

$$\frac{\partial u}{\partial x} = 3x^2 - z^2$$

$$\frac{\partial v}{\partial y} = 3y^2$$

$$\frac{\partial w}{\partial z} = -3x^2 + \frac{1}{3} 3z^2 - 3yz$$

$$3x^2 - z^2 + 3y^2 - 3x^2 + \frac{1}{3} 3z^2 - 3yz = 0$$

Equation of motion: [Energy Equation]

Newton's second law =  $F = ma$

In fluid flow, the following forces are present.

- (i) Force due to Pressure,  $F_p$
- (ii) Force due to gravity,  $F_g$
- (iii) Force due to viscosity,  $F_\mu$
- (iv) Force due to compressibility,  $F_c$
- (v) Force due to turbulence,  $F_t$

$$\therefore \text{Net force} = \boxed{F_x = (F_p)_x + (F_g)_x + (F_\mu)_x + (F_c)_x + (F_t)_x}$$

Reynolds number equation of motion

It is ~~is~~ the force due to compressibility  $F_c$  is negligible.

$$\boxed{F_x = (F_p)_x + (F_g)_x + (F_\mu)_x + (F_t)_x}$$

Navier's Stoke equation of motion:  $F_t$  is negligible

$$\boxed{F_x = (F_p)_x + (F_g)_x + (F_\mu)_x + \cancel{(F_c)_x} + \cancel{(F_t)_x}}$$

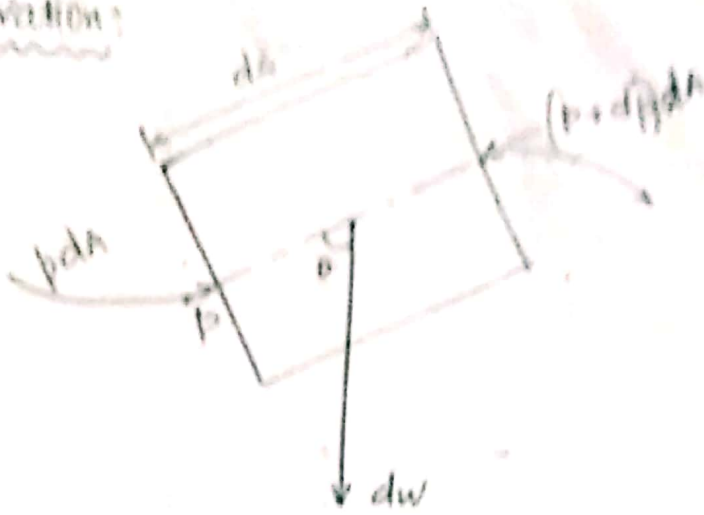
Euler's equation of motion:  $F_\mu$  is negligible

$$\boxed{F_x = (F_p)_x + (F_g)_x + \cancel{(F_\mu)_x} + \cancel{(F_c)_x} + \cancel{(F_t)_x}}$$

Euler's Equation of motion

$$\boxed{F_x = (F_p)_x + (F_g)_x}$$

Derivation:



ds - length  
 dA - cross-section  
 dp - pressure force  
 + weight/gravity force

i) Net pressure in the direction of flow,

$$\begin{aligned}
 &= p dA - (p + dp) dA \quad \text{[Pressure force]} \\
 &= p dA - p dA - dp dA \\
 &= -dp dA
 \end{aligned}$$

ii) Weight of the liquid acting along the direction of flow:

$$\begin{aligned}
 &= -dw \cos \alpha \\
 &= -\rho g dA ds \cos \alpha \\
 &= -\rho g dA ds \frac{dz}{ds} \\
 &= -\rho g dA dz
 \end{aligned}$$



[gravity force]

$$\text{Net force} = -dp dA - \rho g dA dz$$

iii) Mass of the fluid =  $\rho dA ds$

(iv) Acceleration of fluid =  $a = \frac{dv}{dt} = \frac{dv}{dt} \times \frac{ds}{ds}$

$$\begin{aligned}
 &= \frac{dv}{ds} \times \frac{ds}{dt} \\
 &= \frac{dv}{ds} \times v
 \end{aligned}$$

We know that,

$$F = m \times a$$

$$-dp dA - \rho g dA dz = \rho dA ds \times \frac{dv}{ds} \times v$$

Dividing throughout by  $\rho dA$ ,

$$\frac{-\rho dA dz}{\rho dA} - \frac{\rho g dA dz}{\rho dA} = \frac{\rho dA dz}{\rho dA} \times \frac{dv}{v}$$

$$-\frac{dp}{\rho} - g dz = v dv$$

$$\boxed{v dv + \frac{dp}{\rho} + g dz = 0} \rightarrow \text{Euler's equation}$$

$$\boxed{\frac{dp}{\rho} + g dz + v dv = 0} \rightarrow "$$

Bernoulli's equation from Euler's Equation

It is obtained by integrating Euler's equation,

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

$\rho$  is constant when flow is compressible,

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

Dividing 'g' throughout,

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}} \leftarrow \text{Bernoulli's equation}$$

