BOOLEAN POSTULATES & LAWS

Boolean Algebra is a system of mathematical logic which can be used to simplify the design of logic circuits. A Boolean algebra may contain variables, complement of a variables, logical AND operator and logical OR operator.

Ex.

$$Y = A + B \cdot C + \bar{C} \cdot A$$

Variable

AND

OR

$\bar{C}$ is the complement of variable C.

OR OPERATOR.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

AND OPERATOR.

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Laws In Boolean Algebra.

Boolean algebra consist of 3 basic laws as in Ordinary algebra.

* Commutative Law
* Associative Law
* Distributive Law

Commutative Laws

1.) $A+B = B+A$

2.) $A \cdot B = B \cdot A$

1.) $A+B = B+A$

A	B	A+B	B+A
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

2.) $A \cdot B = B \cdot A$

A	B	A·B	B·A
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

Associative Laws.

1.) $A + (B+C) = (A+B) + C$

2.) $(AB)C = A(BC)$

1.) $A + (B+C) = (A+B) + C$

A	B	C	A+B	B+C	L.H.S A+(B+C)	R.H.S (A+B)+C
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

2.) $(AB)C = A(BC)$

A	B	C	AB	BC	L.H.S (AB)C	R.H.S A(BC)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

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Distributive Law

1) $A(B+C) = AB + BC$

A	B	C	B+C	AB	AC	L.H.S A(B+C)	R.H.S AB+AC
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

DEMORGAN'S THEOREM

1.) $\overline{AB} = \overline{A} + \overline{B}$

A	B	$\overline{AB}$	$\overline{A}+\overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

2) $\overline{A+B} = \bar{A}\bar{B}$

A	B	$\overline{A+B}$	$\bar{A}\bar{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

CONSENSUS THEOREM

$$AB + \bar{A}C + BC = AB + \bar{A}C$$

Proof:

$$\begin{aligned} L.H.S &= AB + \bar{A}C + BC \\ &= AB + \bar{A}C + (A+\bar{A})BC \\ &= AB + \bar{A}C + ABC + \bar{A}BC \\ &= AB + ABC + \bar{A}C + \bar{A}BC \\ &= AB(1+C) + \bar{A}C(1+B) \end{aligned}$$

Here $1+C=1$ and $1+B=1$

$\therefore AB + \bar{A}C + BC = AB + \bar{A}C$

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3.) Simplify $xy + x'z + yz$

Solu:

By consensus theorem

$$xy + x'z + yz = xy + x'z + (x+x')yz$$

$$= xy + x'z + xyz + x'yz$$

$$= xy + xyz + x'z + x'yz$$

$$= xy(1+z) + x'z(1+y)$$

Here $1+z = 1$ and $1+y = 1$

$$\therefore xy + x'z + yz = xy + x'z$$

PRINCIPLE OF DUALITY.

The Principle of Duality theorem States that Starting with a Boolean relation we can derive another Boolean relation by

1.) Changing each OR Sign to AND Sign

2.) Changing each AND Sign to OR Sign.

3.) Complementing any 0 or 1 appearing in the expression

Ex. Boolean relation $A + \bar{A} = 1$

Apply Duality Principle we get another relation $A \cdot \bar{A} = 0$

Boolean Relation	Dual
$A+0=A$	$A\cdot 1=A$
$A+1=1$	$A\cdot 0=0$
$A+A=A$	$A\cdot A=A$
$A+\bar{A}=1$	$A\cdot\bar{A}=0$
$A+AB=A$	$A\cdot(A+B)=A$
$A+\bar{A}B=A+B$	$A\cdot(\bar{A}+B)=AB$
$(A+B)(A+C)=A+BC$	$AB+AC=A(B+C)$
$AB+\bar{A}C+BC=AB+\bar{A}C$	$(A+B)(\bar{A}+C)(B+C)=(A+B)(\bar{A}+C$

BOOLEAN EXPRESSION.

MINIMIZATION.

Rule 1:

$0+0=0$

$0+1=1$

$0+A=A$
(or)
$A+0=A$

Rule 2:

$1+0=1$

$1+1=1$

$1+A=1$
(or)
$A+1=1$

Rule 3:

$0+0=0$

$1+1=1$

$A+A=A.$

Rule 4:

$0+1=1$

$1+0=1$

$A+\bar{A}=1$
(or)
$\bar{A}+A=1$

Rule 5:

$0 \cdot 0 = 0$ $\qquad$ $0 \cdot A = 0$

(or)

$0 \cdot 1 = 0$ $\qquad$ $A \cdot 0 = 0$

Rule 6:

$1 \cdot 0 = 0$ $\qquad$ $1 \cdot A = A$

(or)

$1 \cdot 1 = 1$ $\qquad$ $A \cdot 1 = A$

Rule 7:

$0 \cdot 0 = 0$

$1 \cdot 1 = 1$ $\qquad$ $A \cdot A = A$

Rule 8:

$0 \cdot 1 = 0$ $\qquad$ $A \cdot \bar{A} = 0$

(or)

$1 \cdot 0 = 0$ $\qquad$ $\bar{A} \cdot A = 0$

Rule 9:

$\bar{\bar{0}} = 0$ $\qquad$ $\bar{\bar{A}} = A$

$\bar{\bar{1}} = 1$

Rule: 10:

$A + AB = A$

<u>Proof</u>:

$L.H.S = A + AB$ $\qquad$ [Rule 2: $1 + B = 1$]

$= A(1+B)$

$= A$

$R.H.S$

Rule: 11

$$A + \bar{A}B = A + B$$

$L.H.S = A + \bar{A}B$ [Rule 10: $A + AB = A$]

$= A + AB + \bar{A}B$ [Rule 4: $A + \bar{A} = 1$]

$= R.H.S$

Rule: 12

$$(A+B)(A+C) = A + BC$$

$L.H.S = (A+B)(A+C)$

$= AA + AC + AB + BC$ [Rule 7: $A \cdot A = A$]

$= A + AC + AB + BC$

$= A(1+C) + AB + BC$ [Rule 2: $1 + C = 1$]

$= A + AB + BC$

$= A(1+B) + BC$ [Rule 2: $1 + B = 1$]

$= A + BC = R.H.S$