Pressure and Pressure Measuring Devices

INTRODUCTION

The main properties of fluids, which are considered in real-time applications are pressure, velocity and temperature. Pressure and temperatures play a major role in the fields of pipeline constructions, water supply, hydraulics for irrigation, thermal power, hydraulic power, aviation of flights and even in human blood, etc. Fluids under pressure can exert forces that can be used by us to do work. Fluids form the basis for pneumatic and hydraulic systems which are used in industries such as automobile manufacturing (brakes), hoists found in service stations, dentist's chair, aeronautics, submarines, the shipping industry and the list can go on and on.

A small pressure, spread over a very large area, can add up to be very large force. When a fluid moves over or through an object; it gives small pushes on the surface of the object. These pushes, over the entire surface, are defined as pressure and are measured as force per unit area.

To understand the fluid flow, we need to study the fluid properties like pressures, velocity and temperature. And we deal with study of pressure and its ways of measuring, in this section.

PRESSURE OF A FLUID (*P*)

All fluid molecules will be in constant and random motion called "**Brownian motion**", due to which fluid at rest in a vessel, does exerts force on all the walls of the vessel, with which it is in contact,

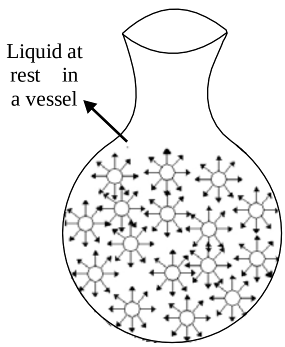


Figure 2.0

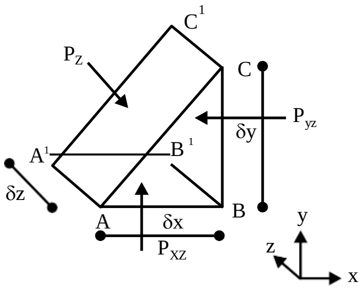


Figure 2.3a

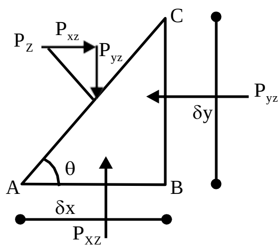


Figure 2.3b

Consider a liquid drop element in shape of a wedge with unit thickness (i.e. $\delta z = 1$), having two triangular faces $ABC, A'B'C'$ and three rectangular faces $AA'CC', AA'BB', BB'CC'$ as shown in the Figures 2.3a and 2.3b.

Let pressure intensities p_{xz}, p_{yz}, p_z be acting along the three rectangular faces.

Let total pressures acting along faces $AA'BB', BB'CC', AA'CC'$ be P_{xz}, P_{yz}, P_z respectively.

Then total pressure $P_{xz} = p_{xz} * (AB * BB')$ since $P_{xz} = p_{xz} * (area)$

$$\Rightarrow P_{xz} = p_{xz} * (\delta x * 1) \qquad (2.3a)$$

Similarly, total pressure $P_{yz} = p_{yz} * (BC * BB')$ since $P_{yz} = p_{yz} * (area)$

$$\Rightarrow P_{yz} = p_{yz} * (\delta y * 1) \qquad (2.3b)$$

Similarly, total pressure $P_z = p_z * (AC * AA')$ since $P_z = p_z * (area)$

$$\Rightarrow P_z = p_z * (AC * 1) \qquad (2.3c)$$

Under equilibrium condition, sum of all forces is equal to zero in horizontal and vertical directions. i.e.

$$\sum F_x = \sum F_y = 0$$

Hence, for sum of forces along horizontal direction is $\sum F_x = 0$

$$\text{i.e. } P_{yz} = P_z * (\sin\theta)$$

$$\Rightarrow p_{yz} * (\delta y * 1) = (p_z * AC * 1) * (\sin\theta)$$

$$\Rightarrow p_{yz} = p_z \qquad (2.4)$$

since $\delta y = AC * \sin\theta$

Similarly, for sum of forces along vertical direction is $\sum F_y = 0$

$$\text{i.e. } P_{xz} = P_z * (\cos\theta)$$

$$\Rightarrow p_{xz} * (\delta x * 1) = \left(p_z * AB * 1\right) * (\cos\theta)$$

$$\Rightarrow p_{xz} = p_z \tag{2.5}$$

since $\delta x = AB * \cos\theta$.

So. from Eq. (2.4) and Eq. (2.5), we get

$$\boxed{p_{xz} = p_{yz} = p_z = p} \tag{2.6}$$

Hence, **Pascal's law is proved** and is independent of the wedge angle θ or independent of surface angular orientation and is a scalar quantity.

Example 2.1: Find the weight that can be lifted by a hydraulic press when the force applied at the plunger is 350 N and has diameters of 250 mm and 40 mm of ram and plunger respectively.

Solution: Given data:

$$F = 350\text{N}, d_{ram} = 0.25\text{m}, d_{plunger} = 0.04\,\text{m}$$

Area of the ram

$$A_{ram} = \frac{\pi\, d_{ram}^{\;2}}{4} = \frac{3.14 * 0.25^2}{4} \quad 0.049\,\text{m}$$

Area of the plunger

$$A_{plunger} = \frac{\pi d_{plunger}^{\;2}}{4} = \frac{3.14 * 0.04^2}{4} \quad 0.001256\,\text{m}$$

Normal pressure intensity due to the force applied

$$p = \frac{F}{A_{plunger}} = \frac{350}{0.001256} = 278.6\,\text{kPa}$$

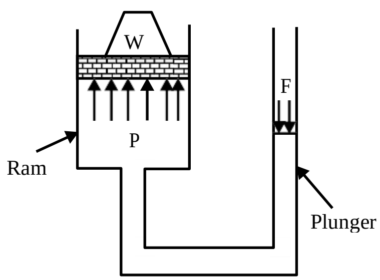


Figure P2.1

As the pressure intensity due to the force in the plunger lifts the weight, so the total pressure acting on the weight in the ram must be equal or greater to the weight, and hence