$\frac{p}{\rho g}$  — pressure energy/weight

$\frac{v^2}{2g}$  — kinetic energy/weight

$z$  — potential energy

## Assumptions made in Bernoulli's equation

(10)

(21)

- (i) The fluid is ideal (no viscosity)
- (ii) The flow is steady and continuous (velocity is uniform)
- (iii) The flow is incompressible
- (iv) The flow is irrotational
- (v) All the frictional losses are neglected.

## Limitations:

- (i) Velocity assumed to be constant. This is not in actual case. Velocity is maximum at centre.
- (ii) Frictional losses will be there. eg. pipe friction
- (iii) Loss of energy in ~~the~~ are not considered.

## Statement of Bernoulli's theorem

It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant.

$$\text{Total energy} = [\text{Pressure energy}] + [\text{Kinetic energy}] + [\text{Potential energy}]$$

$$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}}$$

Energy Equation

- ① Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm<sup>2</sup> and with mean velocity of 2 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Given data

$$d = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

$$p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$$

$$v = 2 \text{ m/s}$$

$$z = 5 \text{ m}$$

(30)

(36)

$$E = \frac{p}{\rho g} + \frac{v^2}{2g} + z$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} + 5$$

$$E = 35.204 \text{ m}$$

- ② Water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm<sup>2</sup>, find the intensity of pressure at section 2.

Given data:

$$d_1 = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$d_2 = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$

$$Q = 35 \text{ l/s} = 35 \times 10^{-3} \text{ m}^3/\text{s}$$

$$z_1 = 6 \text{ m}$$

$$z_2 = 4 \text{ m}$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$p_2 = ?$$

Solution:

$$Q = A_1 v_1 = v_2 A_2 = 35 \times 10^{-3}$$

$$v_1 = \frac{35 \times 10^{-3}}{\frac{\pi}{4} \times d_1^2} = 1.114 \text{ m/s}$$

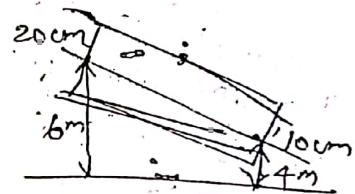
Similarly,

$$v_2 = 4.46 \text{ m/s}$$

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

(31)



(37)

$$\frac{39.24 \times 10^4}{9.81 \times 1000} + \frac{1.114^2}{2 \times 9.81} + 6 = \frac{p_2}{1000 \times 9.81} + \frac{4.46^2}{2 \times 9.81} + 4$$

$$46.063 = \frac{p_2}{9810} + 5.014$$

~~$$p_2 = 451.67 \times 10^3 \text{ N/m}^2$$~~

$$p_2 = 402.69 \times 10^3 \text{ N/m}^2$$

$$= 402.7 \text{ N/cm}^2$$

③ Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm<sup>2</sup> and the pressure at the upper end is 9.81 N/cm<sup>2</sup>. Determine the difference in datum head if the rate of flow through pipe is 40 litres/s.

Given data:

$$d_1 = 300 \text{ mm} = 300 \times 10^{-3} \text{ m}$$

$$d_2 = 200 \text{ mm} = 200 \times 10^{-3} \text{ m}$$

$$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$$

$$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$$

$$Q = 40 \text{ l/s} = 40 \times 10^{-3} \text{ m}^3/\text{s}$$

Solution

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{40 \times 10^{-3}}{\frac{\pi}{4} (300 \times 10^{-3})^2} = 0.57 \text{ m/sec}$$

$$\therefore V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4} (300 \times 10^{-3})^2 \times 0.57}{\frac{\pi}{4} (200 \times 10^{-3})^2} = 1.28 \text{ m/sec}$$

Now,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$z_1 - z_2 = \frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} = \frac{24.525 - 9.81}{1000 \times 9.81} + \frac{(0.57)^2 - (1.28)^2}{2 \times 9.81}$$

$$= \frac{14.715}{9810} - \frac{0.97}{19.62} = 1.5 \times 10^{-3} - 0.0494 = -0.0479 \text{ m}$$

$$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{0.57^2}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

$$25 + 0.017 + z_1 = 10 + 0.083 + z_2$$

$$z_1 - z_2 = 25.017 - 10.083$$

$$= 14.934 \text{ m}$$

- ③ A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm<sup>2</sup> and 22.563 N/cm<sup>2</sup> respectively while the datum head at A and B as 28m and 30m. Find the loss of head between A and B.

