

Boolean Relation

$$A+0=A$$

$$A+1=1$$

$$A+A=A$$

$$A+\bar{A}=1$$

$$A+AB=A$$

$$A+\bar{A}B=A+B$$

$$(A+B)(A+C)=A+BC$$

$$AB+\bar{A}C+BC=AB+\bar{A}C$$

Dual

$$A \cdot 1 = A$$

$$A \cdot 0 = 0$$

$$A \cdot A = A$$

$$A \cdot \bar{A} = 0$$

$$A \cdot (A+B) = A$$

$$A \cdot (\bar{A}+B) = AB$$

$$AB+AC = A(B+C)$$

$$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$$

BOOLEAN EXPRESSION.

MINIMIZATION.

Rule 1:

$$\begin{array}{l} 0 + \boxed{0} = \boxed{0} \\ 0 + \boxed{1} = \boxed{1} \end{array}$$

$$0+A=A$$

(or)

$$A+0=A$$

Rule 2:

$$\begin{array}{l} 1 + \boxed{0} = 1 \\ 1 + \boxed{1} = 1 \end{array}$$

$$1+A=1$$

$$A+1=1$$

Rule 3:

$$\begin{array}{l} \boxed{0} + \boxed{0} = \boxed{0} \\ \boxed{1} + \boxed{1} = \boxed{1} \end{array}$$

$$A+A=A$$

Rule 4:

$$\begin{array}{l} \boxed{0} + \boxed{1} = \boxed{\text{crossed out}} \\ \boxed{1} + \boxed{0} = \boxed{\text{crossed out}} \end{array}$$

$$A+\bar{A}=1$$

(or)

$$\bar{A}+A=1$$

Rule 5:

$$0 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$0 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$0 \cdot A = 0$$

(or)

$$A \cdot 0 = 0$$

Rule 6:

$$1 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$1 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$1 \cdot A = A$$

(or)

$$A \cdot 1 = A$$

Rule 7:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A \cdot A = A$$

Rule 8:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A \cdot \bar{A} = 0$$

(or)

$$\bar{A} \cdot A = 0$$

Rule 9:

$$\begin{bmatrix} \bar{0} \\ \bar{1} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \bar{0} \\ \bar{1} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bar{\bar{A}} = A$$

Rule 10:

$$A + AB = A$$

Proof:

$$\text{L.H.S} = A + AB$$

$$= A(1+B)$$

$$= A$$

$$\text{R.H.S}$$

$$[\text{Rule 2: } 1+B=1]$$

Rule: 11

$$A + \bar{A}B = A + B$$

$$\text{L.H.S} = A + \bar{A}B$$

$$[\text{Rule 10: } A + AB = A]$$

$$= A + AB + \bar{A}B = A + (A + \bar{A})B$$

$$[\text{Rule 4: } A + \bar{A} = 1]$$

$$\therefore A + B = \text{R.H.S}$$

Rule: 12

$$(A+B)(A+C) = A + BC$$

$$\text{L.H.S} = (A+B)(A+C)$$

$$= AA + AC + AB + BC$$

$$[\text{Rule 7: } A \cdot A = A]$$

$$= A + AC + AB + BC$$

$$= A(1+C) + AB + BC$$

$$[\text{Rule 2: } 1+C = 1]$$

$$= A + AB + BC$$

$$= A(1+B) + BC$$

$$[\text{Rule 2: } 1+B = 1]$$

$$= A + BC = \text{R.H.S}$$