MECHANICAL PROPERTIES OF SOLIDS

A rigid body means a hard solid object having a definite shape and size. Its dimensions do not change when a force is applied on it. But in reality, bodies do ~~not~~ undergo changes in their dimensions when some external force is applied on them. This shows that solid bodies are not perfectly rigid.

The property by virtue of which a body tends to regain its original size and shape, when applied force is removed is called <u>elasticity</u>. The elastic behaviour of material plays an important role in engineering designs while constructing bridges, automobiles, rope-ways etc.

Elastic behaviour of solids

Elastic behaviour of solids can be understood by taking microscopic nature of the body. A solid body consists of atoms or molecules, where each atom or molecule is surrounded by other atoms or molecules. These atoms or molecules are bonded together by interatomic or intermolecular forces and stay in equilibrium position.

This can be seen by a model of spring ball system (for fig — refer text). Here the balls represent the atoms while springs represent the interatomic forces. In this system, if any ball is displaced a little from its equilibrium position, the springs attached to that ball will either be stretched or compressed. Due to this, restoring forces will come into play in the springs and they will bring the ball back to its original position. This accounts for the elastic behaviour of the solid body.

STRESS

When a deforming force is applied on a body such that it changes the configuration of the body (by changing the normal positions of the molecules/atoms), then an internal ~~restoring~~ restoring force is developed which tends to bring the body back ~~in~~ to its original configuration. This internal restoring force acting per unit area of a deformed body is called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}.$$

S.I unit N/m^2 or Pascal.

If there is no permanent change in the configuration of the body, the restoring force is equal & opposite to the external deforming force applied.

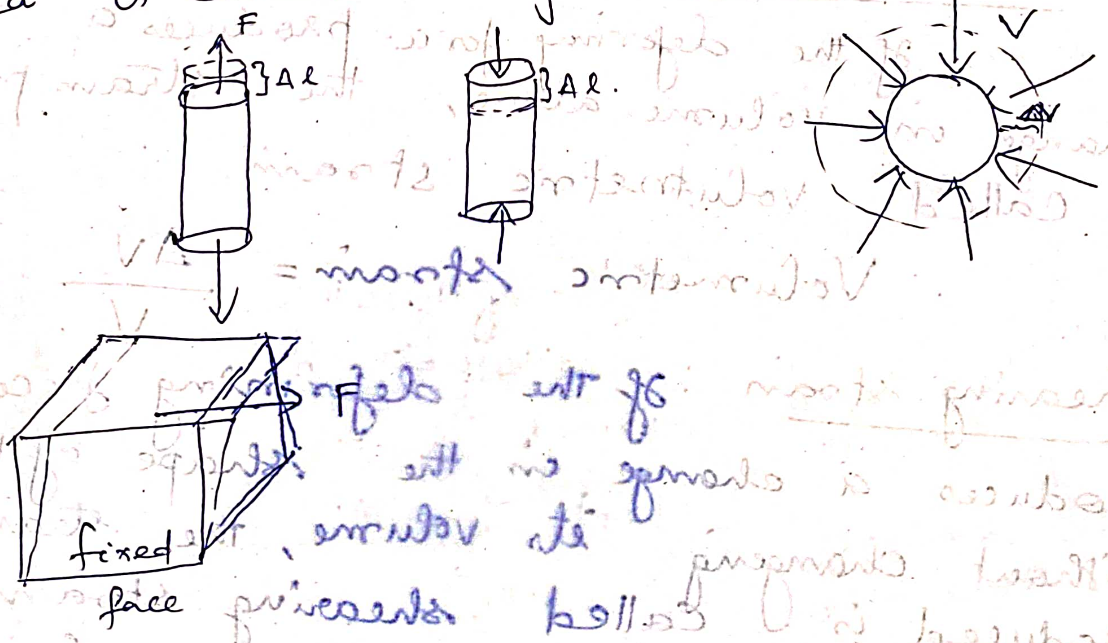
Types of stress

There are 3 diff. ways in which stress can be applied.

(i) Tensile stress : It is the restoring force developed per unit cross sectional area of a body when the length of the body increase in the direction of force. Also known as longitudinal stress. Compressional stress produces a decrease in length of a body due to force applied.

