ELECTROSTATICS

Electrostatics is the study of electric charges at rest.

Applications

(i) In electrostatic loudspeaker.

(ii) Xerox Copying machines

(iii) In the design of Cathode Ray Tube used in TV and Radar.

* Electric charge is an intrinsic property of a material which gives rise to electric force between various objects.
* Two kinds of charges — there are only 2 kinds of charges - positive & negative
* Like charges repel and unlike charges attract each other.
* Atom as such is neutral and bulk matter consists of equal no. of $\bar{e}$s and protons making them neutral.
* If there is an excess of $\bar{e}$s, the body has a negative charge and an excess of protons results in a positive (or deficit of $\bar{e}$s).

II * The cause of charging is the actual transfer of electrons from one material to another.

I * Materials can be charged by
(i) Friction (~~also~~ Insulators) (ii) Conduction (conductors)
(iii) Induction (conductors or insulators)

III charging by friction

~~Ma~~ Materials (usually insulators) can be charged by rubbing it against another material for eg: glass rod rubbed against silk.
plastic scale rubbed against dry hair.

Explanation: An atom consists of a positively charged nucleus around which negatively charged $\bar{e}$s are revolving. As the no. of $\bar{e}$s and protons are equal in an atom or a piece of matter is neutral.

* The ēs in the outermost shell of an at[om] is loosely bound to the nucleus.
* The ~~lowest~~ min amount of energy required to pull off an electron from the surface of a material is called Workfunction.
* When two different bodies are rubbed ag[ainst] each other, ēs are transferred from mat[erial] with ~~higher~~ lower work function to that with higher work function.
* In the case of glass rod rubbed against silk cloth, some ēs from ~~silk~~ glass rod to silk cloth, leaving glass rod +vely charged and silk cloth negatively char[ged]

CONDUCTORS & INSULATORS.

* The substances thro' which electric charges flow easily are called Conductors. The contain a large no. of free ēs which can move easily within the material
Eg: ~~Metal~~ Metals, animal & human body, earth graphite, acids, alkalies etc are conductors.

* The substances through which electric charg[es] cannot flow easily are called insulators.
In such substances, ēs of the outer shell are tightly bound to the nucleus – known as bound charges which are not free to mo[ve]
Therefore they offer high resistance to the flow of electric charges.
Eg: glass, diamond, wood, rubber, mica, plast[ic] nylon etc.

* Most distinguishing feature b/w insulator & Conductor is that when some charge is

transferred to a conductor, it readily gets distributed over its surface, but if some charge is put on an insulator, it stays at in that region

* This is the reason why a metal rod held in hand cannot be charged — 'cos any charge that is developed on the metal is transferred to the earth through our body.
→ So rubbing a metal rod ~~for electrifying~~ with an insulating handle can be electrified.

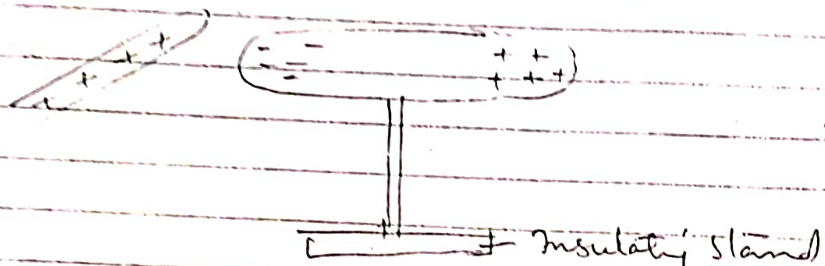
* Earthing –
when a charged body is brought in contact with the earth (thru' a ~~connecting~~ conductor), its entire charge passes to the earth. This process in which excessive charge in a body flows to the earth is called grounding or earthing.
(i) [The earth wire (green) in our main supply is connected to a thick metal plate burried deep into the earth. the metalic bodies of electric appliances such as electric iron, heater, TV etc are connected to the earth wire. When any fault occurs due to which live wire touches ~~neutral wire~~ the metallic body, the charge flows to the earth and the person who touches it is saved from a severe electric shock].
(ii) Wheels of aircraft is made ~~of~~ from a material which is slightly conducting – this is done to transfer any excess charges due to friction while landing.
(iii) Tankers containing inflammable liquids or gases have a metal rope underneath which is in constant touch with the ground to transfer the excessive charges due to friction.

ELECTROSTATIC INDUCTION

- It is the phenomenon of charging a material in which opp. charges appear at its closer end and similar charges appear at its farther end in the vicinity of a charged body.

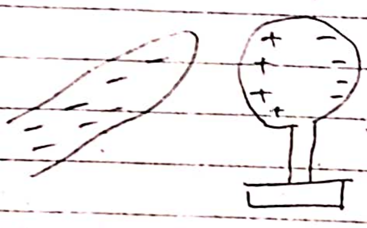
→ based on the law that uncharged object is always attracted by a charged object.

[Eg: when A metal rod is placed on a insulating stand and a +vely charged glass rod is brought near it −ve charges appear at its the end near to the charged rod & equal +ve charges appear ~~near~~ at the farther end]

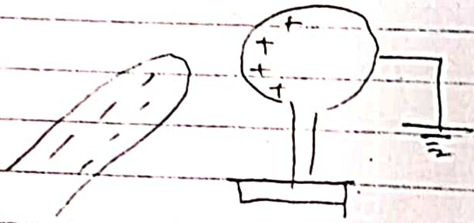


The +ve and −ve charges induced in the metal rod are called induced charges.

