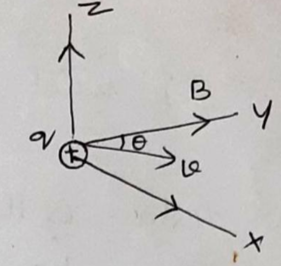
MOVING CHARGES & MAGNETISM

(1)

Force on a moving charge in a magnetic field

The force experienced by a charge 'q' moving with a velocity 'v' in a magnetic field of strength 'B' is magnetic Lorentz force given by $\vec{F} = q(\vec{v} \times \vec{B})$

$$= qvB\sin\theta$$



$\theta \rightarrow$ is the angle b/w $\vec{v}$ and $\vec{B}$

Case (i) If $v = 0$, then $F = 0$

Thus a stationary charged particle doesnot experience any force in a magnetic field.

Case (ii) If $\theta = 0^\circ$ or 180°; $F = 0$ $[\vec{v} \parallel \vec{B}]$

Thus a charged particle moving parallel or antiparallel to a magnetic field doesnot experience any force.

Case (iii) If $\theta = 90^\circ$, then $F = qvB\sin 90 = qvB$

Thus a charged particle experiences maximum force when it moves perpendicular to the magnetic field.

* The direction of force can be found out by Fleming's left hand rule or right hand thumb rule

* Definition of magnetic field

$$F = qvB\sin\theta$$

$$B = \frac{F}{qv\sin\theta}$$

If $q = 1C$, $v = 1 m/s$ and $\theta = 90^\circ$, then $B = F$

Thus the magnetic field at a point may be defined as the force acting on a unit charge moving with unit velocity at right angles to the magnetic field.

$$\text{S.I unit of } B = \frac{1N}{1C \cdot 1ms^{-1}} = \frac{1N}{1A\,1m} = 1\,NA^{-1}m^{-1}$$

$$= 1\,\text{Tesla}$$

One Tesla is that magnetic field in which a charge of 1C moving with a velocity of 1 m/s at right angle to the field experiences a force of 1 Newton

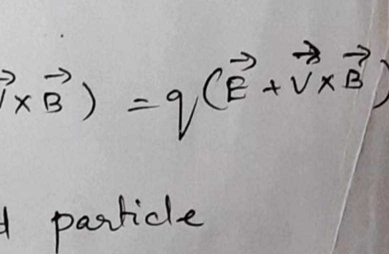
<u>Lorentz force</u>: The total force experienced by a charged particle moving in a region of combined electric and magnetic fields is called Lorentz Force.

The total force, $\vec{F} = \quad + F_{magnetic} = \vec{F}_e + \vec{F}_B$

$$\vec{F}_e = q\vec{E} \text{ and } \vec{F}_B = q(\vec{v} \times \vec{B})$$

$$\therefore \vec{F} = q\vec{E} + q(\vec{V} \times \vec{B}) = q(\vec{E} + \vec{V} \times \vec{B})$$

* Force experienced by a charged particle moving perpendicular to the magnetic field. ($V \perp B$)



A charge 'q' is projected with a velocity 'v' perpendicular to magnetic field (into the plane). A force $F = qvB$ acts $\perp$r to both $\vec{v}$ and $\vec{B}$. (This force continuously deflects the particle sideways and the particle will move along a circle perpendicular to the field). Thus the magnetic force provides the centripetal force. Let 'r' be the radius of the circular path.

Centripetal force, $\frac{mv^2}{r}$ = mag. force, qvB

$$\frac{mv^2}{r} = qvB$$

$$\boxed{r = \frac{mv}{qB}}$$

$v = r\omega$

$$\boxed{v = \frac{qBr}{m}}$$

$$v = \cancel{r}\omega = \frac{qB\cancel{r}}{m}$$

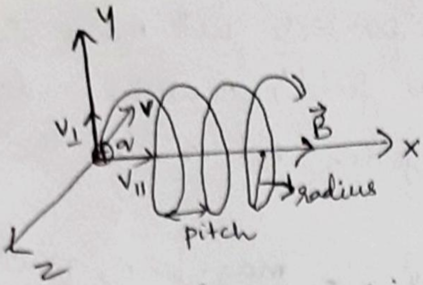
$$2\pi\nu = \frac{qB}{m}$$

$$\boxed{\nu = \frac{qB}{2\pi m}} \; ; \; \boxed{T = \frac{2\pi m}{qB}}$$

This frequency is independent of 'v' and 'r' and this is the principle of particle accelerator 'cyclotron'.

* When the initial velocity makes an arbitary angle θ with the field direction ($\vec{v}$ at an $\angle^{le}$ 'θ' w.r.t $\vec{B}$)

Consider a charged particle 'q' entering a uniform magnetic field 'B' with velocity 'v' inclined at an angle 'θ' w.r.t $\vec{B}$ as show[n]



Helical Motion

The velocity vector 'v' [is] resolved into two components $V_{\parallel}$ – parallel to the mag. field and $V_\perp$ – $\perp$r to B.

