FLUID FLOW :

Fluid flow is described by two methods

(1) Lagrangian Method - Single particle is followed in flow

(2) Eulerian Method. — At a point, all is measured.

Eulerian method is mostly used.

Fluid mechanics

Study about the velocity at any point of flow field at any given time. Velocity is known, then pressure distribution, forces can be determined.

Types of fluid flow

(i) steady and unsteady flow

(ii) Uniform and non-uniform flow

(iii) Laminar and turbulent flow

(iv) Compressible and incompressible flow

(v) Rotational and irrotational flow

(vi), One, two and three - dimensional flow.

Steady and Unsteady flow :

→ Flow through prismatic conduit

→ Type of flow in which fluid characteristics at a point do not change with time. $\left(\frac{\partial v}{\partial t}\right)_{x=0,y=0,z=0} = 0$, $\left(\frac{\partial p}{\partial t}\right)_{x=0,y=0,z=0} = 0$, $\left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0$

In unsteady flow, it changes with time $\neq 0$ ← Valve opened + closed gradually

Uniform and non-uniform flow:

→ straight prismatic conduit

Uniform flow, is a type of flow in which velocity at any given time does not change with respect to space (length of direction of the flow). $\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}} = 0$

Non-uniform flow, is that type of flow in which the velocity at any given time changes with respect to space. $\neq 0$ ← Non-prismatic conduit and pipe with bends

Laminar and turbulent flows

→ In this flow, fluid particles move along well-defined paths (or) stream-lines and all the stream-lines are straight and parallel. Also known as stream-line flow or viscous flow. ← blood in veins, ground water flow

Turbulent flow, fluid particles move is zig-zag way. Due to this, eddies are formed, which result in high energy loss. ← almost all (High velocity flow)

If the Reynold's number (non-dimensional number) - $\frac{VD}{\nu}$ < 2000, flow is laminar; > 4000, flow is turbulent. Whereas. V is mean velocity, D - diameter of pipe and ν - kinematic viscosity.

Compressible and incompressible flow:

In compressible flow, the density of the fluid changes from point to point, i.e density is not constant. ← flow of gases through orifices, nozzles, gas turbines

$$\rho \neq \text{constant}$$

In, incompressibe flow, density is constant.

$$\rho = \text{constant}$$ ← Subsonic aerodynamics

Liquids - incompressible
Gases - Compressible

Rotational and irrotational flow $\boxed{\text{Vorticity} = \frac{\text{circulation}}{\text{unit area}}}$

Rotational, when flowing in stream lines, it also rotates its own axis. flow below boundary layer, river

If, it is not rotating, it is known as irrotational flow. irrotational + steady = potential flow. Ex, flow above wash basin!!!

One, two and three dimensional flows:

One dimensional flow, simply velocity changes in one direction only. $u = f(x)$; $v = 0$; $w = 0$

Two dimensional flow, velocity changes in two directions, i.e $u = f_1(x,y)$; $v = f_2(x,y)$; $w = 0$

Three dimensional flow, velocity changes in all directions, $u = f_1(x,y,z)$; $v = f_2(x,y,z)$; $w = f_3(x,y,z)$

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Rate of flow or Discharge (Q)

Defined as quantity of a fluid flowing per second through a section of a pipe or a channel.

For liquids $Q = m^3/s$

For gases $Q = N/s$

$$Q = A \times V$$

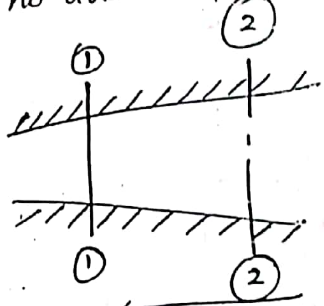
A – area of cross-section of pipe

V = velocity (mean) of fluid across the section.

Continuity equation :

Based on principal of conservation of mass.

It states that for a fluid flowing through the pipe at all the cross section, the quantity of fluid per second is constant, if there is no addition or subtraction.