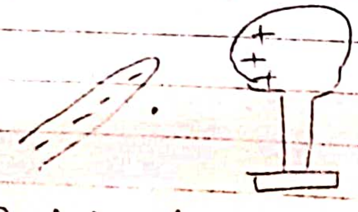
(i) Charging a metal sphere by Induction (+ve charge).



Opposite charges induced



excess negative charges flow to the ground thru' earthing

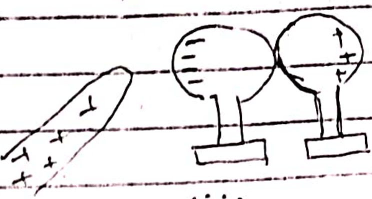


Positive charges in the near end of charged rod

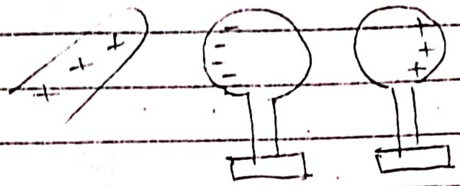
Positive charges distributed over the entire conductor

I Charging 2 spheres by Induction

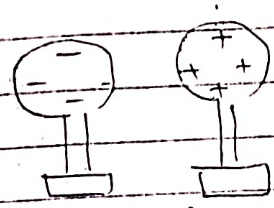
(i)
Two metal spheres in contact.



(ii)
Charge separation in the spheres as a +vely charged object is brought near 1st sphere.



(c) After equilibrium, They are moved away



(d) Rod taken away, the charges gets ~~Re~~distributed over the entire spheres.

Thus 2 metal spheres get oppositely charged by induction.

Charging by Conduction

* In charging by conduction (in conductors) - charges are shared by the bodies.
* Same charges are developed on an uncharged body when it comes in contact with a charged body.

Gold Leaf Electroscope - It is a device used for detecting an electric charge and identifying its polarity.

Three Basic properties of electric charge

(i) Additivity (ii) Quantisation
(iii) Conservation.

(i) Additivity : * Charge is a scalar quantity
* ~~the~~ additivity of charges means that total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.

Eg: ~~suppose~~ The total charge of a system containing four charges $2\mu C, -3\mu C, 4\mu C$ and $-5\mu C$

$$2 + (-3) + 4 + (-5) = -2\mu C$$

(ii) Quantization of charge: It is found experimentally that the electric charge of any body is always an integral multiple of elementary charge. This basic amount of charge ~~has~~ called quantum of charge is denoted by 'e' has a magnitude of $1.6 \times 10^{-19} C$. ~~Thus~~ (Eg: charge on an e^- is $-e$, on a proton is $+e$ and on α-particle is $+2e$)

* The fact that charges can occur in discrete amounts instead of continuous amounts is called quantization of electric charge.

Thus total charge in any system,

$$Q = ne \quad \text{where } n = 0, \pm 1, \pm 2 \ldots$$

[When we deal with macroscopic charges - we can ignore quantization of electric charge - it is considered only in microscopic levels.]

[when $Q = ne$

then 1 ~~coul~~ Coulomb of charge will have
$$n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \; e's .$$

thus Coulomb is a big unit, smaller units like μC, mC, nC etc].

(iii) Conservation of charge:

* the total charge on an isolated system remains constant.
* the electric charges can neither be created nor be destroyed, they can only be transferred from one body to another.

(Eg: when a glass rod is rubbed against a silk cloth, the ~~silk~~ glass rod ~~cloth~~ develops a ~~p~~ positive charge but at the same time, the silk cloth acquires an equal $-ve$ charge, thus the net charge of the glass rod and silk cloth is zero as it was before rubbing)

{ charge conservation is valid in all domains.

Eg. in β_- decay, a neutron gets converted in p inside the nucleus ie

$${}_0^1n \longrightarrow {}_1^1p + \bar{e} + \bar{\nu} + \text{Energy}.$$

β^+ decay 'p' converts into neutron.

$$p \longrightarrow n + e^+ + \nu$$

In both cases, charge is conserved }.

[Acc to Einstein's theory of relativi
the mass of a body increases with its
speed according to the relation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$m_0 \rightarrow$ rest
of the b

$m \rightarrow$ relat
bo

$V \rightarrow$ spe
veloci
bod
movin

$\therefore$ As $V < c$, $\therefore m > m_0$

In contrast to mass, charge on a body is always const and does not change as the speed of the body]

COULOMB'S LAW

It states that the force of attract or repulsion b/w 2 stationary point charg is (i) directly proportional to the prod of the charges and (ii) inversely propo tional to the square of the distance b/w them.

* This force always acts along the line joining the 2 charges.

If 2 point charges q_1 and q_2 are separated by a distance r, the the electrostatic force b/w them is

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{}$$

(5)

$$F = k\frac{q_1 q_2}{r^2}$$

where k is a constant of proportionality called electrostatic force const. The value of 'k' depends on the nature of the intervening medium b/w the charges.