(ii) Hydraulic stress – If a body is subjected to a uniform force from all sides, ~~then the~~ such that the body undergoes change in its volume, then the restoring force developed per unit area is called hydraulic stress.

(iii) Tangential or shearing stress : when a deforming force acts tangentially to the surface of a body producing a change in the shape of the body without any change in its volume, then the tangential force applied per unit area is called tangential stress.



STRAIN

The ratio of change in any dimension ~~configuration~~ to the original dimension is called strain..

$$\text{strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

It is a unitless, dimensionless quantity.

Longitudinal strain

If the deforming force produces a change in length alone, then the strain is referred to as longitudinal strain.

$$\text{Longitudinal strain} = \frac{\text{Change in length } (\Delta l)}{\text{Original length } l}$$

Volumetric strain

If the deforming force produces a change in volume alone, the strain produced is called volumetric strain

$$\text{Volumetric strain} = \frac{\Delta V}{V}$$

Shearing strain: If the deforming force produces a change in the shape of the body without changing its volume, the strain produced is called shearing strain.

It is defined as the angle θ (in radian) through which a face original perpendicular to the fixed face gets ~~twisted~~ turned under the effect of tangential force.

Shear strain, $\tan\theta$

$$= \frac{\text{Relative displacement b/w parallel planes}}{\text{distance b/w parallel planes}} = \frac{\Delta l}{L}$$

ELASTIC LIMIT:

The maximum stress within which the body regains its original size and shape after the removal of deforming force is referred to as elastic limit. If the deforming force exceeds the elastic limit, the body acquires a permanent set or deformation and is said to be 'overstrained'.

HOOKE's LAW

The extension produced in a wire is directly proportional to the load applied.

Hooke's law states that within elastic limit the stress developed is proportional to the strain produced in a body.

$$\text{stress} \propto \text{strain}$$

$$\text{stress} = E \times \text{strain}$$

$$\frac{\text{stress}}{\text{strain}} = E$$

where E is a constant known as Modulus of elasticity of material of a body. (Also known as coefficient of elasticity)

* The value of E depends on the nature of material of the body and the manner in which it is deformed.

*

TYPES OF MODULUS OF ELASTICITY.

(i) Young's Modulus of elasticity: Within elastic limit, the ratio of longitudinal stress to the longitudinal ~~stress~~ Strain is known as the Young's modulus of elasticity (Y).

$$Y = \frac{F/A}{\frac{\Delta l}{l}} = \frac{Fl}{A\Delta l}.$$

For a wire of circular cross section,
$A = \pi r^2$

$$\therefore Y = \frac{Fl}{\pi r^2 \Delta l}.$$

SI unit is Nm^{-2} or Pa.

* Young's Modulus is involved in solids only.

(ii) BULK MODULUS OF ELASTICITY (B):

Within elastic limit, the ratio of normal stress to the volumetric strain is referred to as bulk modulus of elasticity.

$$B = \frac{F/A}{-\frac{\Delta V}{V}} = -\frac{PV}{\Delta V}$$

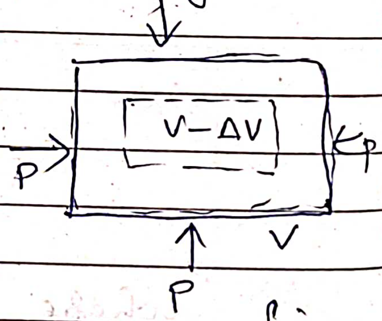


fig.

The negative sign indicates that the volume decreases with increase in stress.

S.I unit is Pa.

Compressibility (k) of a material is the reciprocal of its bulk modulus of elasticity.

$$K = \frac{1}{B}.$$

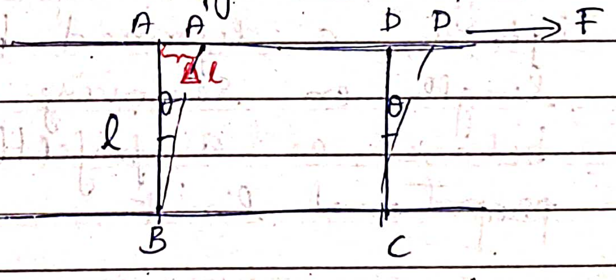
* Bulk modulus is involved in solids, liquids and gases. Bulk modulus is largest for solids. (as all have volume)

(iii) <u>Modulus of rigidity</u> or shear modulus

Within elastic limit, the ratio of tangential stress to shearing strain is called shear modulus of elasticity (η).