$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

↳ Continuity equation (applicable both for compressible + incompressible)

Note: if flow is incompressible, $\rho_1 = \rho_2$

So $\boxed{A_1 V_1 = A_2 V_2}$

$\boxed{\rho_1 A_1 V_1} \longrightarrow$ mass rate of flow.

Limitations

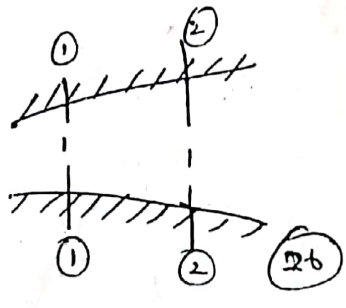
Used only when,

(i) flow is steady & continuous

(ii) ~~flow is incompressible~~

(iii) flow is 1D

① The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.



Given data

$d_1 = 10\text{ cm} = 10 \times 10^{-2}\text{ m}$

$d_2 = 15\text{ cm} = 15 \times 10^{-2}\text{ m}$

$V_1 = 5\text{ m/s}$

$Q = ?\quad V_2 = ?$

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Solution:

Using, continuity equation, for incompressible flow,

$$Q = A_1 V_1 = A_2 V_2$$

$$Q = A_1 V_1$$

$$= \frac{\pi}{4} (10 \times 10^{-2})^2 \times 5 = 0.0392 \ m^3/s$$

$$A_1 V_1 = A_2 V_2$$

$$0.0392 = \frac{\pi}{4} (15 \times 10^{-2})^2 \times V_2$$

Team 1

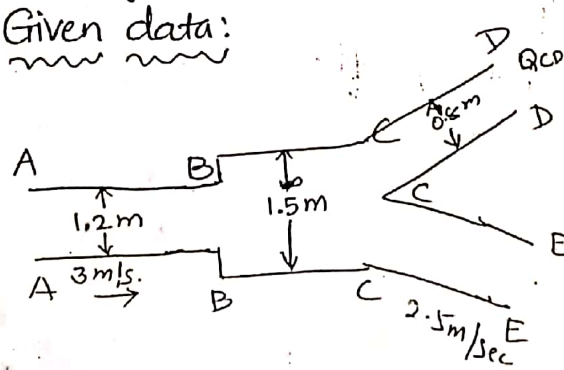
$$V_2 = 2.22 \ m/s.$$

$$\boxed{\begin{aligned} Q &= 0.0392 \ m^3/s \\ V_2 &= 2.22 \ m/s \end{aligned}}$$

(2) Water flows through a pipe AB 1.2m diameter at 3m/s and then passes through a pipe BC 1.5m diameter. At C, the pipe branches. Branch CD is 0.8m in diameter and carries one third of flow in AB. The flow velocity in branch CE is 2.5m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

Given data:

(26)



$Q_{AB} = ?$
$V_{BC} = ?$
$V_{CD} = ?$
$d_{CE} = ?$

$d_{AB} = 1.2m$

$V_{AB} = 3m/s,$

$Q_{CD} = \frac{1}{3} Q$

$d_{CD} = 0.8m$

$V_{CE} = 2.5 \ m/s$

(51)

Solution:

By Continuity equation,

$$Q = A_1 V_1 = A_2 V_2$$

$$Q_{AB} = A_{AB} V_{AB} = \frac{\pi}{4}(1.2)^2 \times 3 = 3.39 \, m^3/s$$

$$A_{AB} V_{AB} = A_{BC} V_{BC}$$

$$\therefore V_{BC} = \frac{A_{AB} V_{AB}}{A_{BC}} = \frac{3.39}{\frac{\pi}{4}(1.5)^2} = 1.92 \, m/s$$

$$Q_{CD} = \frac{1}{3} Q_{AB}$$

$$= \frac{1}{3} \times 3.39 = 1.13 \, m^3/sec$$

$$Q_{CD} = A_{CD} \times V_{CD}$$

$$V_{CD} = \frac{Q_{CD}}{A_{CD}} = \frac{1.13}{\frac{\pi}{4} \times (0.8)^2} = 2.25 \, m/s$$