For 2 charges located in free space (vacuum)

$$k = \frac{1}{4\pi\epsilon_0} = 9\times10^9\ 'N\ m^2/c^2$$

where ϵ_0 is called the permittivity of free space.

$$\therefore\quad F = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}.$$

[when charges are placed in a medium of dielectric constant k or ϵ_r $F' = F/k$].

[If $q_1 = q_2 = 1\,C$; $r = 1\,m$, then

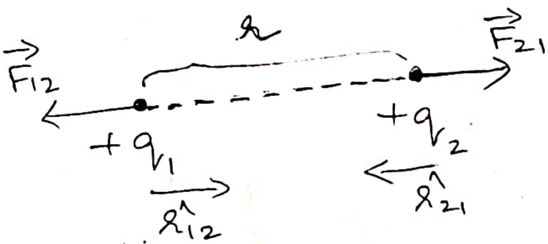
$$F = \frac{1}{4\pi\epsilon_0} = 9\times10^9\,N$$

So one coulomb is that amount of charge that repels equal and similar charge with a force of $9\times10^9\,N$ when placed in vacuum at a distance of one metre from it].

* permittivity of free space $\epsilon_0 = 8.85\times10^{-12}\ C^2/N/m^2$

Coulomb's Law in Vector form

Consider 2 positive point charges q_1 and q_2 placed in vacuum at distance 'r' from each other.



Repulsive force for $q_1 q_2 > 0$

In vector form, Coulomb's law is expressed as

[F_{21} – force on 2 due to 1]

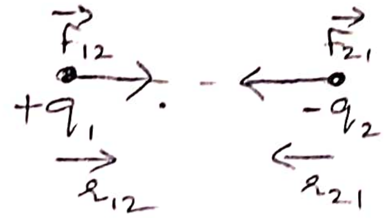
$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

where $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$ is a unit vector pointi[ng from] 1 to 2.

Similarly, $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21}$

$\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$ unit vector pointing from 2 t[o 1]

(Attractive)
Coulomb's forces between 2 unlike charges ie $q_1 q_2 < 0$ will be.



Importance
(i) As $\hat{r}_{12} = -\hat{r}_{21}$ $\therefore \vec{F}_{12} = -\vec{F}_{21}$
which means that the 2 charges exert equal and opposite force on each other ie they obey Newton's third law of motion.
(ii) As the forces act along $\vec{F}_{12}$ or $\vec{F}_{21}$ ie along the line joining the centres of 2 charges, so they are central forces.

Dielectric Const or relative permittivity

$$F_{vac} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

when the 2 charges are placed in any other medium, then

$$F_{med} = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2}.$$

where $\epsilon \rightarrow$ absolute permittivity of a medium.

$$\therefore \frac{F_{vac}}{F_{med}} = \frac{\varepsilon}{\varepsilon_0}$$

The ratio of permittivity of the medium to the permittivity of free space is called relative permittivity (ε_r) or dielectric constant (k) of the medium.

$$\frac{\varepsilon}{\varepsilon_0} = k \quad \text{---(1)}$$

Also from (1)

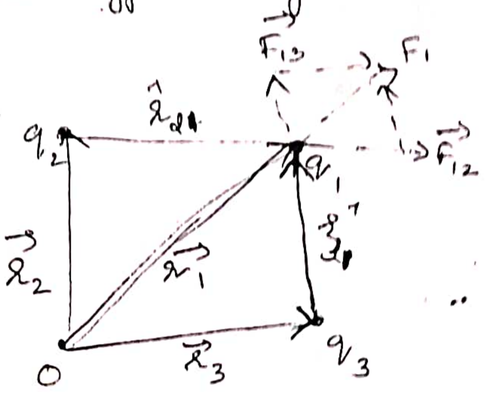
$$F_{med} = \frac{F_{vac}}{k}$$

[K (vaccum) = 1
K (air) = 1.00054 ~ 1
K (water) = 80]

Forces b/w multiple charges: The superposition principle

* The principle of superposition states that when a number of charges are interacting, the total force on a given charge is the vector sum of forces exerted on it due to all other charges.
* The force b/w 2 charges is not affected by the presence of other charges.

Consider a system of 3 charges, q_1, q_2 & q_3 as shown. The force on charge q_1 due to q_2 and q_3 ~~one~~ is obtained by vector addition of force due to each one of these charges.



Thus $$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2}\hat{r}_{21}$$

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{r_{13}^2}\hat{r}_{31}$$

$\therefore$ Total force F_1 on q_1 due to the two charges q_2 & q_3

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{r}_{31}$$

This can be generalised to ~~any~~ a system of any no. of charges. Total force on q_1, due to $q_2, q_3 \ldots q_n$

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots \vec{F}_{1n}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{31} + \ldots \frac{q_1 q_n}{r_{1n}^2}\right.$$

$$= \frac{q_1}{4\pi\epsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{r}_{i1}$$

Electric field

(An Electric field is said to exist at a point if electrostatic force is exerted on a test charge placed at that point.) ~~Quantitatively~~, electric Intensity or electric field strength $\vec{E}$ at a point is defined as the force experienced by a unit positive ~~ch~~ test charge placed at that point.