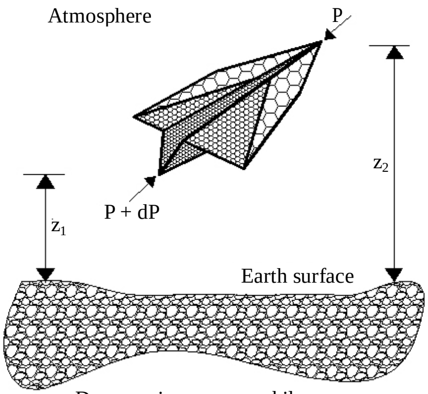
$$p = \frac{W}{A_{ram}}$$

$$\Rightarrow W = p * A_{ram} = 278.6 * 0.049 = 13.65\text{kN}$$

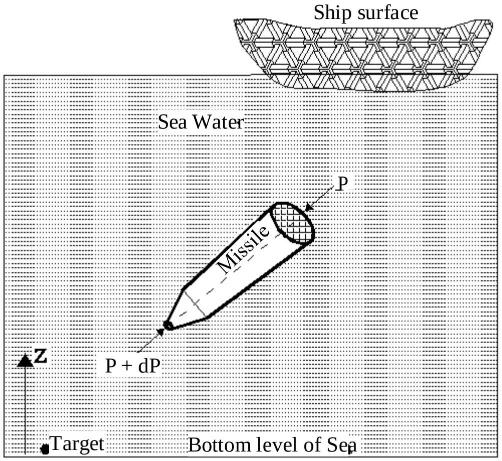
HYDROSTATICS LAW : VARIATION OF PRESSURE IN A STATIC FLUID

Hydrostatic law defines the variation of pressure with respect to the elevation of fluid particles under static condition and states that "as the distance from a datum surface level increases the pressure drops". When an object pushed in a fluid to its bottom, it compresses the fluid. so there arise an increase in pressure force around it and is termed as "**Hydrostatic force**."

A projectile soaring higher altitudes from earth surface experiences decrease in pressure and similarly underwater missiles experience increase in hydrostatic force as it penetrates into the depths of sea waters, as illustrated in the below pictures.



Decrease in pressure while soaring higher altitudes



Increase in pressure while targeting beneath altitudes

Similarly, a diver experiences more pressure as he goes down to the depths of the sea levels and similarly a tumbled mug when pushed to bottom of a water filled bucket, experience more upward force and this can be proved by taking a cylindrical fluid element having an area and length as dA, ds respectively, at some height from the datum, as shown in Figure 2.4.

The cylindrical element will be subjected to pressure force at one end with height z_1 is P and at the other end of height z_2 is $P + dP$ and also the weight of the fluid element dw which acts vertically downwards.

Under equilibrium of the element, the resultant of forces in any direction is zero and resolving the forces in the direction along the central axis gives

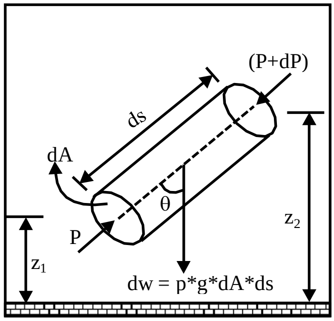


Figure 2.4: Cylindrical fluid element with an arbitrary orientation in a container

$$(P * dA) - (P + dP)dA \quad - (\rho g * dA * ds * \cos\theta) = 0$$

By rearranging

$$(dP) = -(\rho g * ds * \cos\theta)$$

$$\boxed{\Rightarrow \frac{dP}{ds} = -(\rho g * \cos\theta)} \qquad (2.7)$$

Note:

Case 1: If the cylindrical fluid element is vertical, then the angle of orientation $\theta = 0^o$, then Eq. (2.7) becomes.

$$\frac{dP}{dz} = -(\rho g) = -\frac{Pg}{RT} \qquad (2.7a)$$

so on integrating we get

$$\boxed{P = P_0 * e^{-gZ/RT}} \qquad (2.7b)$$

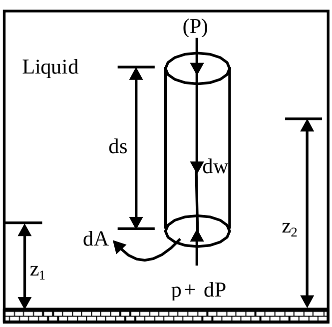


Figure 2.4a: Cylindrical fluid element subjected to variation of pressure through out its length

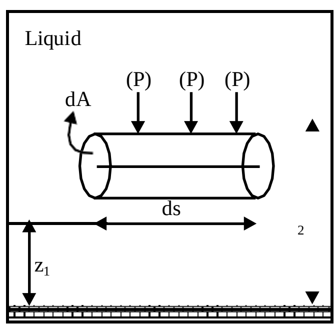


Figure 2.4b: Cylindrical fluid element subjected to uniform pressure through out its length

From Eq. (2.7) it is evident that when a diver dives into sea, the datum elevation will be reducing, as he goes into the depths of the sea and hence the pressure increases at the depths.

Case 2: If the cylindrical fluid element is horizontal, then the angle of orientation θ = 90° then Eq. (2.7) becomes

$$\boxed{\frac{dP}{dz} = 0} \tag{2.7c}$$

and so pressure is constant (i.e. P = constant) throughout in a static liquid, at a given height.

PRESSURE HEAD (h)

When pressure expressed in terms of the height of fluid raised/fallen is called fluid "Pressure head". A diver experiences more pressure as he goes below the sea level, due to reason of hydrostatic forces which increases the level of fluid by an amount equal to pressure raise and is expressed as "mm of Hg" on "meters of water".

If considered a cylindrical bar of length L and cross-section area A immersed in a water/liquid with an inclination θ, as shown in figure.

Under equilibrium conditions sum of all the forces acting on the cylindrical bar is equal to zero and hence there will be three forces acting like pressure forces $\left(P_1A, (P_1 + \Delta P)\,A\right)$ at both the ends and weight force $\gamma\, lA\cos\theta$ due to the weight of the cylindrical bar.

Then $$(P_1 + \Delta P)\,A - P_1A - \gamma\, lA\cos\theta = 0$$

But $$h = L\cos\theta \quad \text{so} \quad P_1A - P_2A - \gamma\, Ah = 0$$

$$\boxed{\Rightarrow h = \frac{\Delta P}{\gamma}} \tag{2.7d}$$

where γ is specific weight of liquid, h is depth or else called **pressure head.**

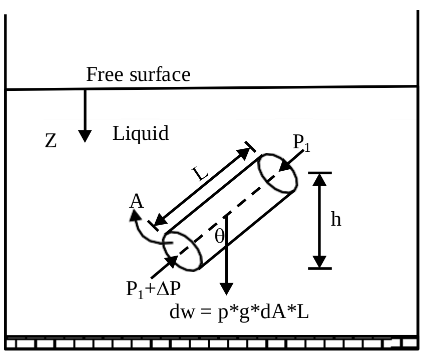


Figure 2.5: Cylindrical bar immersed in a liquid

b) Height of Mercury Barometer $P_{atm} = \rho g h$

$$\Rightarrow 101000 = 13600 * 9.81 * h$$
$$\Rightarrow h = 0.75\,\text{m of mercury}$$

Example 2.6: An oil of specific gravity 0.8 is under pressure of 137.2 kPa, then determine pressure head expressed in terms of meters of oil.