$$Q_{CE} = Q_{AB} - Q_{CD}$$

$$= 3.39 - 1.13$$

$$= 2.26 \, m^3/s$$

$$Q_{CE} = A_{CE} \times V_{CE}$$

$$A_{CE} = \frac{Q_{CE}}{V_{CE}}$$

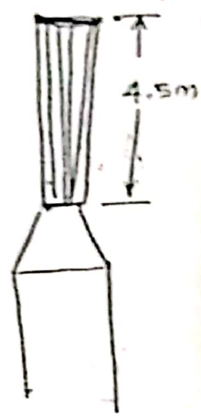
$$\frac{\pi}{4} d^2 = \frac{2.26}{2.5}$$

$$\therefore d = \sqrt{\frac{2.26 \times 4}{\pi \times 2.5}}$$

$$= \cancel{0.038\,m} \; 1.07 \, m$$

Team3

③ A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, ~~that~~ what will be the diameter at a point 4.5m above the nozzle, if the velocity with which the jet leaves the nozzle is 12 m/s. (35)



Given data:

$d_n = 25\ mm = 25 \times 10^{-3}\ m$

$V_1 = 12\ m/s$

$h = 4.5m$

Solution:

Let velocity at the point 4.5m as V_2.

By using the equation of motion,

$$V_2^2 - V_1^2 = 2gh \qquad [V^2 = u^2 + 2as]$$

$$V_2^2 = 2gh + V_1^2$$

$$V_2 = \sqrt{2gh + V_1^2}$$

$$= \sqrt{2(-9.81)\,4.5 + (12)^2}$$

$$V_2 = 7.46\ m/s$$

$S = ut + \frac{1}{2}at^2$

$v = u + at$

By Continuity equation,

$$Q = A_1 V_1 = A_2 V_2$$

Team 4

$$\frac{\pi}{4}(d_n)^2 \times 12 = \frac{\pi}{4}(d_p)^2 \times 7.46$$

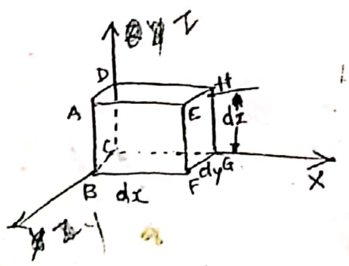
$$d_p^2 = \frac{\cancel{\pi} \times (25\times10^{-3})^2 \times 12 \times \cancel{4}}{\cancel{4} \times \cancel{\pi} \times 7.46}$$

$$d_p = \sqrt{1.00 \times 10^{-3}}$$

$$= 0.0317\ m$$

$$d_p = 31.71\ mm$$

Continuity equation in Three - dimension:



Let dx, dy, dz lengths of a fluid element in the direction of x, y, z.

Let u, v, w are inlet velocity components in x, y and z directions.

Mass of fluid entering the face ABCD per second

$= \rho \times$ velocity in x-direction $\times$ area of ABCD

$= \rho \times u \times (dy \times dz) \longrightarrow (1)$ [Fluid influx]

Mass of fluid leaving the face EFGH

$= \rho u\,dy\,dz + \frac{\partial}{\partial x}(\rho u\,dy\,dz)\,dx \longrightarrow (2)$ [fluid efflux]

$\because$ Gain of mass in x-direction (influx - efflux)

$= (1) - (2)$

$= -\frac{\partial}{\partial x}(\rho u\,dy\,dz)\,dx$

Similarly in y and z direction

$= -\frac{\partial}{\partial y}(\rho v\,dx\,dz)\,dy$ & $-\frac{\partial}{\partial z}\rho w\,(dx\,dy)\,dz$

So, net gain of masses $= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]dx\,dy\,dz \longrightarrow (3)$

Mass neither be created nor be destroyed. So, gain of masses must be equal to rate of increase of mass of fluid in the element. The rate of increase with time is given by $\frac{\partial}{\partial t}(\rho\,dx\,dy\,dz) \longrightarrow (4)$

Rate of change of mass

Equating (3) & (4)

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]dx\,dy\,dz = \frac{\partial \rho}{\partial t}\,dx\,dy\,dz$$

$$\left[\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right] = 0$$

For steady flow, $\frac{\partial \rho}{\partial t} = 0$

$$\boxed{\frac{\partial u}{\partial x}(\rho) + \frac{\partial v}{\partial y}(\rho) + \frac{\partial w}{\partial z}(\rho) = 0}$$

For incompressible flow, ρ is constant,

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

For two-dimensional flow, $\omega = 0$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

1. The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation.
   (i) $u = x^2 + y^2 + z^2$ $\quad v = xy^2 - yz^2 + xy$.
   (ii) $v = 2y^2$, $w = 2xyz$.

