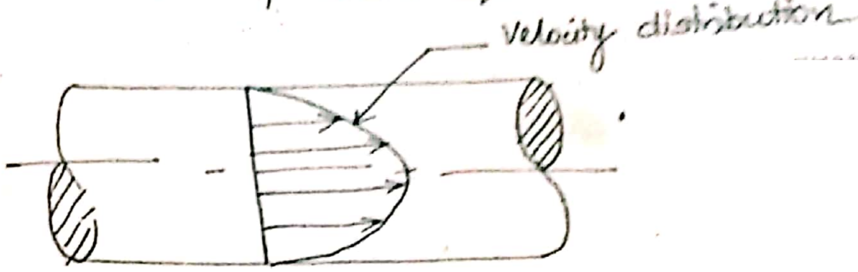
In Equation (3), μ, $\frac{\partial p}{\partial x}$ and R are constant, so velocity u varies with the square of r. Then, the equation may be considered as parabolic equation. Hence



Ratio of maximum velocity to average velocity:

The velocity is maximum, when $r = 0$ (from eq. 3).

Thus,

$$\boxed{u_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2} \longrightarrow (4)$$

$$\text{Average velocity} = \bar{u} = \frac{Q}{\pi R^2}$$

Considering the flow through a circular ring element of radius r and thickness dr. The fluid flowing per second through this elementary ring

$$dQ = \text{velocity at a radius } r \times \text{area of ring element}$$

$$= u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \times 2\pi r\, dr$$

$\pi R^2 - \pi r^2$

$\pi(R^2 - r^2)$

$\pi(R+r)(R-r)$

$\pi(R+r)\, dr$

$\pi(r+dr+r)\, dr$

$\pi(2r+dr)\, dr$

$2\pi r\, dr + \pi dr^2$

$= 2\pi r\, dr$ (0)

$$\because Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r\, dr$$

$$\Rightarrow \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2) r\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \int_0^R (R^2 r - r^3)\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

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$$\Rightarrow \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \left[\frac{4R^4 - 2R^4}{8}\right]$$

$$\Rightarrow \frac{1}{\cancel{4}_2\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \left[\frac{R^4}{4}\right]$$

$$\Rightarrow \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) \pi R^4$$

$$\boxed{Q \Rightarrow \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4} \longrightarrow (5)$$

Average velocity $\bar{u} = \frac{Q}{\text{Area}}$

$$= \frac{\frac{\cancel{\pi}}{8\mu}\left(\frac{-\partial p}{\partial x}\right) \cancel{R^4}}{\cancel{\pi} \cancel{R^2}}$$

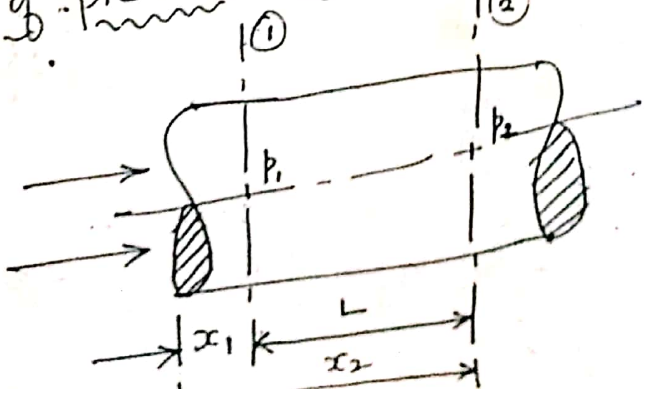
$$\boxed{\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2} \longrightarrow (6)$$

Team!

$$\frac{U_{max}}{\bar{u}} = \frac{\cancel{+}\frac{1}{\cancel{4\mu}}\frac{\partial p}{\partial x}\cancel{R^2}}{\cancel{+}\frac{1}{\cancel{8\mu}}\cancel{\left(\frac{\partial p}{\partial x}\right)}R^2} = 2$$

$$\boxed{\frac{U_{max}}{\bar{u}} = 2}$$

Drop of pressure for a given length (l) of a pipe:



