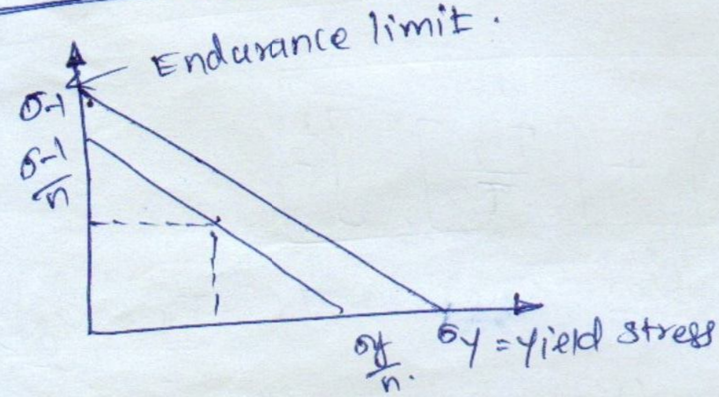
Design of variable loads

1) Soderberg line.
2) Goodman diagram.
3) Gerber parabola.

[From PSG DDB pg.no 7·4, 7·6]

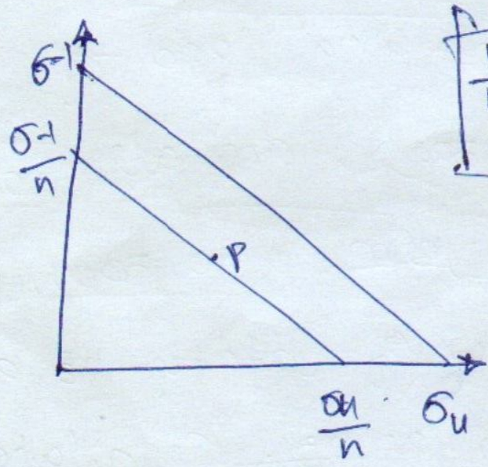
1) Soderberg line:-



$$\boxed{\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + k_t \frac{\sigma_a}{\sigma_{-1}}} \; ; \; \boxed{\frac{1}{n} = \frac{\tau_m}{\tau_y} + k_t \frac{\tau_a}{\tau_{-1}}}$$

For ductile materials.

2) Goodman diagram:-



$$\boxed{\frac{1}{n} = k_t \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1}}\right]} \; ;$$

$$\boxed{\frac{1}{n} = k_t \left[\frac{\tau_m}{\tau_u} + \frac{\tau_a}{\tau_{-1}}\right]}$$

for brittle materials.

27. A machine component is subjected to flutural stress which fluctuates b/w $+300 MN/mm^2$ and $-150 MN/mm^2$. Determine the minimum ultimate strength according to soderberg relation and goodmans.

Take yield strength at $0.55\sigma_u = \sigma_y$. and endurance strength $\sigma_e = 0.5 \times \sigma_u$ at the FOS at 2.

Given data:-

$\sigma_{max} = 300 \times 10^6 N/mm^2$.

$\sigma_{min} = -150 \times 10^6 N/mm^2$.

$\sigma_y = 0.5 \times \sigma_u$.

FOS (or) $n = 2$.

σ_e (or) $\sigma_{-1} = 0.5\,\sigma_u$.

To find

$\sigma_u = ?$

Solution:-

[From PSG DDB Pg. no 7.6].

i) Soderberg relation

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + k_f \cdot \frac{\sigma_a}{\sigma_{-1}}.$$

$$\therefore \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{(300-150)}{2} \times 10^6,$$

$$\boxed{\sigma_m = 75 \times 10^6 N/mm^2}$$

$$\sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2} = \frac{(300+150)\times10^6}{2}$$

$$\boxed{\sigma_a = 225\times10^6 N/mm^2}$$

$$\frac{1}{2} = \frac{75\times10^6}{0.55\sigma_u} + 1.5\left(\frac{225\times10^6}{0.5\times\sigma_u}\right]$$