A charge 'Q' located in space produces an electric field around ~~the spa~~ it. It is given by

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}\hat{r} \quad (1)$$

$\hat{r} = \frac{\vec{r}}{r}$ is a unit vector from the origin to the point r

* Electric field is a vector quantity

* suppose a test charge 'q' is brought to this point r. Then force exerted by the charge Q on q will be

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\cdot\frac{Qq}{r^2}\hat{r} \quad \text{---}(2)$$

$\therefore \dfrac{F_{vac}}{F_{med}} = \dfrac{\cancel{\varepsilon}}{\varepsilon_0}$ ~~The ratio of the~~ ~~P~~ (7)

Subs. (1) in (2)

$$\vec{F}(r) = q\,\vec{E}(r).$$

The charge Q, which produces the electric field is called the source charge & charge 'q' which tests the effect of source charge is called test charge.

$$\text{The } \vec{E} = \frac{\vec{F}}{q} \quad \text{—(3)}$$

* In order that the test charge does not disturb the electric field of Q, ~~it is~~ q is kept vanishingly small, then eqn (3) is rewritten as

$$\vec{E} = \lim_{q \to 0}\left(\frac{\vec{F}}{q}\right)$$

S.I unit is N/C or V/m.

Physical significance of electric field

* Each point $\vec{r}$ is associated with a unique vector $E(r)$. So electric field is an example of vector field.
* By knowing the electric field at any point, force on a charge placed at that point can be determined.

~~* Electric field plays~~

② * the electric field E due to Q, is independent of q

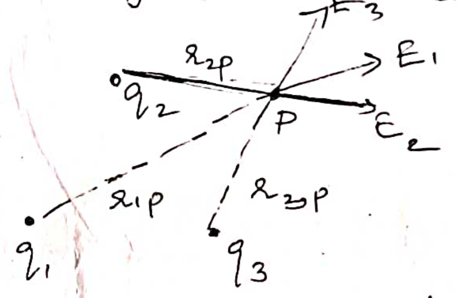
① * For a positive charge, $\vec{E}$ will be directed radially outward from the charge & for negative charges, it is directed radially inward.

③ At equal distances from the charge, Q, E will be same.

* the actual physical significance of $\vec{E}$ is emerges when we treat system of particles beyond electrostatics and consider electromagnetic phenomena.
* the accelerated motion of charge produces electromagnetic wave which then propagate with the speed of light and cause a force on another charge.

Electric field due to a system of charges.

Consider a system of charges $q_1, q_2 \ldots q_n$ with position vectors $r_1, r_2 \ldots r_n$ w.r.t to some origin O. The net Electric field at a point P denoted by position vector $\vec{r}$ is given by



$$E(r) = E_1(r) + E_2(r) + E_3(r)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r_{1P}^2} \hat{r}_{1P} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{2P}^2} \hat{r}_{2P} + \frac{1}{4\pi\epsilon_0} \frac{q_3}{r_{3P}^2} \hat{r}_{3P}$$

This can be generalised to any no. of charges

then $$E(r) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_{iP}^2} \hat{r}_{iP}.$$

Electric flux ~~through~~ through a surface in a region of electric field is the total measure of field lines passing normally through that area.

[* Electric flux is a property of electric field i.e field lines passing thru' an area when in the direction of electric field.]

Electric Flux is denoted by ϕ ~~and its S.I unit is~~ ~~...~~.

Suppose if an electric field $\vec{E}$ passes normally through an area element Δs, the electric flux through this area is

$\Delta\phi = E \times \Delta s$.

Electric field lines

* Electric field lines are hypothetical lines & on ~~which~~ tangent drawn to this line or curve at any point gives the direction of electric field at that point.

* It may be defined as the curve along which a small positive charge would tend to move in an electric field.

Properties

(i) they are continuous smooth curves without any break.

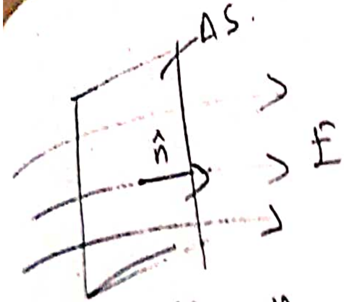
(ii) ~~they do not~~ form any closed loops – they originate from +ve charge and terminate on negative charge. Due to this they do not form closed loops. If there is a single +ve charge, They will start from it and end at infinity.

(iii) The tangent to an electric field line at any pt. gives the direction of electric field at that point.

(iv) Electric field lines never intersect each other. Because if they intersect, then there will be 2 tangents at the point of intersection or two directions for Electric field at same point which is not possible.

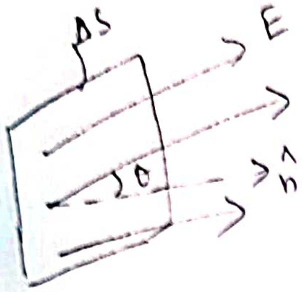
(v) The relative closeness of the field lines gives a measure of strength of the electric field in any region.
ie (a) field lines are close together in a strong field
(b) far apart in a weak field.

(vi) Field lines are equally spaced for uniform electric field.



Electric flux through normal area.