Solution: Given data: $\rho_{water} = 1000\ \text{kg/m}^3$, $P_{gauge} = 137.2\,\text{kPa}$, $S = 0.8$

Pressure head $P_{gauge} = \rho_{oil} g h$

$$\Rightarrow P_{gauge} = S * \rho_{water} g h$$

$$\Rightarrow 137.2 * 1000 = 0.8 * 1000 * 9.81 * h$$
$$\Rightarrow h = 17.48\,\text{m of oil}$$

So pressure head in terms of oil is 17.48 m.

Example 2.7: Find the weight that can be lifted by a hydraulic press when the force applied at the plunger is 350 N and has diameters of 250 mm and 40 mm of ram and plunger respectively, take the specific weight of the liquid in the press as 1000 N/m^3.

Solution: Given data:

$$F = 350\,\text{N}, d_{ram} = 0.25\,\text{m}, d_{plunger} = 0.04\,\text{m}$$
$$\gamma = 1000\,\text{N/m}^3, h = 0.3$$

Area of the ram

$$A_{ram} = \frac{\pi\, d_{ram}^{\ 2}}{4} = \frac{3.14 * 0.25^2}{4} \quad 0.049\,\text{m}$$

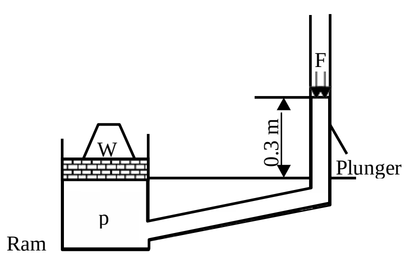


Figure P2.7

Area of the plunger

$$A_{plunger} = \frac{\pi d_{plunger}^{2}}{4} = \frac{3.14 * 0.04^2}{4} \quad 0.001256\,\text{m}$$

Normal pressure intensity due to the force applied in the plunger

$$p = \frac{F}{A_{plunger}} = \frac{350}{0.001256} = 278.6\,\text{kPa}$$

As the level of force and weight are not of the same and so there exists a pressure intensity due to head of 0.3 m height in the plunger then the total pressure intensity on the weight will be

$$p = \frac{F}{A_{plunger}} + \gamma h = \frac{350}{0.001256} + 1000 * 0.3 = 278.96\,\text{kPa}$$

As the total pressure intensity due to the force in the plunger lifts the weight, so the total pressure acting on the weight in the ram must be equal or greater to the weight, and hence

$$p = \frac{W}{A_{ram}}$$

$$\Rightarrow W = p * A_{ram} = 278.96 * 0.049 = 13.669\,\text{kN}$$

Types of Pressure Measuring Devices

a) Manometers
b) Mechanical gages.

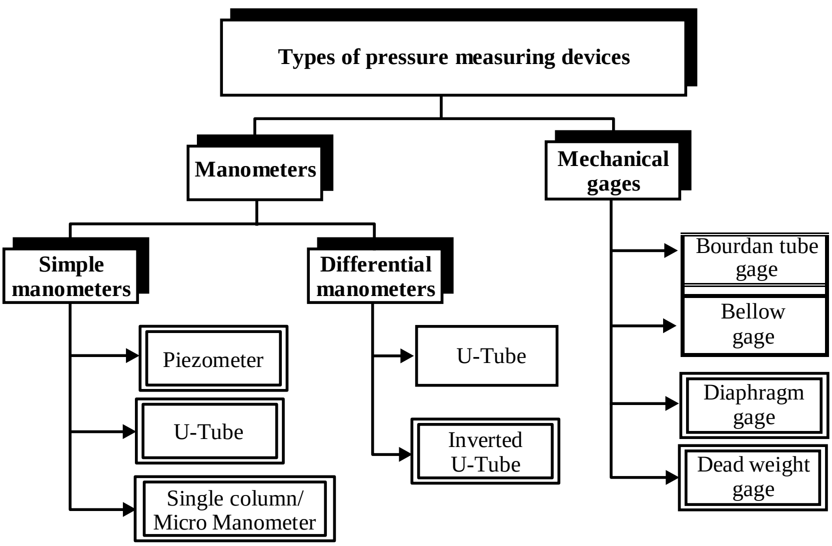


Figure 2.6: Different types of pressure mesuring devices

Manometers

Manometer is a device used to measure pressure at a single or multiple points in a single or multiple pipelines, by balancing the fluid column by the same or another column of fluid. Manometers can be categorized into two types, namely simple manometer and differential manometer. Simple manometric devices measure pressure at a single point in a fluid, whereas differential manometric devices measure pressure at two or more number of points, in a single or multiple flow lines.

The simplest type of manometer is simple manometers, wherein we have three types of simple manometers like **Piezometer**, **U-tube manometer** and **Single column/Micro manometer**. Simple manometers consist of a glass tube straight or bent, whose one end is connected to the point at which the measurement of pressure is required and the other end is left open to the atmosphere.

A. Piezometer

Piezometer is a simple manometric device which measures pressure at a point in a fluid, without balancing any other fluid column.

Consider a cylindrical vessel having a fluid filled in it and is open to atmosphere. Connect a piezometer glass tube as shown in Figure 2.7, at the point where pressure is to be measured, such that the other end of the piezometer tube is open to the atmosphere. As soon as the piezometer glass tube is connected to the cylindrical tank, some amount of the fluid rushes into the tube, due to atmospheric pressure outside the cylindrical tank. But as the other end of the piezometer glass tube is exposed to atmosphere, atmospheric pressure acts at that open end also and hence fluid raises up to some level in the glass tube and does not overflow from the tube.

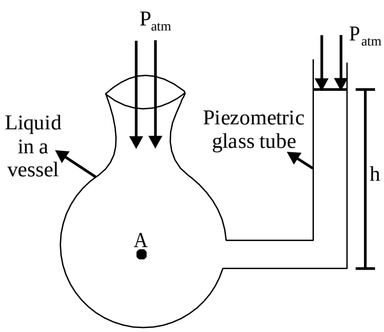


Figure 2.7: Piezometer

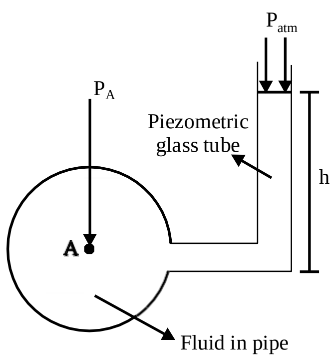


Figure 2.7a: Piezometer connected to pipeline

If pressures considered in the cylindrical vessel $\left(P_{atm} + P_A\right)$ and piezometer tube $\left(P_{atm} + \gamma h\right)$ then they will be equal, due to which the overflow of fluid does not take place,

i.e. $$P_{atm} + P_A = P_{atm} + \gamma h$$

$$\boxed{\Rightarrow P_A = \gamma h} \tag{2.8}$$

where P_A is the pressure at the point "A" where pressure is required to measure in an open container and (γh) is the specific weight of the fluid raised in the piezometer column.