Given data:

$$d = 400 \text{ mm} = 400 \times 10^{-3} \text{ m}$$

$$V = 25 \text{ m/s}$$

$$p_1 = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$$

$$p_2 = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$$

$$z_1 = 28 \text{ m}$$

$$z_2 = 30 \text{ m}$$

Solution:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = E$$

$$E_A = \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28 = 30 + 31.86 + 28 = 89.86 \text{ m}$$

$$E_B = \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.86 + 30 = 84.86 \text{ m}$$

$$E_A - E_B = 89.86 - 84.86 = 5 \text{ m}$$

33

Team 5

(A) A pipe line carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4 metres at a higher level. If the pressures at A & B are 9.81 N/cm<sup>2</sup> and 5.886 N/cm<sup>2</sup> respectively and the discharge is 200 litres/sec. Determine the loss of head and direction of flow. (16)

Given data

- $S_o = 0.87$
- $d_A = 200 \text{ mm} = 200 \times 10^{-3} \text{ m}$
- $d_B = 500 \text{ mm} = 500 \times 10^{-3} \text{ m}$
- $Z_A = 0$
- $Z_B = 4 \text{ m}$
- $p_A = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$
- $p_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$
- $Q = 200 \text{ litres/s} = 200 \times 10^{-3} \text{ m}^3/\text{sec}$

Loss of head? ( $h_L$ )

Solution:

$$Q = A_A V_A$$

$$V_A = \frac{Q}{A_A} = \frac{200 \times 10^{-3}}{\frac{\pi}{4} (200 \times 10^{-3})^2} = 6.37 \text{ m/s}$$

$$A_A V_A = A_B V_B$$

$$V_B = \frac{A_A V_A}{A_B} = \frac{(200 \times 10^{-3})^2}{(500 \times 10^{-3})^2} \times 6.37 = 1.02 \text{ m/sec}$$

$$E_A = \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.37)^2}{2 \times 9.81} + 0 = 11.49 + 2.07 + 0 = 13.56 \text{ m}$$

$$E_B = \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{(1.02)^2}{2 \times 9.81} + 4 = 6.90 + 0.05 + 4 = 10.95 \text{ m}$$

$h_L = E_A - E_B = 2.61 \text{ m}$ , Direction of flow =  $E_A > E_B$ , so

(F) A to B.

$E_A = E_B + h_L$



# Practical applications of Bernoulli's Equation

1. Venturimeter — Q
2. Orifice meter — Q
3. Pitot-tube. — V

## Venturimeter:

Venturimeter is a device used for measuring the rate of a flow of a fluid ~~flow~~ flowing through a pipe. By simple observations on pressure, it is easy to find out.

## Convergent Cone:

- \* Short conical pipe =  $l = 2.5 D_1$
- \* Convergent angle =  $15^\circ$  to  $20^\circ$
- \* Converge from  $d_1$  to  $d_2$

## Throat:

- \* Length = Diameter of itself =  $d_2$
- \* Connects convergent and divergent cone

## Divergent cone:

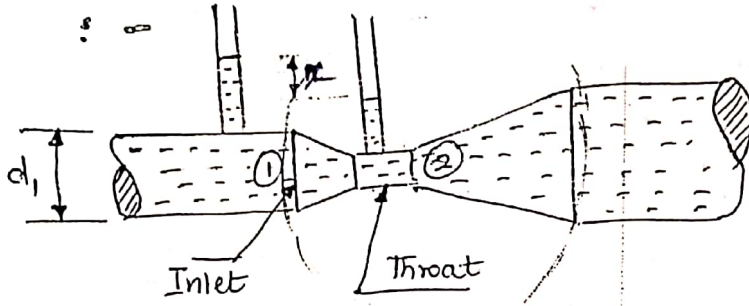
- \* Length =  $7 D_1$  (minimise the loss of energy)
- \* Angle =  $5^\circ$  to  $7^\circ$
- \* Diverges from  $d_2$  to  $d_1$

## Overall throat ratio:

$$\frac{d_2}{d_1} = \frac{1}{3} \text{ to } \frac{1}{2}$$

Expression for rate of flow through venturimeter

(48)



Let,

$d_1, d_2$  = diameter at inlet or at section (1), (2)

$p_1, p_2$  = pressure at section (1), (2)

$v_1, v_2$  = Velocity of fluid at section (1), (2)

$a_1, a_2$  = area at section (1), (2)