* If the normal drawn to the area element ΔS makes an angle θ with the uniform field $\vec{E}$ then the component of $\vec{E}$ normal to ΔS will be $E\cos\theta$ or vector normal to the surface ΔS is $\Delta S\cos\theta$



$\therefore \Delta\phi_E$ = Normal Component of E × surface area

$= E\cos\theta \times \Delta S$

$= E\Delta S\cos\theta$

$$\boxed{\Delta\phi_E = E \cdot \Delta s}$$

If the field is ~~non~~ uniform, and if a closed surface S is lying inside the field, then the surface is divided into small area elements $\vec{\Delta S_1}, \vec{\Delta S_2}, \vec{\Delta S_3} \cdots \vec{\Delta S_n}$

Total flux $\phi_E = E\cdot\Delta S_1 + E\cdot\Delta S_2 + \cdots E\cdot\Delta S_n$

$$\phi_E = \sum_{i=1}^{n} \vec{E_i}\cdot\vec{\Delta S_i}$$

~~when~~ ~~[illegible]~~ when no. of area elements tend to ∞, and $\Delta s \to 0$, the above expression is taken as the surface integral over the closed surface

$$\text{ie}\quad \phi_E = \lim_{\substack{n\to\infty \\ \Delta s\to 0}} \sum_{i=1}^{n} \vec{E}\cdot\vec{\Delta S_i} = \oint_S \vec{E}\cdot\vec{ds}$$

* Electric flux is a scalar quantity.
* S.I unit of Electric flux $= N\,C^{-1}m^2$ or Nm^2C^{-1} or V-m.

[* Since the electric field decreases as the square of the distance from a point charge, & the area enclosing the charge increases as the square of the distance, the no. of field lines crossing the enclosing area remains const.

omit

Electric Dipole

The pair of equal and opposite charges separated by a small vector distance is called an electric dipole.

* Dipole moment → it measures the strength of an electric dipole.
* Dipole moment is the product of magnitude of either charge and distance between the opposite charges.
* Dipole moment is denoted by $\vec{p}$

$\vec{p} = q \times 2\vec{a}$

It is a vector quantity, directed from negative to positive along the dipole axis.

SI unit of dipole moment is C-m.

Eg. of electric dipoles:- molecules like H_2O, HCl, C_2H_5OH, CH_3COOH etc, the centres of +ve charges and centre of negative charges do not coincide - such molecules ~~are electric~~ have permanent dipole moment dipoles & can be treated as electric dipoles.

Point dipoles are those where ~~2q~~ $2a \to 0$ and charge $q \to \infty$, such that $\vec{p}$ has finite value.

Eg: dipoles associated with individual atoms/molecules can be treated as point dipoles..

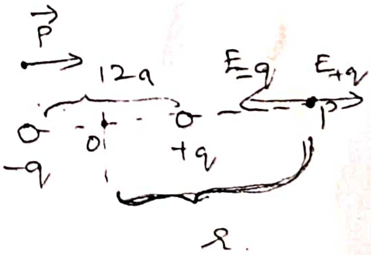
The Electric field due to a dipole

The electric field of a dipole at any point in space can be determined using Coulomb's law and superposition principle.

We can consider 2 cases.

(i) The Electric field at any point along the axial line of a dipole

Consider a dipole and let 'p' be any point at a distance 'r' from the centre of the dipole on the side of +q as shown in fig



Then $\vec{E}_{-q} = -\dfrac{q}{4\pi\epsilon_0 (r+a)^2}\hat{p}$

$\vec{E}_{+q} = \dfrac{+q}{4\pi\epsilon_0 (r-a)^2}\hat{p}$ where $\hat{p}$ is a unit vector directed from -q to +q

The total field at P is

$$\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \frac{q}{4\pi\epsilon_0} \frac{1}{(r-a)^2}\hat{p} - \frac{q}{4\pi\epsilon_0}\frac{1}{(r+a)^2}\hat{p}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{\cancel{r^2}+\cancel{a^2}+2ra-\cancel{r^2}-\cancel{a^2}+2ra}{(r^2-a^2)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{p}$$

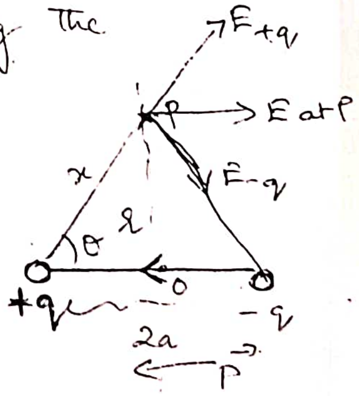
for $r >> a$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \cdot \frac{4\cancel{r}a}{r^{\cancel{4}3}}\hat{p} = \frac{2q}{4\pi\epsilon_0} \cdot \frac{2a}{r^3}$$

$$= \cancel{\frac{12\vec{p}}{4\pi\epsilon_0}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\vec{p}}{r^3}\hat{p}$$

ie Electric field at any point along the axis of a dipole is along the dipolemoment.