If the container is not an open one, for instance like a pipeline and if a piezometer is connected to it, as shown in the Figure 2.7a, to find the pressure inside the pipeline, the hydrostatic pressure equation varies. i.e.

$$P_A = P_{atm} + \gamma h \tag{2.8a}$$

And if needed in terms of pressure heads, dividing the Eq. (2.8a) with the specific weight of water (γ_{water}), we get

$$\Rightarrow h_A = h_{atm} + Sh = 10.3 + Sh \tag{2.8b}$$

Hence, Eqs. (2.8), (2.8a) and Eq. (2.8b) are the equations of **hydrostatic pressure** and **hydrostatic pressure head**.

B. U-tube Simple Manometer

U-tube manometer is a simple manometric device used to measure pressure at a point in a fluid, by balancing the fluid column by the same or another column of fluid. It has a glass tube bent in "U" shape with some amount of same or other type of fluid, called manometric fluid like mercury, as shown in Figures 2.8a and 2.8b.

Consider a pipe carrying a fluid called "fluid 1" with specific weight γ_1 and let pressure P_A to be measured at a point "A" in the pipe. Let the manometric fluid have specific weight γ_2- As the U-tube manometer is connected to the pipe, the "fluid 1" rushes into the U-tube and pushes the manometric fluid upwards in the other limb of the U-tube, due to which the initial manometric fluid level as shown in Figure 2.8a is disturbed and lowers in the left limb and raised up with a pressure head "h_2" in the right limb of U-tube.

Let the fluid 1 in pipe occupy the U-tube column in the left limb, of height "h_1" and let the manometric fluid in U-tube be raised up to a height "h_2", in the right limb.

Due to the push given by the fluid 1 onto the manometric fluid, the manometric fluid may overflow but it does not happen due to the reason that the atmospheric pressure acts on the open end of the U-tube, as shown in Figure 2.8b.

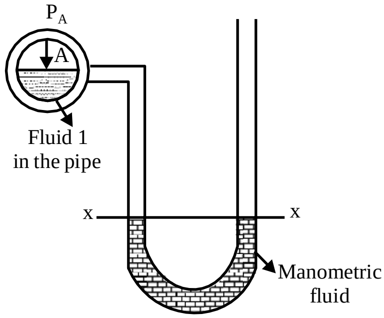


Figure 2.8a: Just before connecting the U-Tube

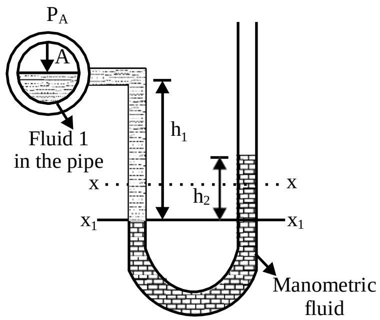


Figure 2.8b: After connecting the U-Tube

Under equilibrium the pressures on the left limb of U-tube must be equal to the pressures in the right limb, so

$$P_A + \gamma_1 h_1 = \gamma_2 h_2 + P_{atm} \tag{2.9}$$

$$\Rightarrow P_A + \gamma_w S_1 h_1 = \gamma_w S_2 h_2 + P_{atm} \tag{2.9a}$$

$$\Rightarrow \frac{P_A}{\gamma_w} = h_2 S_2 - h_1 S_1 \quad \text{meters of water} \tag{2.9b}$$

$$\boxed{\therefore h = S_2 h_2 - h_1 S_1} \tag{2.10}$$

or

If hydrostatic pressure is considered, then

$$h = S_2 h_2 - h_1 S_1 + 10.3 \tag{2.10a}$$

Where *h* is the pressure head at the point "A" where pressure is required.

Example 2.8: Pipeline carrying water has a connection of a mercury manometer connected to it and the readings are measured as shown in Figure P2.8. Find the pressure of the water flowing in the pipeline.

Solution: Given data:

$$h_1 = 0.1\text{m},\ h_2 = 0.8\,\text{m},\ S_{water} = 1,\ S_{Mercury} = 13.6$$

Equating the pressure heads on both sides of the limbs of the manometer, we have

$$h + h_1 S_{water} = h_2 S_{Mercury}$$

$$\Rightarrow h = 0.8*13.6 - 0.1*1 = 10.78\,\text{m}$$

So pressure in the pipeline will be $P_A = \rho_{water} gh = 1000 * 9.81*10.78 = 105.75\,\text{kPa}$

If hydrostatic pressure is considered, then we have $P_A = \rho_{water}\, g\,(h + 10.3) = 206.75$ kPa

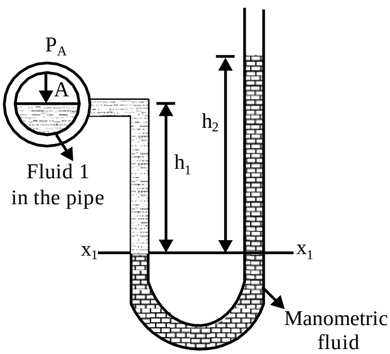


Figure P2.8

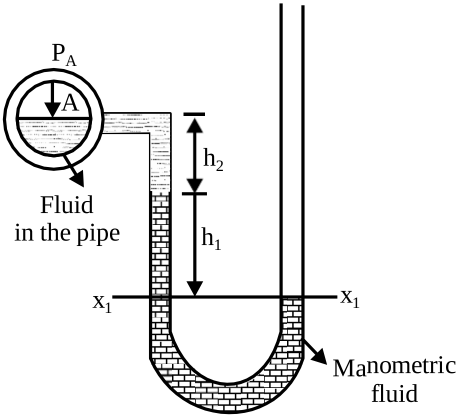


Figure P2.10

$$h + h_1 S_{water} + h_2 S_{Mercury} = 0$$

$$\Rightarrow h = -0.3*1 - 0.5*13.6 = -7.1\text{m}$$

So pressure in the pipeline will be

$$P_A = \rho_{water} gh = 1000*9.81*7.1 = 69.651\text{kPa (vaccum)}$$

If hydrostatic pressure considered, then we have

$$P_A = \rho_{water} g\left(h+10.3\right) = 1000*9.81*3.2 = 31.4 \text{ kPa}$$

Example 2.11: A funnel connected to a U-tube mercury manometer having water in it, as shown in Figure P2.11a. The other end of the manometer is exposed to atmosphere. The level of mercury is