Applying Bernoulli's equation at (1) & (2)

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal,  $[z_1 = z_2]$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

We know that  $h = \frac{p}{\rho g} = \frac{p_1 - p_2}{\rho g}$ ,

$$\therefore h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

By continuity equation,

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

(42)

Applying value of  $v_1$ ,

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g}$$

$$= \frac{v_2^2}{2g} - \frac{a_2^2 v_2^2}{2g a_1^2}$$

$$= \frac{v_2^2}{2g} \left[ 1 - \frac{a_2^2}{a_1^2} \right]$$

$$h = \frac{v_2^2}{2g} \left[ \frac{a_1^2 - a_2^2}{a_1^2} \right]$$

$$v_2^2 = 2gh \left[ \frac{a_1^2}{a_1^2 - a_2^2} \right]$$

$$v_2 = \sqrt{2gh \left[ \frac{a_1^2}{a_1^2 - a_2^2} \right]}$$

$$v_2 = \sqrt{2gh} \frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

We know that  $Q = a_2 v_2$

$$= a_2 \sqrt{2gh} \frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

$$Q_{theo} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

Actual discharge:

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$C_d$  - Co-efficient of venturiment  $< 1$

Practical discharge

A. Measurement

M. Measurement

Method

1. U-tube manometer

2. U-tube manometer

3. Pitot-static probe

§ If the differential manometer used to measure the pressure and it contains a liquid which is heavier than the liquid flowing through pipe, then

$$h = x \left[ \frac{S_h}{S_o} - 1 \right]$$

$S_h$  - Specific gravity of heavier liquid  
 $S_o$  - " " " " liquid flowing through pipe  
 $x$  - difference of height of the heavier liquid in manometer

If the same liquid lighter, then

$$h = x \left[ 1 - \frac{S_l}{S_o} \right]$$

$S_l$  - Specific gravity of lighter liquid.

Inclined Venturimeter & Vertical Venturimeter

$$h = \left[ \frac{p_1 + \cancel{z_1}}{\cancel{\rho g}} \right] - \left[ \frac{p_2 + z_2}{\cancel{\rho g}} \right] = x \left[ \frac{S_h}{S_o} - 1 \right] = x \left[ 1 - \frac{S_l}{S_o} \right]$$

An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter.  $C_d = 0.98$

Given data:

- $S_o = 0.8$
- $d_1 = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$
- $d_2 = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$
- $x = 25 \text{ cm} = 25 \times 10^{-2} \text{ m}$
- $C_d = 0.98$

Solution:

$$h = x \left[ \frac{S_h}{S_o} - 1 \right] = 25 \times 10^{-2} \left[ \frac{13.6}{0.8} - 1 \right]$$

$$= 4 \text{ m}$$

$$Q = C_d \times a_1 a_2 \sqrt{2gh}$$

$$= 0.98 \times \frac{\pi}{4} (20 \times 10^{-2})^2 \times \frac{\pi}{4} (10 \times 10^{-2})^2$$

$$a_1 = \frac{\pi}{4} \times (20 \times 10^{-2})^2 = 0.03 \text{ m}^2$$

$$a_2 = \frac{\pi}{4} \times (10 \times 10^{-2})^2 = 0.008 \text{ m}^2$$

$$\therefore = 0.98 \times \frac{31.42 \times 10^{-3} \times 7.65 \times 10^{-3}}{0.03 \times 0.008} \times \sqrt{2 \times 9.81 \times 4}$$

$$= 0.98 \times \frac{2.46 \times 10^{-4}}{0.024} \times 8.86$$

$$= 0.98 \times 0.029 \times 8.86$$

$$= 0.45 \text{ m}^3/\text{sec} = 0.07365 \frac{\text{m}^3}{\text{s}} = 73.65 \frac{\text{l}}{\text{s}}$$

A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is  $17.658 \text{ N/cm}^2$  and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter in litres/s

Take  $C_d = 0.98$

Given data:

$$d_1 = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$d_2 = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$

$$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$$

$$p_2 = 30 \text{ cm of mercury (vacuum)} = -0.3 \text{ m of mercury}$$

$$C_d = 0.98$$

Solution:

$$h_w S_w = h_m S_m$$

$$h_w = \frac{h_m S_m}{S_w} = \frac{0.3}{13.6} \times 13.6 = -4.08 \text{ m of water}$$