(ii) Electric field at a point along the equatorial line



The magnitudes of the electric fields due to charges $+q$ and $-q$ are given by,

$$E_{+q} = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{(r^2+a^2)} \qquad \left[\because x^2 = a^2+r^2\right.$$

$$E_{-q} = \frac{+q}{4\pi\epsilon_0}\frac{1}{(r^2+a^2)}$$

The directions of E_{+q} and E_{-q} are as shown in fig.
The components normal to the dipole axis cancel each other. The components along the dipole axis add up. The net electric field is opposite to $\hat{p}$.

$$\vec{E} = -(E_{-q} + E_{+q}) \cos\theta \, \hat{p} \qquad \cos\theta = \frac{a}{(r^2+a^2)^{1/2}}$$

$$= -\frac{2q}{4\pi\epsilon_0(r^2+a^2)} \cdot \frac{a}{(r^2+a^2)^{1/2}}$$

$$= -\frac{q \cdot 2a}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}$$

$$= -\frac{p}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}$$

At large distances ($r \gg a$), this reduces to

$$\vec{E} = -\frac{p}{4\pi\epsilon_0 r^3} \hat{p}$$

∴ Electric field due to a dipole at any point along the equitorial axis of the dipole is $\vec{E}_{eq} = -\frac{p}{4\pi\epsilon_0 r^3} \hat{p}$

* the field is opposite to the direction of dipolemoment $\vec{p}$

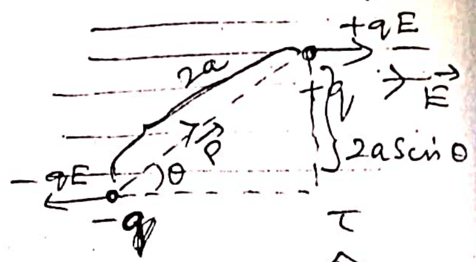
Also by comparing E_{axial} and $\vec{E}_{eq}$

$$E_{axial} = 2E_{eq}.$$

* Also the field due to a dipole falls off rapidly as $1/r^3$ at large distance than that due to single charge which falls off as $1/r^2$.

Dipole in a uniform external field

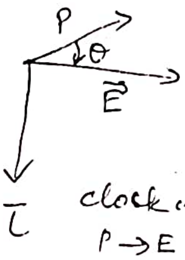
Consider an electric dipole of dipole moment $\vec{p}$ kept in an external field $\vec{E}$ as shown in the fig.



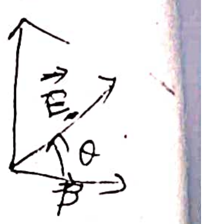
Force exerted on charge +q by field $\vec{E}$ $= q\vec{E}$ (along $\vec{E}$)

Force exerted on charge −q by field $\vec{E}$ $= -q\vec{E}$ (opp. to $\vec{E}$)

$$\vec{F}_{total} = +q\vec{E} - q\vec{E} = 0$$



clockwise
P → E
τ down



P → E
anticlockwise
τ − up

* Hence net translatory force on a dipole in a uniform electric field is zero.
* As there are two equal & opposite forces having different line of action, they constitute a couple which exerts a torque

Torque = Either force × Perpendicular distance b/w the forces.

$$\vec{\tau} = qE \times 2a\sin\theta$$

$$\vec{\tau} = (q\,2a)\,E\sin\theta = pE\sin\theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

~~The~~ The direction of torque is given by right handed screw rule or Fleming's L.H. rule.

* ~~If the~~ The torque will tend to rotate the dipole & align in the direction of the external field.

* The torque is max when the dipole is held $\perp r$ to $\vec{E}$ ie $\theta = 90^\circ$

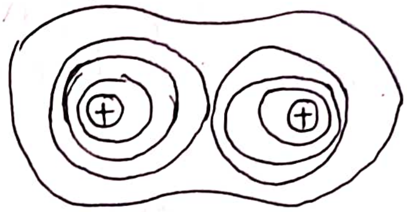
$$\therefore \tau_{max} = pE \sin 90^\circ = pE$$

* Torque is min. or zero, when the dipole is held parallel to $\vec{E}$, ie $\theta = 0^\circ$ or 180°

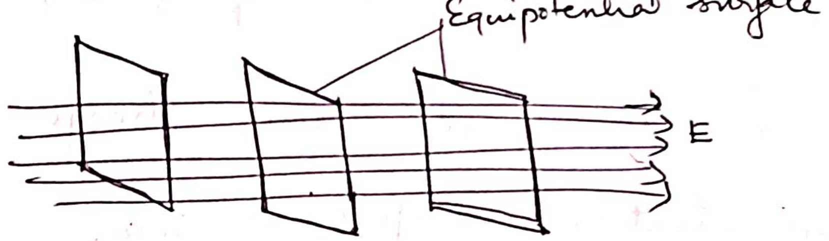
$$\therefore \tau = pE \sin 0 = 0$$

Important points

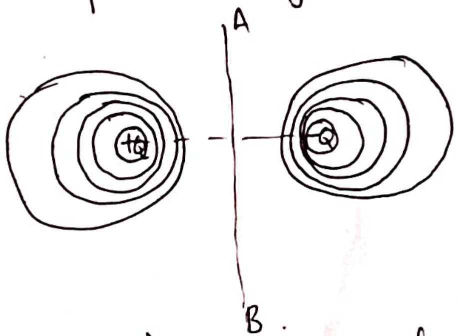
Equipotential surfaces for a system of 2 identical positive point charges placed at a distance 'd' apart



Equipotential surface in a uniform electric field.