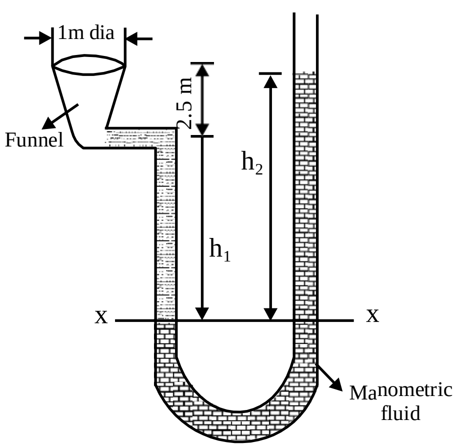


Figure P2.11a

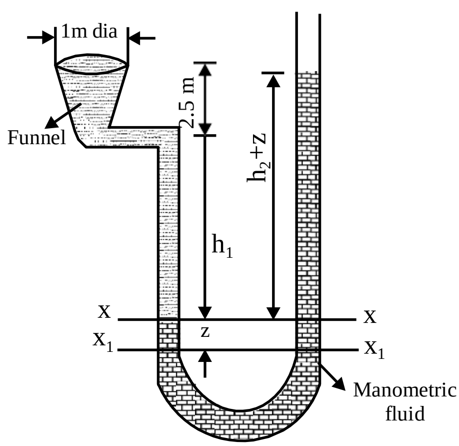


Figure P2.11b

0.25 m from the reference line. If the funnel is filled with water, find the level of manometer readings.

Solution: Given data:

$$h_2 = 0.25\,\text{m}, S_{water} = 1, S_{Mercury} = 13.6$$

When the funnel is empty: Observe Figure P2.11a.

Equating the pressure heads on both sides of the limbs of the manometer, we have

$$h_1 S_{water} = h_2 S_{Mercury}$$
$$\Rightarrow h_1 = 0.25 * 13.6 / 1 = 3.4 \text{ m}$$

Hence, the level of the water column above the reference is 3.4 m, when the funnel is empty.

When the funnel is filled completely: Observe Figure P2.11b. The reference line moves to new position to X_1-X_1 and so by equating the pressure heads on both sides of the limbs of the manometer, we have

$$h_1 S_{water} + z + 2500 = (h_2 + 2z)\, S_{Mercury}$$
$$\Rightarrow 3.4 * 1 + z + 2500 = 0.25 * 13.6 + 2 * z * 13.6$$
$$\Rightarrow z = 95.4\,\text{mm}$$

Hence, the reading of manometer is 95.4 mm.

Example 2.12: Figure below shows a gasoline tank in a car, which reads proportional to the bottom gage. The tank is 30 cm deep and accidentally contains 1.8 cm of water in addition to the gasoline. Determine the height of air remaining at the top when the gauge erroneously reads full. Assume

$$\gamma_{water} = 0.0118\,\text{kN/m}^3, \gamma_{mercury} = 6.65\,\text{kN/m}^3$$

Solution: Given data: $h = 0.3\text{m}, \gamma_{water} = 0.0118\,\text{kN/m}^3, \gamma_{mercury} = 6.65\,\text{kN/m}^3$

When the tank is full of gasoline, then $P_{gauge} = \gamma_{gasoline} * h = 6.65 * 1000 * 0.3 = 1.995\,\text{kPa}$. When the tank contains 1.8 cm of water then the sum of pressure created due to presence of air, gasoline and water must be equal to tank with full gasoline, so as to have erroneous reading and hence

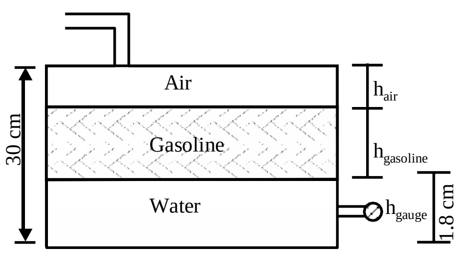


Figure P2.12

$$P_{Air} + P_{gasoline} + P_{water} = P_{Gage}$$

$$\Rightarrow \gamma_{Air} * h_{Air} + \gamma_{gasoline} * h_{gasoline} + \gamma_{water} * h_{water} = 1.995\,\text{kPa}$$

$$\Rightarrow 0.0118 * h_{Air} + 6.65 * (0.3 - 0.018 - h_{air}) + 9.81 * 0.018 = 1.995\text{kPa}$$

$$\Rightarrow h_{Air} = 0.857\,\text{cm}$$

Hence, the height of air remaining at the top is 0.857 cm.

c) Single Column/Micro-manometer

Single column/micro-manometers are just similar to the U-tube manometer, only with a difference of having a small reservoir in the tube, as shown in Figures 2.9a and 2.9b.

Micro-manometer has a glass tube bent in "U" shape or inclined and has a small reservoir in the tube, having some amount of same or other type of fluid called manometric fluid. Consider a pipe

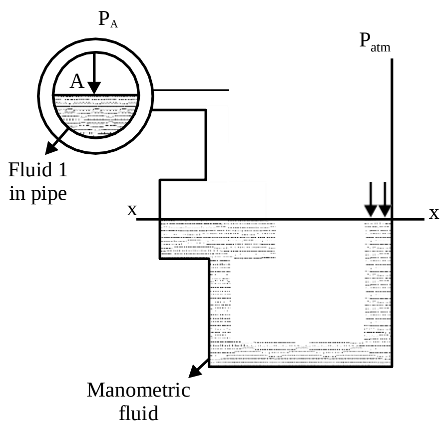


Figure 2.9a: just before connecting micro-manometer

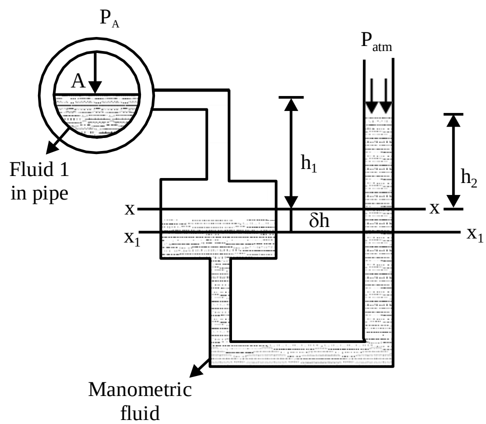


Figure 2.9b: After connecting micro-manometer

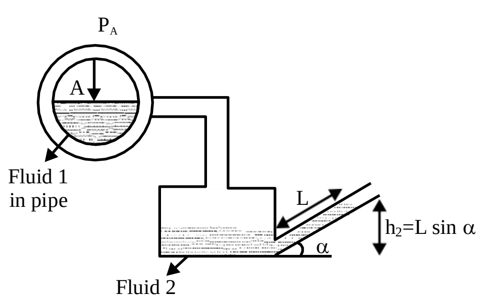


Figure 2.9c: Inclined micro-manometer

If a<<<<A, then $\frac{a}{A} \approx 0$ which give

$$h = h_2 S_2 - h_1 S_1 \tag{2.13b}$$

If the tube is **inclined** to the reservoir at angle α as shown in Figure 29c, then $h_2 = l\sin\alpha$ and so

$$\boxed{h = \frac{a}{A} l\sin\alpha\,(S_2 - S_1) + l\sin\alpha\, S_2 - h_1 S_1} \tag{2.14}$$

And if $\frac{a}{A} \approx 0$ then

$$h = l\sin\alpha\, S_2 - h_1 S_1 \tag{2.14a}$$

Example 2.13: A micro-manometer having a ratio of reservoir to limb areas as 40 was used to determine the pressure in a pipe containing water. Determine the pressure in the pipe for manometer readings, as shown in Figure P2.13.