$$\frac{V_2^2}{2g} \left[ 1 - \frac{0.8}{16} \right] - 0.12 = 0$$

$$\frac{V_2^2}{2g} [0.95] = 0.12$$

$$V_2 = \sqrt{\frac{2 \times 9.81 \times 0.12}{0.95}} = 1.57 \text{ m/s}$$

$$Q = a_2 V_2$$

$$= \frac{\pi}{4} (0.15)^2 \times 1.57 = 0.0278 \text{ m}^3/\text{sec}$$

$$= 0.0285 \text{ m}^3/\text{sec} = 27.82 \text{ litres/sec.}$$

$$= 28.54 \text{ litres/sec.}$$

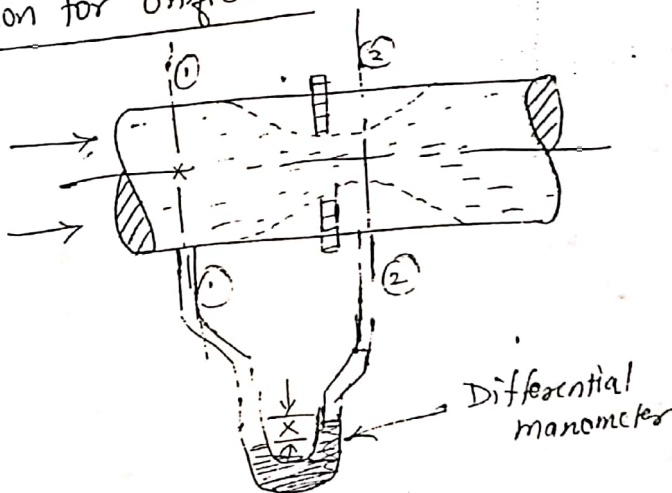
Orifice meter (or) Orifice plate:

It is a device used for measuring the rate of flow of a fluid through a pipe.

Works on same principle as that of venturimeter. Cheaper device. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with pipe. Loss of head is min.

$$d_0 = 0.5d_1 \quad (0.4 \text{ to } 0.8 \text{ times})$$

Expression for orifice meter:



An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp gr 0.9 when the coefficient of discharge of the meter = 0.64.

Given data:

- $d_o = 15 \text{ cm} = 0.15 \text{ m}$
- $d_i = 30 \text{ cm} = 0.3 \text{ m}$
- $x_h = 50 \text{ cm} = 0.5 \text{ m}$
- $S_o = 0.9$
- $C_d = 0.64$
- $Q = ?$

Solution:

$$Q = C_d \times \frac{a_1 a_0}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$a_0 = \frac{\pi}{4} \times 0.15^2 = 0.0177 \text{ m}^2$$

$$a_1 = \frac{\pi}{4} \times (0.3)^2 = 0.0707 \text{ m}^2$$

$$h = x \left[ \frac{S_h}{S_o} - 1 \right] = 0.5 \left[ \frac{13.6}{0.9} - 1 \right] = 7.06 \text{ m of oil}$$

$$Q = 0.64 \times \frac{0.0707 \times 0.0177}{\sqrt{(0.0707)^2 - (0.0177)^2}} \times \sqrt{2 \times 9.81 \times 7.06}$$

$$= 0.64 \times \frac{1.25 \times 10^{-3} \times 11.769}{0.0685}$$

$$= 0.137 \text{ m}^3/\text{sec} = 137.45 \text{ litres/sec,}$$

Pitot-tube:

It is device used to measure the velocity of flow at any point in a pipe or a channel.



Principle:

If the velocity at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy.



$p_1$  = intensity of pressure at ①  
 $p_2$  = " " at ②  
 $v_1, v_2$  = velocity at ① & ②  
 $H$  = depth of tube in liquid  
 $h$  = rise of liquid above free surface

Expression:

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad \text{--- (1)}$$

$z_1 = z_2 = 0$  and  $v_2 = 0$  and  $H = \frac{p_1}{\rho g}$ ,  $h = \frac{p_2}{\rho g}$

ie pressure head at ① =  $H = \frac{p_1}{\rho g}$   
 " " " ② =  $(h+H) = \frac{p_2}{\rho g}$

Substituting above in (1),

$$H + \frac{v_1^2}{2g} = h + H$$

$$h = \frac{v_1^2}{2g} + H - H$$

$$h = \frac{v_1^2}{2g}$$

$$v_1 = \sqrt{2gh}$$

Theoretical velocity

Actual velocity  $v_1 = C_v \sqrt{2gh}$

$C_v$  - Coefficient of pitot-tube

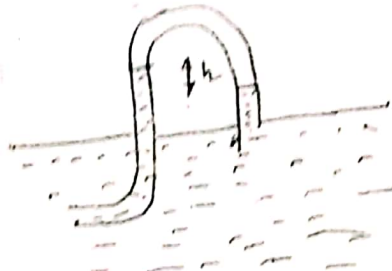


# Velocity of flow in a pipe:

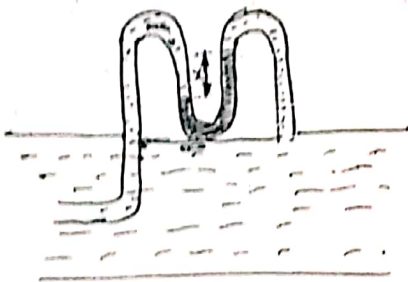
Above can be measure by following (18)



