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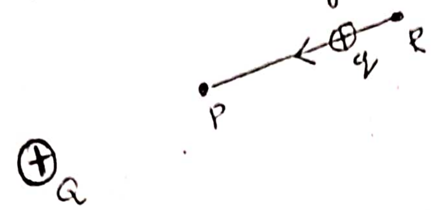
ELECTROSTATIC POTENTIAL AND CAPACITANCE

①

· Potential Energy difference:

Consider a positive charge Q placed at the origin. Now we bring a positive test charge from a point R to P against repulsive force on it due to Q as shown in the fig.



Then the W.D in moving the charge 'q' against the electrostatic forces gets stored as the P.E of q.

$$W = \int_R^P F_{ext} \cdot dr = -\int_R^P F_E \cdot dr$$

q is moved ~~which~~ in such a way that ext. force just balances the electrostatic force. Therefore, the W.D is equal to Pot. energy difference b/w the points R and P

$$W = \Delta U = U_P - U_R.$$

2. * Potential energy is taken to be zero at infinity ie $U_\infty = 0$

* Therefore potential energy of charge q at a point is the work done by the external force in bringing the charge from infinity to that point.

$$W = U_p - U_\infty = U_p$$

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Electrostatic potential

Consider a charge Q placed at the origin. Imagine a unit positive test charge q being moved from point R to P against the electrostatic force of Q. The workdone per unit ~~positive~~ test charge is

$$= V_P - V_R = \frac{U_P - U_R}{q}$$

is which is equal to electrostatic potential difference.

or electric

Therefore, electrostatic potential difference b/w two points is defined as the work done by an external force in bringing a unit positive charge from one point to another against electrostatic forces.

Electric ~~static~~ potential (V) at a point is the W.D in bringing a unit positive test charge from infinity to a point

$$V = \frac{W}{q}$$

* The electrostatic potential is the physical quantity which determines the direction of charge flow b/w two bodies when brought in contact. The positive charge always flow from a body at higher potential to that at lower potential (vice versa for negative charges).

* S.I of potential is volt or J/C.

* It is a scalar quantity.

The electric potential at a point is 1 volt if one joule of work has to be done in moving a positive charge of 1 coulomb from ∞ to that point.

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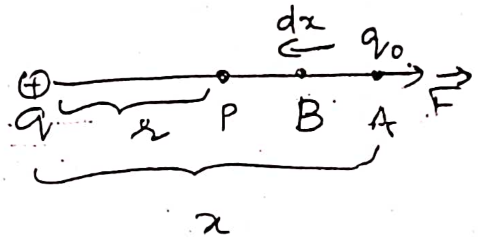
Electric potential due to point charge

Consider a positive charge q placed at the origin O. Suppose a test charge q_0 is placed at point A at a distance x from O. By Coulomb's law, the electrostatic force acting on charge q_0 is



$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{x^2}$$

The small W.D. in moving the test charge q_0 from A to B thru' small displacement $\vec{dx}$ against the electrostatic force is

$$dW = \vec{F}\cdot\vec{dx} = F\,dx \cos 180^\circ = -F\,dx.$$

[$\theta = 180^\circ$ as F is directed away from q_0]

∴ Total W.D. in moving the charge q_0 from ∞ to an arbitrary point P at distance 'r' from the origin is

$$W = \int dW = -\int_\infty^r F\,dx = -\int_\infty^r \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{x^2}\,dx$$

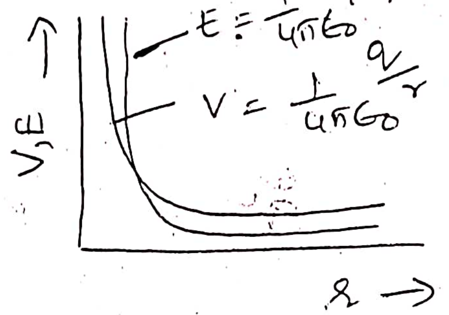
$$= -\frac{qq_0}{4\pi\epsilon_0}\int_\infty^r x^{-2}\,dx = \frac{-qq_0}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_\infty^r$$

$$= \frac{qq_0}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r}$$

By definition electric potential at a point is $V = \frac{W}{q_0}$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}.$$

* As r increases, V decreases.
* As $V \propto \frac{1}{r}$. Thus electric potential due charge is spherically symmetric as it dep r.



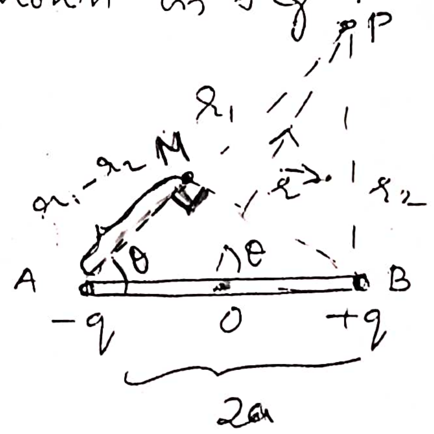
Variation of V and E with distance r from a point charge q.