$$\Rightarrow \frac{1}{2} = \frac{136.36\times10^6}{\sigma_u} + \frac{675\times10^6}{\sigma_u}$$

$$\boxed{\sigma_u = 1.62\times10^9 N/mm^2}$$

ii). Good man's equation

$$\frac{1}{n} = k_t\left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1}}\right]$$

$$\Rightarrow \frac{1}{2} = 1.5\left[\frac{75\times10^6}{\sigma_u} + \frac{225\times10^6}{0.5\sigma_u}\right]$$

$$\therefore \sigma_u = \left[112.5\times10^6 + 6.75\times10^6\right]2$$

$$\boxed{\sigma_u = 1.58\times10^9 N/mm^2}$$ //

37. A hot rolled steel shaft is subjected to a torsional moment that varies from 330 N-m clockwise to 110 N-m counter clockwise and an applied bending moment at a critical section varies from 440 N-m to -220 N-m. The shaft its of uniform cross-section and no key way is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 MN/m² and a yield strength of 410 MN/m². Take the endurance limit as half the ultimate strength. FOS = 2. ~~Size~~ Size factor of 0.85 and a surface factor of 0.62. [2008, 2013]

Solution:-

$T_{max} = 330$ N-m (clockwise).

$T_{min} = -110$ N-m (counter clock wise.)

$M_{max} = 440$ N-m

$M_{min} = -220$ N-m.

To find:-

d = diameter of the shaft = ?.

$\sigma_u = 550\ MN/m^2 \Rightarrow 550 \times 10^6\ N/m^2$.

$\sigma_y = 410\ MN/m^2 = 410 \times 10^6\ N/m^2$.

$\sigma_{-1} = \frac{1}{2}\ \sigma_u = 275 \times 10^6\ N/m^2$.

F.O.S = 2.

$K_{sz} = 0.85$

Solution:-

W.K.T,

$$T_m = \frac{T_{max}+T_{min}}{2} = \frac{330+(-110)}{2} = 110\ \text{N-m}$$

$$T_a = \frac{T_{max}-T_{min}}{2} = \frac{330-(-110)}{2} = 220\ \text{N-m}$$

$\tau_m = ?$

$$\frac{T}{J} = \frac{\tau}{r}. \quad \frac{T_m}{\pi/32\, d^4} = \frac{\tau_m}{d/2} \quad \boxed{\tau_m = \frac{16 T_m}{\pi d^3}}$$

$$\tau_m = \frac{16 T_m}{\pi d^3} \Rightarrow \frac{16\times110}{\pi d^3} = \frac{560}{d^3}\ \text{N/m}^2$$

$\tau_a = ?$

$$\frac{T}{J} = \frac{\tau}{r}.$$

$$\tau_a = \frac{16 T_a}{\pi d^3} = \frac{16\times220}{\pi d^3} = \frac{1120}{d^3}\ \text{N/m}^2$$

$\tau_{-1} = 0.55\,\sigma_{-1}$ [From DDB. pg.no 1.42]

$\tau_{-1} = 0.55\times 27\times10^6 \Rightarrow 151.25\times10^6\ \text{N/m}^2$.

$\tau_y = \sigma_y/2$

$\tau_y = 410/2 = 205\times10^6\ \text{N/m}^2$.

$k_f = ?$

$$\tau_{eq} = \frac{\tau_y}{n} = \tau_m + k_f \cdot \frac{\tau_a \cdot \tau_y}{\tau_{-1} \times k_{sz} \times k_{sur}}$$

$k_f = (1 + q(k_t - 1))$.
Assume $q = 1$.
$k_f =$
Assume $k_f = 1$

$$\tau_{eq} = \frac{560}{d^3} + 1 \cdot \frac{1120/d^3 \times 205 \times 10^6}{151.25 \times 10^6 \times 0.85 \times 0.62}$$

$$\tau_{eq} = \frac{560}{d^3} + \frac{1120/d^3 \times 205 \times 10^6}{151.25 \times 10^6 \times 0.85 \times 0.62}$$

$$\tau_{eq} = \frac{560}{d^3} + \frac{2880}{d^3}$$

$$\boxed{\tau_{eq} = \frac{3440}{d^3} \; N/m^2}$$

$$\sigma_{eq} = \sigma_m + k_f \frac{\sigma_a \times \sigma_y}{\sigma_{-1} \times k_{sz} \times k_{sur}}$$

$\sigma_m = ?$ $\quad \frac{M}{I} = \frac{\sigma}{y}$; $\sigma = \frac{M \times y}{I}$ $\quad Z = I/y$.