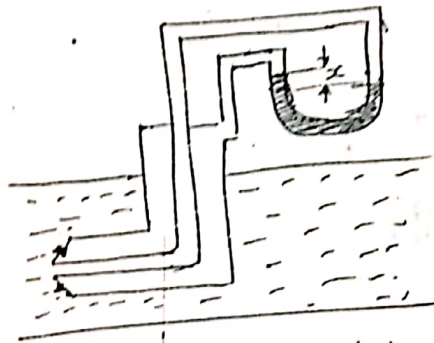
Pitot tube + Piezometer



Pitot tube connected with piezometer



Pitot tube connected with piezometer and U-tube manometer



Pitot static tube

A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other  $\perp$  to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if pressure difference between the two orifices is 60 mm of water. take  $C_v = 0.98$ .

Given data:

$$d = 300 \text{ mm} = 0.3 \text{ m}$$

$$\text{Mean velocity} = 0.80 \text{ of central velocity}$$

$$h = 60 \text{ mm} = 0.06 \text{ m}$$

$$C_v = 0.98$$

$$Q = ?$$

Solution:

$$V = C_v \sqrt{2gh} \quad \& \quad Q = AV_m$$

$$V = 0.98 \sqrt{2 \times 9.81 \times 0.06} = 1.063 \text{ m/s}$$

$$\therefore \text{Mean velocity} = 0.8 \times 1.063 = 0.851 \text{ m/s.}$$

~~30~~ 40

(52)

$$\begin{aligned}
 \therefore Q &= A V_{\text{mean}} \\
 &= \frac{\pi (0.3)^2 \times 0.851}{4} \\
 &= 0.060 \text{ m}^3/\text{sec} \\
 &= 60.154 \text{ litres/sec}
 \end{aligned}$$

A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the submarine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the submarine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh water.

Given data:

$$x = 170 \text{ mm} = 0.17 \text{ m}$$

$$S_m = 13.6$$

$$S_o = 1.026$$

$$V = ?$$

Solution

$$V = C_v \sqrt{2gh}$$

$$h = x \left[ \frac{S_m}{S_o} - 1 \right] = 0.17 \times \left[ \frac{13.6}{1.026} - 1 \right]$$

$$= 2.08 \text{ m}$$

$$V = \sqrt{2 \times 9.81 \times 2.08} = 6.39 \text{ m/s}$$

$$= 6.39 \times 60 \times 60 = 23016.512 \text{ m/hr}$$

$$= 23.01 \text{ km/hr}$$

Note:

Static pressure = pressure in pipe  $[H]$   
 Stagnation pressure = pressure at pitot tube  $[H+h]$

$$h = \text{Stagnation pressure} - \text{Static pressure}$$

$$= H+h - H$$

Impulse-momentum equation

College of Engineering  
University of Baghdad

90

Based on the law of conservation of momentum

or on the momentum principle.

It states that the net force acting on fluid mass is equal to the change in momentum of flow per unit time in that direction.