Electric potential at any general point due to a dipole

Consider an electric dipole consist of 2 point charges $-q$, and $+q$ separated by a distance $2a$ as shown in fig.

Let $AP = r_1$ and $BP = r_2$

The ~~vector~~ $\vec{r}$ makes an angle θ with the dipole axis.



Net potential at P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\epsilon_0} \frac{-q}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{q}{r_2}$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{r_1 - r_2}{r_1 r_2} \right]$$

the point 'P' lies far away from the dipole, then
$r_1 - r_2 \approx AB\cos\theta = 2a\cos\theta$ and $r_1 r_2 \approx r^2$

$$V = \frac{q}{4\pi\epsilon_0} \cdot \frac{2a\cos\theta}{r^2} \qquad [P = q2a]$$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{P\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{P}\cdot\vec{r}}{r^3} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{P}\cdot\hat{r}}{r^2}$$

<u>Special Cases</u> $\left[\hat{r} = \frac{\vec{r}}{|r|}\right]$

(i) when the point P lies on the axial line of the dipole, $\theta = 180°$ or $0°$

then $V = \pm \frac{1}{4\pi\epsilon_0} \cdot \frac{P}{r^2}$

ie potential has greatest +ve or -ve value.

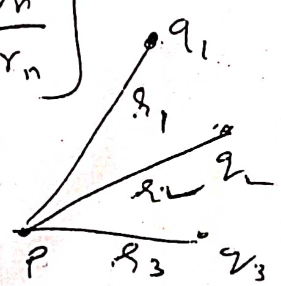
(ii) when the point 'p' lies on the equatorial line, Then $\theta = 90°$ and $V = 0$. ie potential at any point on the equatorial line of the dipole is zero though Electric field is non zero.

<u>Electric potential due to a System of charges</u>

The net electric potential due to a system of charges $q_1, q_2, q_3 \ldots q_n$ at a point P is

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots \frac{q_n}{r_n}\right]$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$



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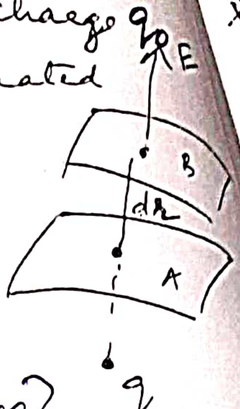
Relation between Electric field and potential

Consider the electric field due to charge q. Let A and B be two adjacent points separated by a distance dr which is the ⊥r distance b/w the equipotential surfaces A and B.



The ext force required to move a test charge q_0 from B to A

$$\vec{F} = +q_0\vec{E}$$

$(F dr \cos 180) = -F dr$

$\therefore$ W.D $= F.dr = -q_0 E.dr.$

$= \text{charge} \times \text{P.d} = q_0(V_B - V_A) = q_0 dV$

$\therefore -q_0 E.dr = q_0 dV$

$\therefore E = -\frac{dV}{dr}.$

$-\frac{dV}{dr} \rightarrow$ rate of change of potential with distance $\rightarrow$ potential gradient

* The electric field at any point is equal to negative of potential gradient at that point
* negative sign shows that the direction of $\vec{E}$ is in the direction of decreasing potential

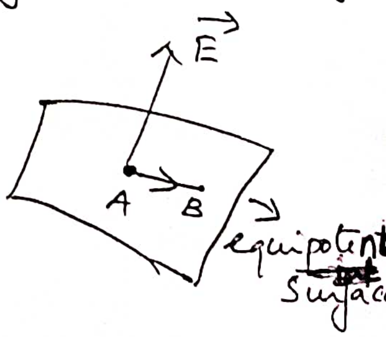
Equipotential surfaces and their properties.

Any surface that has constant electric potential at every point on it is called equipotential surface.

Properties

(i) No work is done to moving a test charge over an equipotential surface.

Let A and B two points over an equipotential surface as shown in fig. If a test charge q_0 is moved from A to B along the surface, the W.D will be



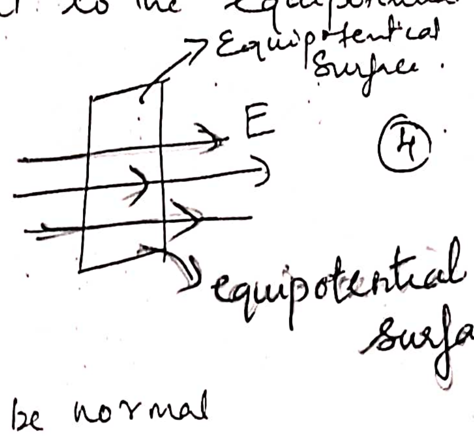
$W_{AB} = \text{charge} \times \text{pot. diff}.$

$= q(V_B - V_A) = 0$

$[\because V_B - V_A = 0$ on an equipotential surface.$]$

(ii) Electric field is always normal to the equipotential surface at every point.

If the $\vec{E}$ were not normal to the equipotential surface, it would have a non-zero component along the surface & work would have to be done against this component but there is no p.d. along the surface and hence $\vec{E}$ has to be normal to the surface.