Tangential stress $= F/A$

where F is the ~~tangential~~ deforming force applied parallel/tangential to a surface then the solid deforms with the surface twisting through an angle 'θ' as shown in fig.



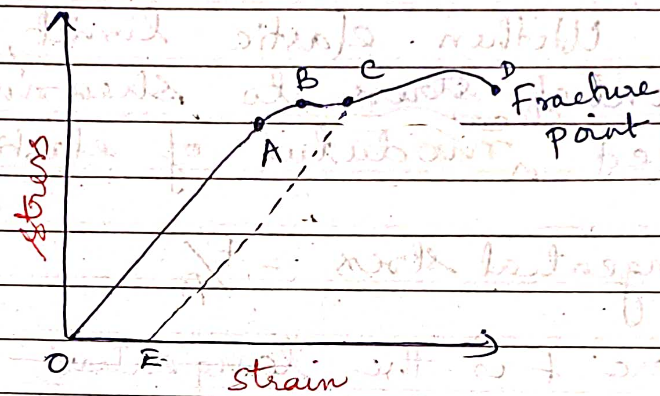
$$\text{shear strain} = \theta \approx \tan\theta = \frac{AA'}{AB} = \frac{\Delta l}{l}.$$

$$\therefore \eta = \frac{F/A}{\theta} = \frac{F}{A\theta}.$$

shear modulus of a material is smaller than its Young's modulus. This shows that it is easier to slide layers of atoms of solids over one another than to pull them apart or to squeeze them close together.

STRESS - STRAIN CURVE FOR A LOADED METALLIC WIRE.

A metallic wire suspended from a rigid support is ~~loaded using~~ different subjected to different amount of stress ie loaded using different weights. A graph is plotted for the stress given and the corresponding strain produced as shown below:



* For fairly large value of stress, the stress-strain relation is linear or Hooke's law is obeyed - OA shows this region.
Upto A, a deformed body regains its original length on removing deforming forces hence behaves as a perfectly elastic body - A is the proportional limit

* After A - stress is not proportional to strain. If load is removed at any point b/w O and B, curve is retraced along BAO or wire attains original length. OB is the elastic region. Point B is the elastic limit or yield point.

* Beyond B, the strain increases rapidly than stress. If load is removed at any point at C, the wire does not come back to original length but traces the dashed line CE. Even on reducing the stress to zero, a residual

strain equal to OE is left in the wire. The material is said to have acquired permanent set.

* Beyond C, there is a large increase in the strain. In this regions, constrictions develop at few points along the length of the wire and the wire ultimately breaks at the point D, called the fracture point. In this region, the length of the wire increases without even addition of load - known as the plastic region - the material is said to undergo plastic flow.

* The stress corresponding to breaking point is called ultimate strength or tensile strength of the material.

* Ductile materials show large plastic range beyond elastic limit. Eg Copper, silver, iron etc. Pure gold is brittle.
* Brittle materials have small plastic region beyond elastic limit. For such materials, breaking point lies close to the elastic limit eg: Cast iron, glass etc.

Elastomers : Materials like rubber behave differently under stress. It can be extended to several times its length and on removing the load, it regains its original shape. For such materials, the plastic region is not well defined. They are called elastomers and their stress - strain relationship is not linear within elastic limit.
Eg: Rubber, elastic tissue of aorta etc are egs.

Elastic After effect : Some materials return to their origenal state immediately after the removal of deforming force while some take longer time to do so. The delay in regaining the origenal state by a body on the removal of deforming force is called elastic after effect.
Eg: In galvanometers, suspensions made from quartz are used because its elastic after effect is very small.