$\sigma = \frac{M}{Z}$.

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{440 + (-220)}{2} = 110 \, N\text{-}m.$$

$$M_a = \frac{M_{max} - M_{min}}{2} = \frac{440 - (-220)}{2} = 330 \, N\text{-}m.$$

$Z = \pi/32 \, d^3$.

$$\boxed{Z = 0.0982 d^3 \, m^3}$$

$$Z = I/y = \frac{\pi/64 \, d^4}{d/2}$$

$$\sigma_m = \frac{M_m}{z} = \frac{110}{0.0982 d^3} = \frac{1120}{d^3} \ N/m^2.$$

$$\sigma_a = \frac{M_v}{z} = \frac{330}{0.0982 d^3} = \frac{3360}{d^3} \ N/m^2.$$

$$\sigma_{eq} = \sigma_m + k_f \cdot \frac{\sigma_a \cdot \sigma_y}{\sigma_{-1} + K_{sz} + K_{sur}}$$

$$\sigma_{eq} = \frac{1120}{d^3} + 1 \times \frac{3360 + 410 \times 10^6}{d^3 \times 275 \times 10^6 + 0.62 \times 0.85}$$

$$\boxed{\sigma_{eq} = \frac{10626}{d^3} \ N/m^2.}$$

W.k.t, Max. equ shear stress

$$\tau_{eq}(max) = \frac{\tau_y}{FOS} = \frac{1}{2}\sqrt{(\sigma_{eq})^2 + 4(\tau_{eq})^2}$$

$$\frac{205 \times 10^6}{2} = \frac{1}{2}\sqrt{\left[\frac{10625}{d^3}\right]^2 + 4\left[\frac{3440}{d^3}\right]^2}.$$

$$205 \times 10^6 \times d^3 = \sqrt{113 \times 10^6 + 4 \times 11.84 \times 10^6} = 12.66 \times 10^3.$$

$$d^3 = \frac{12.66 \times 10^3}{205 \times 10^6} = \frac{0.0617}{10^3}.$$

$d = 0.395m$. $\boxed{d = 39.5mm}$ (or) $\boxed{d = 40mm.}$

~~4) A cantilever rod of circular section~~

4) A cantilever beam made of cold drawn carbon steel of circular cross-section as shown in fig. is subjected to a load which varies from -F to 3F. Determine the max. load that this member can withstand for an indefinite life using a factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9. Assume the following values.

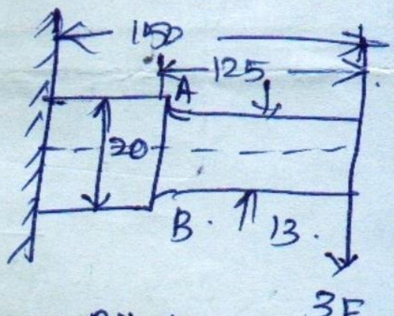
Ultimate stress = 550 MPa.

Yield stress = 470 MPa.

Endurance limit = 275 MPa.

Size factor = 0.85.

~~Size~~ surface finish factor = 0.89.



All dimensions are in mm.

Sol:-

Wmin = -F;

Wmax = 3F;

FOS (or) n = 2.

$K_t = 1.42$.

$q = 0.9$.

$\sigma_u = 550 MPa = 550 N/mm^2$.

$\sigma_y = 470 MPa = 470 N/mm^2$.