Solution:

Given data:

$$S_{water} = 1, S_{mercury} = 13.6, P_{atm} = 101\text{kPa}, \frac{A}{a} = 40$$

$$h_1 = 50 \text{ mm}, h_2 = 80 \text{ mm}$$

Equating the pressure heads on both sides of the manometer limbs, we have

$$h = \frac{a}{A} h_2 (S_{mercury} - S_{water}) + h_2 S_{mercury} - h_1 S_{water}$$

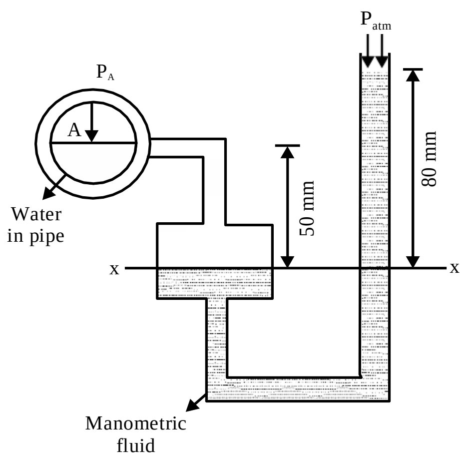


Figure P2.13

$$\Rightarrow h = \frac{1}{40}\left[0.08(13.6-1)\right] + 0.08*13.6 - 0.05*1$$

$$\Rightarrow h = 1.06\,\text{m of water}$$

So pressure in the pipe will be

$$P = \gamma_{water} h = 9.81*1.06 = 10.4\,\text{kPa}$$

Example 2.14: An inclined micro-manometer having a ratio of reservoir to limb areas as 10 was used to determine the pressure in a pipe containing water. Determine the pressure in the pipe for manometer readings, if the inclination of the manometer limb has a slope of 4:1, as shown in Figure P2.14.

Solution:
Given data:

$$S_{water} = 1, S_{mercury} = 13.6, \frac{A}{a} = 10, l = 0.25\,\text{m}$$

$$h_1 = 50\,\text{mm}$$

$$h_2 = l \sin\alpha = 0.25 * \sin 14° = 0.06\,\text{m}$$

Equating the pressure heads on both sides of the manometer limbs, we have

$$h = \left(\frac{a}{A}\right)\left[l \sin\alpha (S_2 - S_1)\right] + l \sin\alpha S_2 - h_1 S_1$$

$$\Rightarrow h = \frac{1}{10}\left[0.06(13.6-1)\right] + 0.06*13.6 - 0.05*1$$

$$\Rightarrow h = 0.84\,\text{m of water}$$

So pressure in the pipe will be

$$P = \gamma_{water} h = 9.81*0.84 = 8.25\,\text{kPa}$$

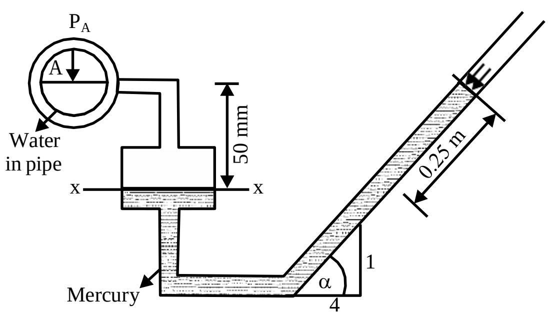


Figure P2.14:

where γ is the specific weight of fluids.

$$\Rightarrow P_A + \gamma_w S_1 h_1 = \gamma_w S_2 h_2 + \gamma_w S_3 h_3 + P_B \qquad (2.15)$$

since $S = \gamma / \gamma_w$ where γ_w, S_1, S_2 are specific weight of water and specific gravities of fluids respectively. By rearranging and dividing by γ_w throughout, we get

$$\frac{(P_A - P_B)}{\gamma_w} = h_3 S_3 + h_2 S_2 - h_1 S_1 \qquad (2.16)$$

meters of water. When written in terms of pressure heads, we get

$$\boxed{(h_A - h_B) = h_2 S_2 + h_3 S_3 - h_1 S_1} \qquad (2.17)$$

If the "fluid 1" and "fluid 2" are of same kind, then $S_1 = S_3$.

$$\boxed{(h_A - h_B) = h_2 (S_2 - S_1)} \qquad (2.17a)$$

Example 2.15: Find the pressure difference at two points A and B in the pipe carrying water and connected to a mercury differential manometer and also has oil filled in between it, as shown in figure P2.15. Take $\gamma_{water} = 10\,\text{kN/m}^3, \gamma_{mercury} = 136\,\text{kN/m}^3, \gamma_{oil} = 8.5\,\text{kN/m}^3$

Solution: Given data: $\gamma_{water} = 10\,\text{kN/m}^3, \gamma_{mercury} = 136\,\text{kN/m}^3, \gamma_{oil} = 8.5\,\text{kN/m}^3$

Equating using differential manometer equations, we have

$$P_A + \gamma_{water} * 1.08 - \gamma_{mercury} * 0.72 + \gamma_{oil} * 0.48 - \gamma_{mercury} * 0.6 - \gamma_{water} * 0.36 = P_B$$

$$\Rightarrow P_A - P_B = 10 * 1.08 - 136 * 0.72 + 8.5 * 0.48 - 136 * 0.6 - 10 * 0.36 = 168.2\,\text{kPa}$$

Hence, the pressure difference between two points is 168.2 kPa.

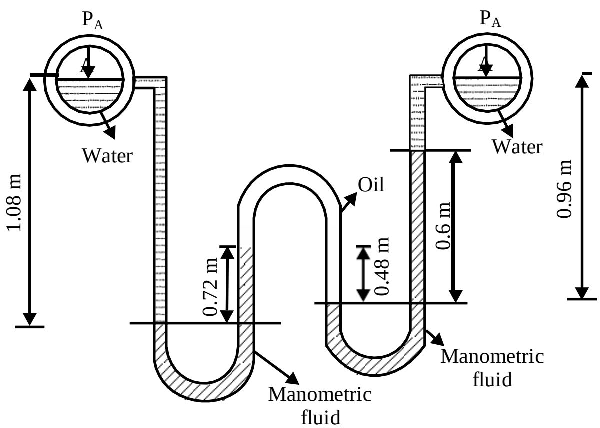


Figure P2.15

Example 2.16: Two pipes A and B contain carbon tetrachloride of specific gravity 1.59 at 103 kPa pressure and oil of specific gravity 0.8 at 171.6 kPa respectively. The manometric liquid is mercury. Find the difference "h" between the levels of mercury.