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We know that

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$$

$$\frac{-\partial p}{\partial x} = \frac{8\mu\bar{u}}{R^2}$$

Integrating $\left(\frac{-\partial p}{\partial x}\right)$ w.r.t. x,

$$-\int_2^1 \partial p = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx$$

$$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}\int_2^1 dx$$

$$= \frac{8\mu\bar{u}}{R^2}\left[x\right]_2^1$$

$$= \frac{8\mu\bar{u}}{R^2}\left[x_1 - x_2\right]$$

$$p_1 - p_2 = \frac{8\mu\bar{u}}{R^2}\left[x_2 - x_1\right]$$

$$= \frac{8\mu\bar{u}}{R^2}L \qquad \left[\because x_2 - x_1 = L\right]$$

$$= \frac{8\mu\bar{u}}{\left(\frac{D}{2}\right)^2}L$$

$$(p_1 - p_2) = \frac{32\mu\bar{u}}{D^2}L \qquad (p_1 - p_2) = \text{drop of pressure}$$

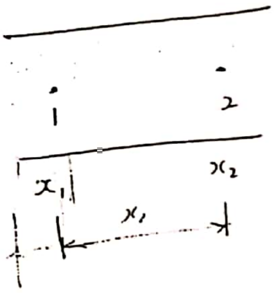
$\therefore$ Loss of pressure head $= \frac{p_1 - p_2}{\rho g} = h_f \quad [p = \rho g h]$

$$\therefore \boxed{h_f = \frac{32\mu\bar{u}L}{\rho g D^2}} \longrightarrow \text{Hagen Poiseuille Equation}$$

(b)

An oil of viscosity 0.1 $\frac{Ns}{m^2}$ and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300 m. The rate of flow through the pipe is 3.5 $\frac{litres}{sec}$. Find the pressure drop in a length of 300m and also the shear stress at the pipe wall. 36

Given data:



$\mu = 0.1 \frac{NS}{m^2}$

$S = 0.9$

$d = 50\ mm = 0.05\ m$

$l = 300\ m$

$Q = 3.5 \frac{l}{s} = 3.5 \times 10^{-3} \frac{m^3}{s}$

pressure drop $= (p_1 - p_2) = ?$

Shear stress $= \tau = ?$

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Solution:

$$\boxed{p_1 - p_2 = \frac{32 \mu \bar{u} L}{d^2}}$$

$$\bar{u} = \frac{Q}{Area} = \frac{3.5 \times 10^{-3}}{\frac{\pi}{4} \times (0.05)^2} = 1.78 \frac{m}{s}$$

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$$\therefore p_1 - p_2 = \frac{32 \times 0.1 \times 1.78 \times 300}{(0.05)^2} = 683.52 \times 10^3 \frac{N}{m^2}$$

$$\boxed{\tau = \frac{-\partial p}{\partial x} \frac{r}{2}} \qquad \boxed{r = R} \rightarrow \text{pipe}$$

$$\frac{-\partial p}{\partial x} = -\frac{(p_1 - p_2)}{x_2 - x_1} = \frac{p_2 - p_1}{x_2 - x_1} = \frac{p_2 - p_1}{L} = \frac{683.52 \times 10^3}{300}$$

$$= 2278.4 \frac{N}{m^3}$$

Team 2

$$\tau = 2278.4 \times \frac{0.05}{4} = 28.48 \frac{N}{m^2}$$

A fluid of viscosity $0.7 \frac{Ns}{m^2}$ and specific gravity 1.3 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as $196.2 \frac{N}{m^2}$, find (a) the pressure gradient (b) the average velocity and (c) Reynold's number of the flow.

Given data:

$\mu = 0.7 \frac{Ns}{m^2}$

$S = 1.3$

$d = 100\,mm = 0.1\,m$

$\tau = 196.2 \frac{N}{m^2}$

$\frac{\partial p}{\partial x} = ?$ $\bar{u} = ?$ $Re = ?$

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Team B

Solution:

$$\tau = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$$\frac{\partial p}{\partial x} = -\frac{\tau \times 2}{r} = -\frac{196.2 \times 2}{\frac{0.1}{2}} = -7848 \frac{N}{m^3}$$

$$\bar{u} = \frac{1}{8\mu} * \left(-\frac{\partial p}{\partial x}\right) R^2$$

$$= \frac{1}{8 \times 0.7} \times -(-7848) \times \left(\frac{0.1}{2}\right)^2$$

$$= 3.50 \frac{m}{s}$$

$$Re = \frac{\rho V d}{\mu} = \frac{1300 \times 3.50 \times 0.1}{0.7}$$

$$= 650$$

$S = \frac{\rho_l}{\rho_w}$

$\Rightarrow \rho_l = S \times \rho_w = 1.3 \times 1000$

$= 1300$