$\sigma_{e-1} = 275 MPa = 275 N/mm^2$.

$K_{sz} = 0.85$.

$K_{sur} = 0.89$.

$$\sigma_m = \frac{M_m}{Z} = \frac{110}{0.0982d^3} = \frac{1120}{d^3} \ N/m^2$$

$$\sigma_a = \frac{M_v}{Z} = \frac{330}{0.0982d^3} = \frac{3360}{d^3} \ N/m^2.$$

$$\sigma_{eq} = \sigma_m + k_f \cdot \frac{\sigma_a \cdot \sigma_y}{\sigma_1 + K_{sz} \cancel{+} K_{sur}}$$

$$\sigma_{eq} = \frac{1120}{d^3} + 1 \times \frac{3360 + 410 \times 10^6}{d^3 \times 275 \times 10^6 + 0.62 \times 0.85}$$

$$\boxed{\sigma_{eq} = \frac{10626}{d^3} \ N/m^2.}$$

W.k.t, Max. equ shear stress

$$\tau_{eq}\ (max) = \frac{\tau_y}{F_{os}} = \frac{1}{2}\sqrt{(\sigma_{eq})^2 + 4(\tau_{eq})^2}$$

$$\frac{205 \times 10^6}{2} = \frac{1}{2}\sqrt{\left[\frac{10625}{d^3}\right]^2 + 4\left[\frac{3440}{d^3}\right]^2}$$

$$205 \times 10^6 \times d^3 = \sqrt{113 \times 10^6 + 4 \times 11.84 \times 10^6} = 12.66 \times 10^3.$$

$$d^3 = \frac{12.66 \times 10^3}{205 \times 10^6} = \frac{0.0617}{10^3}.$$

$$\underline{d = 0.395m} \quad \boxed{d = 39.5mm} \ (or) \ \boxed{d = 40mm.}$$

The beam is subjected to a reversed (37)

bending load only. Since the point A at the change of cross section is critical, therefore we shall find the bending moment at point A.

Max. bending moment at point A

$$M_{max} = W_{max} \times 125 = 3F \times 125 = \underline{375F \text{ N-mm}}$$

Min. bending moment at point B

$$M_{min} = W_{min} \times 125 = -F \times 125 = \underline{-125F \text{ N-mm}}$$

$$M_m = \frac{M_{Max} + M_{Min}}{2} = \frac{375 + (-125F)}{2}$$

$$\boxed{M_m = 125F \text{ N-mm}}$$

$[d = 13 \text{mm}]$

$$M_a = \frac{M_{max} - M_{min}}{2} = 250F \text{ N-mm}$$

Section modulus, $Z = \pi/32 \, d^3 = \pi/32 \, (13)^3 = \underline{215.7 \text{ mm}^3}$

$$\sigma_m = \frac{M_m}{Z} = \frac{125F}{215.7} = \underline{0.58F \text{ N/mm}^2}$$

$$\sigma_a = \frac{M_a}{Z} = \frac{250F}{215.7} = \underline{1.16F \text{ N/mm}^2}$$

Fatigue stress concentration factor,

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(1.42 - 1) = \underline{1.378}$$

Goodman's formula.

$$\frac{1}{F.S} = K_t \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1} \times K_{sur} \times K_{sz}} \right]$$

$$~~\tfrac{1}{2} = 0.58~~ \quad 1/2 = 1.42 \left[\frac{0.58F}{550} + \frac{1.16F \times 8}{275 \times 0.89 \times} \right]$$

$$\boxed{F = 57.3N}$$

Soderberg's formula

$$\frac{1}{F.S} = \frac{\sigma m}{\sigma y} + Kt \frac{\sigma a}{\sigma_{-1} \cdot k_{sz} \cdot k_{sur}}$$

$$\frac{1}{2} = \frac{0.58F}{470} + \frac{1.16F}{275 \times 0.89 \times 0.85} \times 1.42$$

$$\boxed{F = 56N}$$

taking larger of the two values,

$$\boxed{F = 57.3N}$$ //.