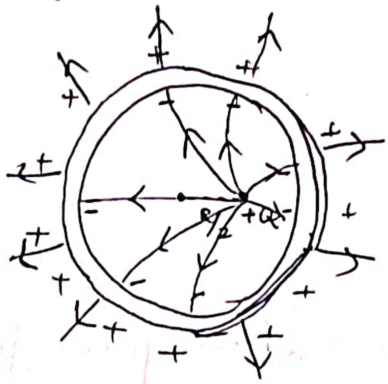


Equipotential surface for a system of 2 charges Q, −Q separated by a distance 'd' in air.



* Electric potential is zero at all points in the plane passing the dipole equator AB.

* Electric field lines for a system with a point charge +Q, located at a distance R/2 from the centre of a spherical metal shell.



* W.D. to move a charge of any magnitude or sign along the equitorial line of a dipole is zero!.

* W.D. to move any charge along a closed path is zero.

$$W = q(V_B - V_A)$$

* W.D. to move any charge along an equipotential surface is zero.

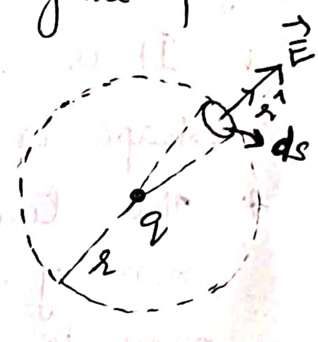
Gauss's Law

Gauss's law states that the total electric flux(ϕ) through any closed surface (in free space) is equal to $1/\varepsilon_0$ times the net charge enclosed by the surface.

Mathematically, $\phi = \int_s \vec{E} \cdot \vec{ds} = q/\varepsilon_0$.

Proof :

Consider the total flux through a sphere of radius 'r', which encloses a point charge q at its centre. Consider a small area element 'ds' on the surface of the sphere. The flux through ds is,



$$\Delta\phi = E \cdot ds = \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} \cdot \vec{ds}$$

[E for a point charge, $E = q/4\pi\varepsilon_0 r^2 \, \hat{r}$

$\hat{r}$ is a unit vector along the outward normal of ds].

As the unit vector $\hat{r}$ is always along the radius from the centre to any area element, $\hat{r} \cdot \vec{ds} = ds$

ie
$$\Delta\phi = \frac{q}{4\pi\varepsilon_0 r^2} ds.$$

The total flux through the sphere is obtained by summing the flux through all area elements,

$$\phi = \sum_{\text{all } ds} q/4\pi\varepsilon_0 r^2 \, \Delta s$$

Since $\sum \Delta s = 4\pi r^2$ (Surface area of sphere)

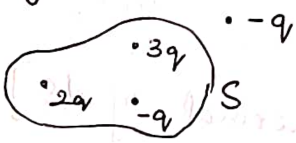
$$\phi = \frac{q}{\cancel{4\pi}\varepsilon_0 \cancel{r^2}} \cdot \cancel{4\pi r^2}$$

$$\phi = \frac{q}{\varepsilon_0}$$

Implications of Gauss's law (Only Concepts).

* The total flux through a closed surface is zero if charge is enclosed by the surface. Eg: Flux due to a dipole in a Gaussian surface is zero, as net cha of a dipole is zero.
* It is true for any closed surface, irrespective of shape or size
* The term 'q' on the R.H.S of Gauss's law includ sum of all charges enclosed by the surface. The charge may be anywhere within the surface.

Eg:



Total flux through S,

$$\phi = \frac{2q + 3q - q}{\varepsilon_0} = \frac{4q}{\varepsilon_0}$$

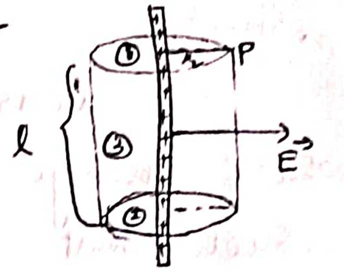
* The Gauss's law is applicable to both discrete charges (provided the surface does not pass through the surface as well as for continuous charge distribution (as in the case of uniformly charged wire, sheet or shell.)
* It gives a simplified method to calculate the electric field when the system possess some symmetry.
* It is based on inverse square law.

Applications of Gauss's Law

I Field due to an infinitely long straight uniformly charged wire

Consider an infinitely long thin straight wire with uniform linear charge density λ. The wire is symmetrical about its axis.

To find the electric field at P, distant 'r' from the wire, imagine a cylindrical Gaussian Surface with 'r' as radius and of finite length 'l' around the charged wire. We may consider three areas marked ①, ② and ③ in the figure.



For areas ① and ②, flux due to the charged wire is zero, because $d\phi = \vec{E}\cdot\vec{ds} = 0$ [∵ outward normal of the areas makes an $\angle^{le}$ 90° w.r.t $\vec{E}$].

Thus only the curved surface area ③ contributes to the flux. [as $\hat{n}$ and $\vec{ds}$ are in the same direction for all area elements over the curved surface]

Applying Gauss's law,

$$\phi = \int_S \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

$$E \times 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0}$$

[$\int ds = 2\pi r l$ curved surface area for a cylinder. Also $\lambda = q/l$, $q = \lambda l$].