$V_{\parallel} = V\cos\theta$ makes it move along the direction of B and $V_\perp = V\sin\theta$ makes it move along a circular path $\perp$r to the field. Hence the trajectory will be helical.

$$r = \frac{mV_\perp}{qB} = \frac{mV\sin\theta}{qB}$$

The distance moved along the magnetic field in one rotation is called pitch of the helical path.

$$\text{Pitch} = V_{\parallel} \times T$$

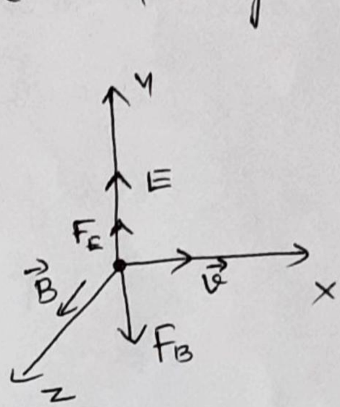
$$T = \frac{2\pi m}{qB} \; ; \; V_{\parallel} = V\cos\theta$$

$$\therefore p = V\cos\theta \times \frac{2\pi m}{qB}.$$

$$\boxed{p = \frac{2\pi m V\cos\theta}{qB}}$$

2. Force acting on a charged particle moving in the presence of crossed electric and magnetic field (Velocity selector).

A charge 'q' moving with velocity $\vec{v}$ in the presence of both electric and magnetic fields experiences a force given by



$F = (q\vec{E} + \vec{v} \times \vec{B}) = F_E + F_B$

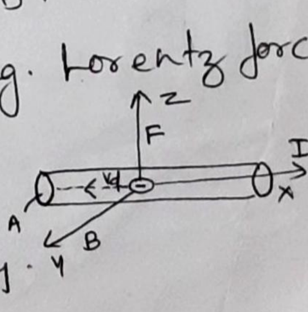
Here $\vec{E}$ and $\vec{B}$ are $\perp$r to each other and also $\perp$r to the velocity of the particle as shown in fig.

$F_E = qE\hat{j}$; $F_B = q(v\hat{i} \times B\hat{k}) = -qvB\hat{j}$

Thus electric and magnetic forces are in opposite directions as shown in fig. Therefore when a beam of charged particles possessing a range of speeds pass through a region of crossed electric and magnetic field, only those particles whose velocity satisfies the above condition moves undeflected. That is, the arrangement can be used as a velocity selector or velocity filter

* Force on a current carrying conductor kept in a mag. field

Consider a conductor of length 'l' and area cross section 'A' carrying a steady current 'I' kept in an external mag. field 'B'. Each $\bar{e}$ in the conductor experiences a mag. Lorentz force given by $F = -e(\vec{v_d} \times \vec{B})$



where v_d is the drift velocity.

If 'n' is the number density of $\bar{e}$s in the conductor, then total no. of $\bar{e}$s is

$$N = nV = nAl.$$

$\therefore$ Total force on the conductor

$$F = nAl\left[-e\left[\vec{V_d} \times \vec{B}\right]\right]$$

$-l v_d = v_d \vec{l}$ (as the $\bar{e}$s drift opp. to the direction of conventional current).

$$F = neAv_d(\vec{l} \times \vec{B})$$

$$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$$

Magnitude of force, $F = IlB\sin\theta$.
where θ is the angle b/w $\vec{B}$ and direction of current.

a) If $\theta = 0°$ or $180°$, $F = IlB\sin 0 = 0$. (it does not experience any force when current flows parallel to B)

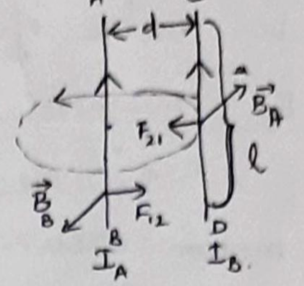
b) If $\theta = 90°$

$$F_{max} = IlB$$

force experienced is maximum when current flows $\perp$r to the direction of B. The direction of force is given by Fleming's LH rule or right hand thumb rule.

Force between two straight parallel conductors carrying steady current and definition of ampere.

Consider two long parallel wires AB and CD carrying currents I_A and I_B resp. in the same direction. Let 'd' be the separation b/w them.



The mag. field produced by I_A at any point on the wire CD is

$$B_A = \frac{\mu_0 I_A}{2\pi d}$$

This field acts perpendicular to the wire CD (into the plane). It exerts a force on the wire CD having length 'l' will be,

$$\vec{F}_{21} = I_B l B_A = I_B l \frac{\mu_0 I_A}{2\pi d} = \frac{\mu_0}{2\pi} \cdot \frac{I_A I_B}{d} \cdot l$$

According to Fleming LH rule, this force acts ~~at~~ $\perp^r$ to CD towards AB (in the plane). Similarly, an equal and opposite force is exerted on the wire AB by the mag. field produced by CD. This is given by,

$$\vec{F}_{12} = \frac{\mu_0}{2\pi} \cdot \frac{I_A I_B}{d} l. \quad [\text{acting towards CD}]$$

Force per unit length is

$$f = \frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_A I_B}{d}$$

Hence when the currents in the two wires are in the same direction, the forces b/w them are attractive. Also when the current are in opp. direction (antiparallel), the forces b/w them are repulsive.