Take $S_{ccl2} = 1.594, S_{mercury} = 13.6, S_{oil} = 0.8$

Solution: Given data:

$$S_{ccl2} = 1.594, S_{mercury} = 13.6, S_{oil} = 0.8$$
$$P_A = 103\text{kPa}, P_B = 171.6\,\text{kPa}$$

Equating the pressure heads on both sides of the manometer limbs, we have

$$h_A + 4 * S_{ccl2} + h * S_{mercury} = h * S_{oil} + S_{oil} * 1.5 + h_B$$

$$\frac{P_A}{\gamma_{water}} + 4 * S_{ccl2} + h * S_{mercury} = h * S_{oil} + S_{oil} * 1.5 + \frac{P_B}{\gamma_{water}}$$

$$\frac{103}{9810} + 4 * 1.59 + h * 13.6 = h * 0.8 + 0.8 * 1.5 + \frac{171.6}{9810}$$

$$\frac{103}{9.81} + 4 * 1.59 + h * 13.6 = h * 0.8 + 0.8 * 1.5 + \frac{171.6}{9.81}$$

$$\Rightarrow h = 402 \text{ mm}$$

Hence, the difference in mercury levels is 402 mm.

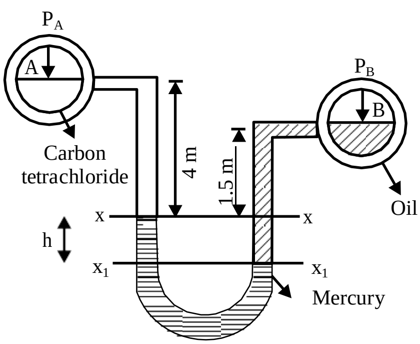


Figure P2.16

Example 2.17: Two pipes A and B contain water and air above in pipe A with 78.5 kPa pressure of air and oil of specific gravity 0.8 in pipe B respectively. The manometeric liquid is mercury. Find the pressure in pipe B (absolute). Take $P_{atm} = 101\text{kPa}$

Solution: Given data: $S_{water} = 1$, $S_{mercury} = 13.6$, $S_{oil} = 0.8$

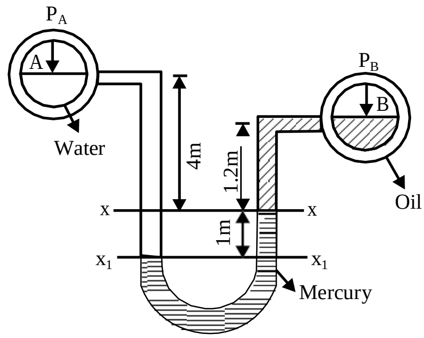


Figure P2.17

$$P_A = 78.5 \text{ kPa}, P_{atm} = 101 \text{ kPa}$$

Equating the pressure heads on both sides of the manometer limbs, we have

$$h_A + 5 * S_{water} = 1 * S_{mercury} + 1.2 * S_{oil} + h_B$$

$$\frac{P_A}{\gamma_{water}} + 5 * S_{water} = 1 * S_{mercury} + 1.2 * S_{oil} + \frac{P_B}{\gamma_{water}}$$

$$\frac{78.5}{9.81} + 5*1 = 1*13.6 + 1.2*0.8 + \frac{P_B}{9.81}$$

$$\Rightarrow P_B = -15.3 \text{ kPa } (vacuum)$$

$$\Rightarrow P_{B(absolute)} = P_{atm} + P_B = 101 - 15.3 = 85.7 \text{ kPa}$$

Hence, the pressure inside the pipe B is 85.7 kPa (absolute).

b) Inverted Differential U-tube Manometer: Inverted differential U-tube manometer is a device just as same that of U-tube differential manometer but is inverted in construction. It measures pressure difference ΔP at two different points "A" and "B" in a single pipe or different pipes, carrying two fluids of same or different kinds, as shown in Figure 2.11a. As small change in

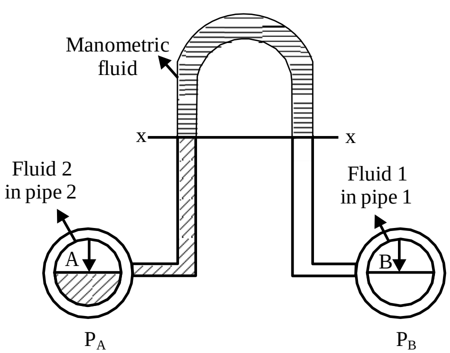


Figure 2.11a: Just before connecting inverted differential U-tube

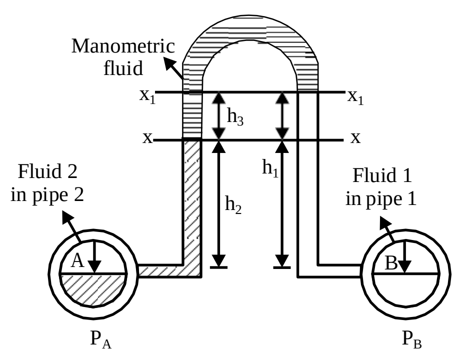


Figure 2.11b: After connecting inverted differential U-tube

Example 2.18: Find the absolute pressure in the pipe A carrying an oil having specific gravity of 0.8 and has a mercury manometric fluid, as shown in figure P2.18. Take

$$h_1 = 0.66\,\text{m}, h_2 = 0.3\,\text{m}, h_3 = 0.165\,\text{m}$$

$$h_4 = 0.11\text{m}, P_{atm} = 105\,\text{kPa}$$

Solution: Given data:

$$S_{oil} = 0.8, S_{Mercury} = 13.6, S_{Water} = 1$$

$$h_1 = 0.66\,\text{m}, h_2 = 0.3\,\text{m}, h_3 = 0.165\,\text{m}$$

$$h_4 = 0.11\text{m}, P_{atm} = 105\,\text{kPa}$$

Equating using differential manometer equations, we have

$$P_A = \gamma_{oil}(h_4 + h_3) - \gamma_{mercury} h_3 - \gamma_{mercury} h_2 + \gamma_{water} h_1 + P_{atm}$$

$$P_A = 0.88*9.81*(0.11+0.165) - 13.6*9.81*0.165 - 1.6*9.81*0.33 + 9.841*0.66 + 105 = 88.6kPa$$

Hence, pressure in the pipe is 88.6 kPa.