Given data:

~~We know~~ (i) $u = x^2 + y^2 + z^2$
$v = xy^2 - yz^2 + xy$

(ii) $v = 2y^2$
$w = 2xyz$

Solution:

We know that the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \longrightarrow (1)$$

(i) $u = x^2 + y^2 + z^2$ $\quad v = xy^2 - yz^2 + xy$

$$\frac{\partial u}{\partial x} = 2x \; ; \quad \frac{\partial v}{\partial y} = 2xy - z^2 + x$$

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$\longrightarrow$

Substituting $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in (1)

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = (-3x - 2xy + z^2)$$

$$\partial w = (-3x - 2xy + z^2)\,\partial z$$

$$\therefore w = \int(-3x - 2xy + z^2)\,\partial z$$

$$= -3xz - 2xyz + \frac{z^3}{3} + \text{constant of integration } (C)$$

C = is function of x and y $f(x,y)$

$$\therefore w = -3xz - 2xyz + \frac{z^3}{3} + f(x,y)$$ //

(ii) $v = 2y^2$

$w = 2xyz$

$$\frac{\partial v}{\partial y} = 4y \quad \frac{\partial w}{\partial z} = 2xy$$

$$\therefore \frac{\partial u}{\partial x} + 4y + 2xy = 0$$

$$\partial u = \int(4y + 2xy)\,\partial x$$

$$= -4xy - 2y\frac{x^2}{2} + f(y,z)$$

$$u = -4xy - x^2y + f(y,z)$$ //

(2) For an incompressible fluid the velocity components are $u = x^3 - y^3 - z^2x$, $v = y^3 - z^3$, $w = -3x^2z - 3y^2z + \frac{z^3}{3}$. Determine whether the continuity equation is satisfied.

Given data:

Continuity equation $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$\longrightarrow$

$$\frac{\partial u}{\partial x} = 3x^2 - z^2$$

$$\frac{\partial v}{\partial y} = 3y^2$$

$$\frac{\partial w}{\partial z} = -3x^2 + \frac{1}{3} 3z^2 - 3y^2$$

$$\cancel{3x^2} - \cancel{z^2} + \cancel{3y^2} - \cancel{3x^2} + \frac{1}{\cancel{3}} \cancel{3z^2} - \cancel{3y^2} = 0$$

Equations of motion: [Energy Equation]

Newton's second law = $F = ma$

In fluid flow, the following forces are present,

(i) Force due to Pressure, F_p
(ii) Force due to gravity, F_g
(iii) Force due to viscosity, F_μ
(iv) Force due to Compressibility, F_c
(v) Force due to turbulence, F_t

∴ Net force = $\boxed{F_x = (F_p)_x + (F_g)_x + (F_\mu)_x + (F_c)_x + (F_t)_x}$

Reynolds ~~number~~ equation of motion

It is the force due to compressibility F_c is neglible.

$$\boxed{F_x = (F_p)_x + (F_g)_x + (F_\mu)_x + (F_t)_x}$$

Navier's stoke equation of motion: F_t is neglible

$$\boxed{F_x = (F_p)_x + (F_g)_x + (F_\mu)_x + \cancel{(F_t)_x}}$$

Euler's equation of motion: F_μ is neligble

$$\boxed{F_x = (F_p)_x + (F_g)_x + \cancel{(F_c)_x} + \cancel{(F_t)_x}}$$

Euler's Equation of motion

$$\boxed{F_x = (F_p)_x + (F_g)_x}$$

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