Definition of ampere: when $I_A = I_B = 1A$ and $d = 1m$

then $$f = \frac{\mu_0}{2\pi} = \frac{{}^{2}\cancel{4\pi} \times 10^{-7}}{\cancel{2\pi}} = 2 \times 10^{-7}\ N/m$$

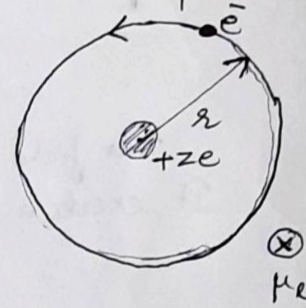
One ampere is that value of steady current, whi[ch] when maintained in two parallel long straight c[onductors] of negligible cross section, and placed one metre ap[art] in vacuum, would produce a force equal to 2×10^{-7} New[ton] per metre of length.

The magnetic dipole moment of a revolving electron

The electron of charge $(-e)$ performs uniform circular motion around a stationary heavy nucleus of charge $+Ze$. This can be considered as a current loop

$$I = \frac{e}{T} \quad \text{---(1)}$$

where T is the period of revolution. 'r' is the orbital radius and 'v' the orbital speed,



then $T = \dfrac{2\pi r}{v}$

Substituting in (1)

$$I = \frac{ev}{2\pi r}$$

The magnetic moment associated with this circulating current is $\mu_e = I\pi r^2 = \dfrac{ev}{2\cancel{\pi}\cancel{r}} \cdot \cancel{\pi} r^{\cancel{2}} = \dfrac{evr}{2}$

[The direction of μ_e is into the plane by right hand thumb rule. As the $\bar{e}$ moves in a.c.w direction, conventional current is in c.w. direction hence μ_e is into the plane].

$$\mu_e = \frac{evr \times m_e}{2 \times m_e} = \frac{e(m_e v r)}{2m_e} = \frac{el}{2m_e}.$$

where l is the orbital angular momentum.

$$\vec{\mu}_e = -\frac{e}{2m_e}\vec{l}$$

[$-ve$ sign indicates $\vec{l}$ is opposite to $\vec{\mu}_e$].

The ratio of magnetic moment to the orbital angular momentum is called gyromagnetic ratio and is a constant

$$\frac{\mu_l}{l} = \frac{e}{2m_e}$$

Bohr magneton is defined as the magnetic moment associated with an $\bar{e}$ due to its orbital motion in the first orbit of hydrogen atom. It is the minimum value of μ_l by putting $n=1$. Bohr magneton,

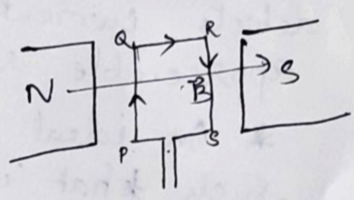
$l = mvr = \frac{nh}{2\pi}$

$$(\mu_l)_{min} = \frac{eh}{4\pi m_e} = 9.27 \times 10^{-24}\ Am^2$$

$(\mu_l)_{min} = \frac{enh}{4\times 2\pi \times m_e}$

Torque on a rectangular coil kept in an external magnetic field

Consider a rectangular coil PQRS placed in a uniform mag. field $\vec{B}$ with its axis perpendicular to the field.



$I \rightarrow$ current thru' the coil

$a, b \rightarrow$ sides of the coil

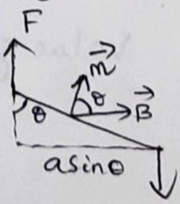
$A = ab =$ area of the coil

$\theta = \angle^{le}$ b/w the direction of $\vec{B}$ and normal to the area (area vector $A\hat{n}$)

Acc. to Fleming's LH rules forces on sides PQ and RS are equal and opp (along the axis), so their resultant is zero. These equal and opposite forces constitute a couple which rotates the coil.

$\tau =$ Force $\times$ perpendicular distance

$= IbB \times a\sin\theta = IB\underbrace{A}_{Area}\sin\theta$.



If the loop has N turns,

$\tau = NIAB\sin\theta$

But $NIA = m$, magnetic moment of the ~~loop~~ loop $\therefore \tau = mB\sin\theta$

Vectorially $\vec{\tau} = \vec{m} \times \vec{B}$

($\vec{m}$ is a vector whose direction is given by dir of area vector $A\hat{n}$. It is also given by direc of current by using Right hand thumb Rule ie of current is a.c.w, $\vec{m}$ is upward, if I is c. $\vec{m}$ is downward).

* when $\theta = 0°$, $\tau = 0$ ie torque is minimum when plane of the loop is $\perp$r to the magnetic field (or $A\hat{n} \parallel B$)
* when $\theta = 90°$, τ_{max} ie torque is max. when plane of the loop is parallel to mag. field (or $A\hat{n} \perp B$)

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