$$F = m \times a$$

$$F = m \times \frac{dv}{dt}$$

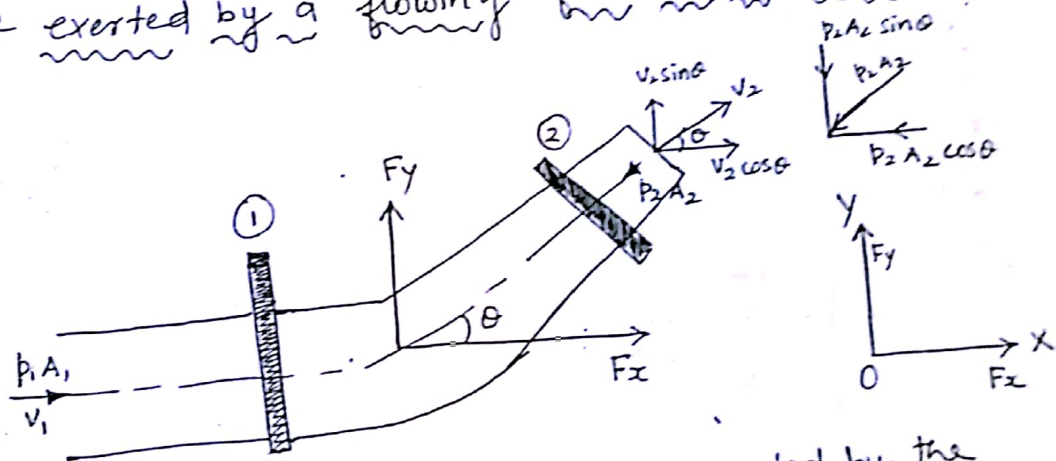
$$F = \frac{d(mv)}{dt}$$

Momentum principle

$$F \cdot dt = d(mv)$$

Impulse-momentum equation (used to find the resultant force exerted by a flowing fluid on a pipe bend)

Force exerted by a flowing fluid on a pipe bend:



$F_x, F_y$  - Components of the forces exerted by the flowing fluid on the bend in x and y directions respectively.

Force by the bend =  $-F_x, -F_y$   
 Other forces =  $p_1 A_1$  and  $p_2 A_2$

54

Momentum equation in x-direction,

Net force acting on fluid = rate of change of momentum in x-direction

$$p_1 A_1 - p_2 A_2 \cos \theta - F_x = \rho Q \left( \frac{\text{mass}}{\text{sec}} \right) \times (V_2 \cos \theta - V_1)$$

change in vel.

~~$F_x = \rho Q (V_2 \cos \theta - V_1)$~~

$$F_x = \rho Q (V_1 - V_2 \cos \theta) + p_1 A_1 - p_2 A_2 \cos \theta \quad \text{--- (1)}$$

Similarly in y-direction

$$0 - p_2 A_2 \sin \theta - F_y = \rho Q (V_2 \sin \theta - 0)$$

$$F_y = \rho Q (-V_2 \sin \theta) - p_2 A_2 \sin \theta \quad \text{--- (2)}$$

Resultant force ( $F_R$ ) acting on bend

$$F_R = \sqrt{F_x^2 + F_y^2}$$

Angle made by resultant force with horizontal direction:

$$\tan \theta = \frac{F_y}{F_x}$$

A  $45^\circ$  reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is  $8.829 \text{ N/cm}^2$  and rate of flow of water is 600 litres/s.

Given data:

$$\theta = 45^\circ$$

$$d_1 = 600 \text{ mm} = 0.6 \text{ m}$$

$$d_2 = 300 \text{ mm} = 0.3 \text{ m}$$

$$p_1 = 8.829 \text{ N/cm}^2 = 8.829 \times 10^4 \text{ N/m}^2$$

$$Q = 600 \text{ litres/s} = 600 \times 10^{-3} \text{ m}^3/\text{s}$$

(41)

Solution:

(19)

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$F_x = \rho g (V_1 - V_2 \cos \theta) + p_1 A_1 - p_2 A_2 \cos \theta$$

$$F_y = \rho g (-V_2 \sin \theta) - p_2 A_2 \sin \theta$$

$$Q = a_1 V_1$$

$$V_1 = \frac{Q}{a_1} = \frac{600 \times 10^{-3}}{\frac{\pi}{4} \times (0.6)^2} = 2.122 \text{ m/s}$$

$$V_2 = \frac{Q}{a_2} = \frac{600 \times 10^{-3}}{\frac{\pi}{4} \times (0.3)^2} = 8.488 \text{ m/s}$$

Applying Bernoulli's equation,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$z_1 = z_2$$

$$\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{(2.122)^2}{2 \times 9.81} = \frac{p_2}{1000 \times 9.81} + \frac{(8.488)^2}{2 \times 9.81}$$

$$9 + 0.23 = \frac{p_2}{1000 \times 9.81} + 3.672$$

$$(9.023 - 3.672) \times 1000 \times 9.81 = p_2$$

$$\therefore p_2 = 52.49 \times 10^3 \text{ N/m}^2$$

Now,

$$\begin{aligned} F_{xc} &= 1000 \times 600 \times 10^{-3} (2.122 - 8.488 \times \cos 45^\circ) \\ &\quad + (8.829 \times 10^4 \times \frac{\pi}{4} \times (0.6)^2) - (52.49 \times 10^3 \times \frac{\pi}{4} \times (0.3)^2 \cos 45^\circ) \\ &= -2327.95 + 24963.40 - 2623.57 \\ &= 20011.87 \text{ N} \end{aligned}$$

$$F_y = 1000 \times 600 \times 10^{-3} \left[ -8.488 \sin 45^\circ \right] - \left[ 5.45 \times 10^{-4} \times \frac{\pi}{4} (0.3)^2 \sin 45^\circ \right]$$

$$= -3601.15 - 2724.04$$

$$= -6325.19 \text{ N,}$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(20011.87)^2 + (6325.19)^2}$$

$$F_R = 20987.69 \text{ N,}$$

$$\tan \theta = \frac{F_x}{F_y}$$

$$\therefore \theta = \tan^{-1} \left( \frac{F_x}{F_y} \right) = \tan^{-1} \left( \frac{6325.19}{20011.87} \right)$$

$$= 17^\circ 32' //$$

Moment of momentum equation:

Derived from ~~moment of momentum principle~~ <sup>it</sup>, states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

Derivation:

$r_1$  &  $r_2$  - radius of curvature

$$\text{Momentum of fluid at section (1)} = \text{mass} \times \text{velocity} \\ = \rho Q \times v_1$$

$$\text{Moment of momentum} = \rho Q \times v_1 \times r_1$$

$$\text{Similarly for section (2)} = \rho Q \times v_2 \times r_2$$

$$\therefore \text{Rate of change of momentum} = \rho Q (v_2 r_2 - v_1 r_1)$$

$$\therefore \text{Resultant torque } T = \rho Q (v_2 r_2 - v_1 r_1)$$

$T = \rho Q (V_2 r_2 - V_1 r_1)$  moment of momentum equation

(66)

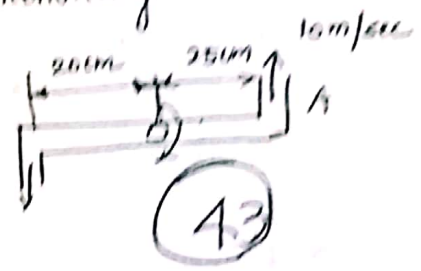
Applications:

- (1) For analysis flow problems in turbines & centrifugal pumps
- (2) for finding torque exerted by water on sprinklers

A lawn sprinkler shown in fig has 0.8 cm diameter nozzle at the end of a rotating arm and discharges water at the rate  $q$ , 10 m/s velocity. Determine the torque required to hold the rotating arm stationary.

Given data:

$d = 0.8 \text{ cm} = 0.8 \times 10^{-2} \text{ m}$   
 $V = 10 \text{ m/s}$   
 $T = ?$   
 $r_2 = 0.25 \text{ m}$   
 $r_1 = 0.20 \text{ m}$



Solution

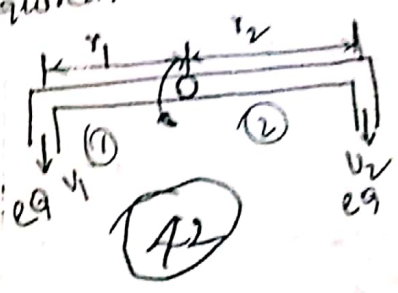
$T = \rho Q (V_2 r_2 + V_1 r_1)$  Same direction  
 $Q = AV = \frac{\pi}{4} (0.8 \times 10^{-2})^2 \times 10$   
 $= 5.027 \times 10^{-4} \text{ m}^3/\text{s}$

$T = 1000 \times 5.027 \times 10^{-4} (10(0.25) + 10(0.2))$   
 $= 2.26215 \text{ Nm}$

A lawn sprinkler with two nozzles of diameter 4 mm each is connected across a tap of water as shown in fig. The nozzles are at a distance of 30 cm and 20 cm from the centre of the tap. The rate of flow of water through tap is  $120 \text{ cm}^3/\text{s}$ . The nozzles discharge water in the downward direction. Determine the torque required.

Given data:

$d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$   
 $r_2 = 30 \text{ cm} = 30 \times 10^{-2} \text{ m} = 0.3 \text{ m}$   
 $r_1 = 20 \text{ cm} = 0.2 \text{ m}$   
 $Q = 120 \text{ cm}^3/\text{s} = 120 \times 10^{-6} \text{ m}^3/\text{s}$



→ (58)

Solution:

$$T = \rho Q (v_2 r_2 - v_1 r_1)$$
$$= 1000 \times 120 \times 10^{-6} (v_2 (0.2) - v_1 (0.2))$$

$$Q = a v$$

$$v = \frac{Q}{a} = \frac{120 \times 10^{-6}}{\frac{\pi}{4} \times (4 \times 10^{-3})^2} = \frac{120 \times 10^{-6}}{1.26 \times 10^{-5}}$$

$$= 9.55 \text{ m/s}$$

$$\therefore T = 1000 \times 120 \times 10^{-6} \times 9.55 (0.1)$$
$$= 0.1146 \text{ Nm,}$$