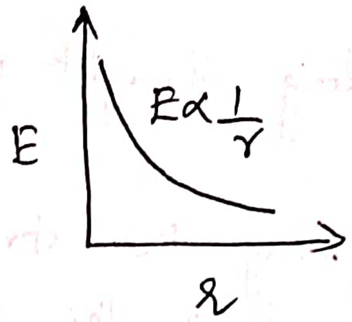
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Vectorially

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{r}$$

[where $\hat{r}$ is away from the wire for positively charged wire].

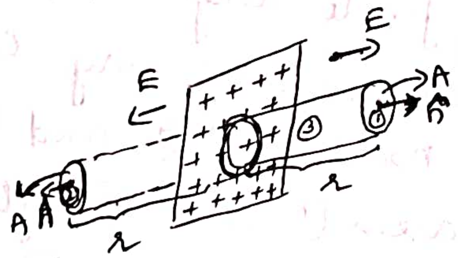
Graph b/w $\vec{E}$ and distance from the wire.



II Electric field due to uniformly charged infinite sheet

Consider a very thin and infinite plane where charge q is distributed uniformly over such that the surface charge density $\sigma = q/A$

To find the electric field at point $\vec{P}$ at a distance 'r' from the sheet, imagine a cylindrical gaussian surface of cross sectional area A of length 2r as shown in fig. The curved surface area does not contribute to the flux because $\hat{n}\cdot\vec{ds} = 0$



Applying Gauss's law for circular areas ① and ② in which $\vec{E}$ and $\hat{n}$ are in the same direction,

$$\phi = \int_{①} E\cdot ds + \int_{②} E\cdot ds = \frac{q}{\varepsilon_0}$$

$$= EA + EA = \frac{\sigma A}{\epsilon_0}$$

[$\hat{n}$ is away from the charged for positively charged sheet]

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0}$$

$$\boxed{E = \sigma/2\epsilon_0}$$

Vectorially $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$

subs. $\sigma = \frac{q}{4\pi R^2}$ in the above eqn.

~~Subs~~

$$E = \frac{q}{4\pi \cancel{R^2}} \cdot \frac{\cancel{R^2}}{\varepsilon_0 r^2}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$ which is the same as the expression for electric field for point charge 'q' at centre O.

Therefore for a charged spherical shell, for any point outside the shell, the electric field is as though the whole charge is concentrated at the centre.

(ii) <u>Field ~~outside~~ inside the shell ($r<R$)</u>

To find the electric field at a point P which is at a distance 'r' from the centre such that ($r<R$), imagine a spherical Gaussian surface of radius 'r' with 'O' as centre.

In this case, the charge enclosed by the Gaussian surface is zero, as the charges reside on the surface of the shell.

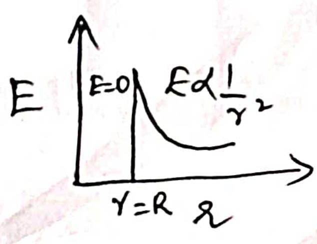
Applying Gauss's law,

$$\phi = E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \times 0 = 0.$$

$$\therefore E = 0$$

Thus there is no electric field inside a uniformly charged shell.

<u>Graph b/w E and r for a shell</u>

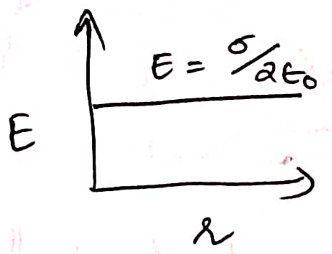


For $r = R$,

$$E = \frac{\sigma \cancel{R^2}}{\varepsilon_0 \cancel{R^2}} = \frac{\sigma}{\varepsilon_0}.$$

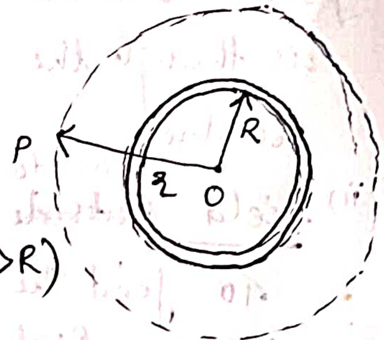
Graph b/w $\vec{E}$ and distance from the sheet

As $\vec{E}$ does not depend on r, $\left(E = \frac{\sigma}{2\epsilon_0}\right)$



III Field due to a uniformly charged thin spherical shell

Consider a thin spherical shell of radius R which has a surface charge density σ.
The system possess spherical symmentry



(i) Field ~~inside~~ ~~outside~~ Outside the sphericall shell. (r>R)

To find the electric field at point P distant r from the centre of the shell ($r>R$) imagine a Gaussian surface which is a sphere of radius r with centre O as of the shell as shown in the fig.

Applying Gauss's law,

[$\int ds$ = area of Gaussian Surface $= 4\pi r^2$]

$$\phi = \int \vec{E}\cdot\vec{ds} = q/\varepsilon_0 .$$

$$E\int ds = E \times 4\pi r^2 = \frac{\sigma \times 4\pi R^2}{\varepsilon_0}$$

$$\left[\sigma = \frac{q}{\text{Area of shell}} = q/4\pi R^2\right]$$

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

Vectorially, $\vec{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\hat{r}$

[$\hat{r}$ is a unit vector radially outward for a +vely charged shell]