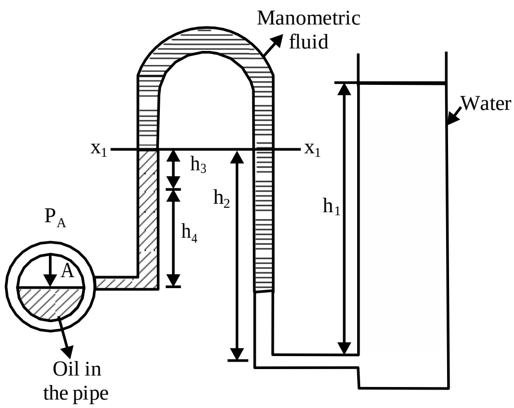


Figure P2.18

Example 2.19: A pipe has two points A and B and carries water and is connected to an inverted U-tube differential manometer having manometeric liquid as mercury. Find the difference in pressures at the two points A and B in absolute. Take atmospheric pressure as 101 kPa, as shown in Figure P2.19

Solution: Given data: $S_{water} = 1, S_{mercury} = 13.6$

Equating the pressure heads on both sides of the manometer limbs, we have

$$h_A + 1.7*S_{water} + 0.8*S_{water} = 0.8 + S_{mercury} + 0.7*S_{water} + h_B$$

$$h_A + 4 * S_{water} + 1 * S_{mercury} = 1 * S_{oil} + 1.2 * S_{oil} + h_B$$

$$\frac{P_A}{\gamma_{water}} + 4 * S_{water} + 1 * S_{mercury} = 1 * S_{oil} + 1.2 * S_{oil} + \frac{P_B}{\gamma_{water}}$$

$$\frac{78.5}{9.81} + 4 * 1 + 1 * 13.6 = 1 * 0.8 + 1.2 * 0.8 + \frac{P_B}{9.81}$$

$$\Rightarrow P_B = 139.7 \, \text{kPa}$$

Hence, the pressure at the point B in the second pipe is 139.7 kPa.

Example 2.21: An inverted differential U-tube manometer having an oil of specific gravity 0.8 as manometeric liquid is connected to two different pipes carrying water under pressure. Determine the pressure in pipe B, if the pressure head in the pipe A is 2.0 m of water as shown in Figure P2.21.

Solution: Given data: S_{oil} = 0.8, H_A = 2.0 m

Equating the pressure heads on both sides of the manometer limbs, we have

$$(h_B - h_A) = 0.15 * S_{oil} + 0.1 * S_{water} - 0.3 * S_{water}$$

$$\Rightarrow (h_B - 2.0) = 0.15 * 0.8 + 0.1 * 1 - 0.3 * 1$$

$$\Rightarrow h_B = 1.92 \, \text{m}$$

$$\Rightarrow P_B = h_B * \gamma_{water} = 1.92 * 9.81 = 18.8 \, \text{kPa}$$

Hence, the pressure at the point B in the second pipe is 18.8 kPa.

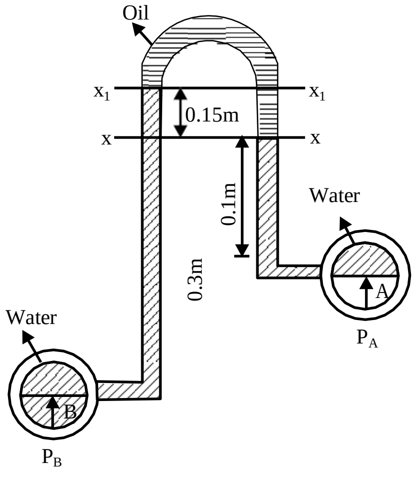


Figure P2.21

EXERCISE

Objective

1. In a static fluid
   a) Resistance to shear stress is small
   b) Fluid pressure is zero
   c) Linear deformation is small
   d) Only normal stresses can exist
   e) Viscosity is nil
2. Liquids transmit pressure equally in all the directions. This is according to the
   a) Boyle's law
   b) Archimedes' principle
   c) Pascal's law
   d) Newton's formula
3. Manometer is used to measure
   a) Pressure in pipes, channels, etc.
   b) Atmospheric pressure
   c) Very low pressures
   d) Difference of pressure between two points
   e) Velocity
4. Gage pressure is equal to
   a) Absolute pressure + atmospheric pressure
   b) Absolute pressure – atmospheric pressure
   c) Atmospheric pressure – absolute pressure
   d) Atmospheric pressure – vacuum
   e) Atmospheric pressure + vacuum
5. Piezometer is used to measure
   a) Pressure in pipes, channels
   b) Atmospheric pressure
   c) Very low pressures
   d) Difference of pressure between two points
   e) Flow
6. Differential manometer is used to measure
   a) Pressure in pipes channels, etc.
   b) Atmospheric pressure
   c) Very low pressure
   d) Difference of pressure between two points
   e) Velocity in pipes
7. 10 m of water column is equal to
   a) 10 kN/m^2
   b) 1 kN/m^2
   c) 100 kN/m^2
   d) 0.1 kN/m^2
   e) None of the above

8. Determine the pressure in bar at a depth of 10 m in oil of relative density 0.750.
   a) 735575
   b) 0.075
   c) 735575×10^5
   d) 73.575
   e) 98100
9. What depth of oil (in m), relative density 0.75, will give a gage pressure of 275000 Pa?
   a) 37.38
   b) 367
   c) 0.027
   d) 20.2×10^4
   e) 28.03
10. Express the pressure head of 15 m of water in meters of oil of relative density 0.75.
    a) 110.36
    b) 11.25
    c) 11250
    d) 15.0
    e) 20.0

Answers

1. d	2. c	3. c	4. b	5. a	6. d
7. c	8. b	9. b	10. c		

Theory

1. Explain different types of pressures.
2. Give the relation between different types of pressures.
3. State and prove the Pascal's law.
4. Explain theory behind hydrostatic law.
5. Write a detailed note on pressure head.
6. Write a detailed note on manometers.
7. Explain the differences between Simple and Differential manometers.
8. Write the merits and demerits of manometers.

Problems

1. A manometer connected to a pipe indicates a negative gage pressure of 60 mm of mercury. What is the absolute pressure in pipe, if the atmosphere pressure is 101 kPa?
2. Calculate the pressure due to column of 0.4 m of (a) Water (b) Oil of specific gravity 0.9 (c) Mercury.
3. A simple manometer (U-Tube) containing mercury is connected to a pipe in which an oil of specific gravity 0.8 is flowing. The pressure in the pipe is vacuum. The other end of the manometer is open to atmosphere. Find the vacuum pressure in the pipe, if the difference