Elastic fatigue: It is defenied as the loss in strength of a material caused due to repeated alternating stress-strain cycles to which material is subjected. This causes reduction in elastic behaviour.
Eg: A wire can be broken by bending it repeatedly in opposite directions.

APPLICATIONS OF ELASTICITY

* The metallic parts of the machinery are never subjected to a stress beyond elastic limit, otherwise they will be permanently deformed.

* While using a material, the working stress is always kept much lower than that of breaking stress so that safety factor may have a large value.
The thickness of metallic ropes used in cranes to lift very heavy load is decided from the knowledge of elastic ~~behaviour~~ limit of the material of rope and factor of safety.

Ultimate stress = $\frac{\text{Load to be lifted}}{\text{area of cross section of rope}}$

Ultimate stress for steel is $3 \times 10^8 \, N m^{-2}$

$$= \frac{mg}{\pi r^2}$$

$\therefore r \simeq 1 cm$ for a load of $10^5 N$

Thus for a load of $10^5 N$ (or $10^4 kg$), the diameter of the wire should be of several centimeters. A wire of large diameter would be a rigid rod so in order to impart flexibility, several thin wires are braided together to have this diameter.

* The bridges are designed in such a way that they do not bend much or break under the load of heavy traffic, force of strong winds & its own weight.

Consider a bar of length 'l', breadth 'b' and depth 'd' supported horizontally at its 2 ends. Let Y be the young's modulus of material of the bar. When a load W is attached at its middle, the depression 'δ' produced in the bar (as shown in fig (i))

$$\delta = \frac{W l^3}{4 Y b d^3}$$

The most effective way to reduce depression in the beam of given length and material is to make depth d of the beam large as compared to its breadth b because

$$\delta \propto \frac{1}{d^3}$$

But increasing depth too much, the beam may bend as shown in fig. (a) and (b). This bending is called buckling. To check this buckling, a compromise between breadth and depth is made by making the cross section of the beam as I shaped as shown in fig (c) which has a large load bearing surface.

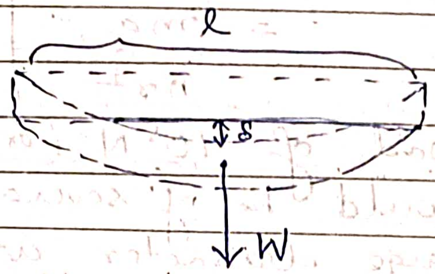
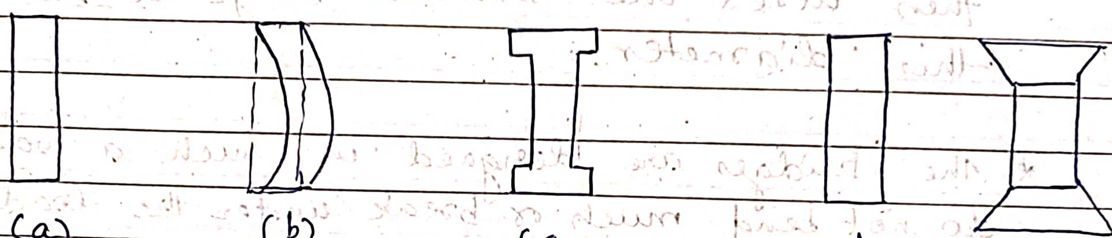


fig (i)



(a) (b) (c) (d) (e)

A pillar with a rounded base at the ends in figure (d) supports less load than that with a distributed shape at the ends fig (e).

Note for evaluation

* [At the bottom of a mountain of height 'h' the pressure due to the weight of the mountain is $h\rho g$ where 'ρ' is the density of material of mountain & 'g' – acceleration due to gravity. The material of mountain experiences this force in the vertical direction while the sides are free due to which a shear stress arises – which is approximately $h\rho g$ itself. Now the elastic limit for a rock is $30\times10^7\ Nm^{-2}$; ($\rho = 3\times10^3\ kgm^{-3}$)

$$h\rho g = 30\times10^7\ Nm^{-2}$$

$$h = \frac{30\times10^7}{(3\times10^3\times10)} = 10\ km.$$

which is nearly the height of Mt. everest]