(4)

(iii) Equipotential surfaces are closer together in regions of strong field & far apart in regions of weak field.

$$E = -\frac{dV}{dr} \quad \text{or} \quad dr = -\frac{dV}{E}$$

If dV is const, then $dr \propto \frac{1}{E}$.

(iv) No 2 equipotential surfaces can intersect each othere. If they intersect, there will be 2 values of potential at the point of intersection which is not possible.

* Refer text for diagrams (equipotential surfaces) → for single charge, dipoles, like charges

P.E of a system of charges

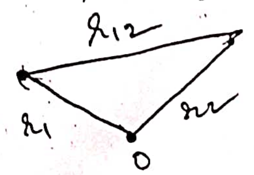
The potential energy of a system of charges is equal to the work done in building up the configuration of charges.

Consider two charges q_1 and q_2 initially at infinity and the work done by an ext. agency in bringing these to the given locations gives the P.E of the sys.

Suppose the first charge q_1 is brought from ∞ to r_1, as there is no external field, the W.D in the process is zero.

This charge produces a potential in space given by

$$V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r_{1P}}$$



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[r_{1P} is the distance of a point P from the

Now workdone in bringing charge q_2 from

to the point r_2 is

$$= \text{charge} \times \text{potential due to } q_1$$

$$= q_2 \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r_{12}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance b/w points 1 and 2.

ie $$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

Consider the system,

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* Due to conservative nature of electrostatic force, the P.E is characteristic of the state of configuration, and not the way the state is achieved.

P.E in an external field

P.E of q at r in an external field

$$= q\, V(r)$$

where $V(r)$ is the external potential at the point r.

* If an electron with charge, $e = 1.6 \times 10^{-19}\,C$ is accelerated by a potential difference of $\Delta V = 1\,volt$, then it would gain energy of $q\Delta V = 1.6 \times 10^{-19}\,J$.

ie $1\,eV = 1.6 \times 10^{-19}\,J$.

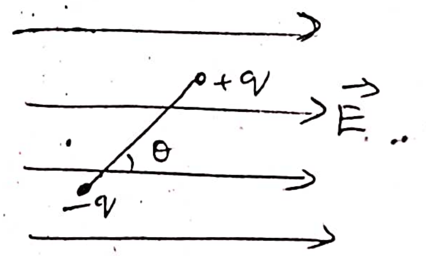
P.E of a system of 2 charges in an external field

$$W = U = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}.$$

for a system of N charges, $U = \frac{1}{4\pi\varepsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$

P.E of a dipole in an external field

Consider a dipole with charges $+q$ and $-q$ separated by a distance $2a$ kept in an external electric field as shown in fig.



In an external field, the dipole experiences a torque,

$$\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta$$

($\theta \rightarrow \angle^{le}$ b/w $\vec{P}$ and $\vec{E}$)

The torque tends to align the dipole in the direction of $\vec{E}$. If the dipole is rotated thru' a small angle $d\theta$ against the torque acting on it, then small W.D in doing so is,

$$dW = \tau\, d\theta.$$

iii) The total workdone in rotating the dipole from an initial position θ_i to a final position θ_f will be

$$W = \int dW = \int_{\theta_i}^{\theta_f} \tau(\theta)\, d\theta = \int_{\theta_i}^{\theta_f} pE\sin\theta\, d\theta$$

$$= pE\left[-\cos\theta\right]_{\theta_i}^{\theta_f} = pE\left[\cos\theta_i - \cos\theta_f\right]$$

This W.D. is stored as its P.E.

$$\therefore\ U = pE(\cos\theta_i - \cos\theta_f).$$

If initially the dipole is oriented perpendicular to the direction of the field ie $\theta_i = 90^\circ$ and then brought to another orientation ~~with~~ making $\angle^{le}$ θ $(\theta_f = \theta)$ with the field, then P.E will be

$$U = pE(\cos 90^\circ - \cos\theta)$$

$$= -pE\cos\theta$$

$$\text{or } U = -\vec{p}\cdot\vec{E}$$

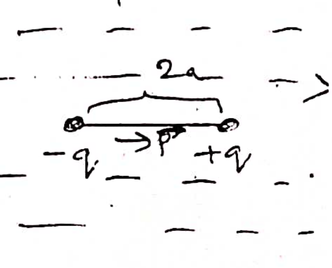
This gives the P.E at any orientation of the dipole.

Special Cases

* Position of stable equilibrium

when $\theta = 0^\circ$; $U = -pE\cos 0^\circ = -pE$

P.E of the dipole is min when its dipole moment is parallel to the ext field.

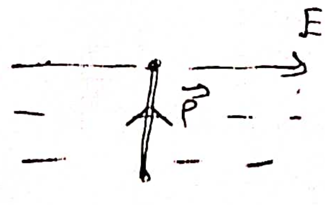


* Position of zero energy.

when $\theta = 90^\circ$

$$U = -pE\cos 90^\circ = 0$$

P.E is zero when $p \perp E$



position of unstable equilibrium

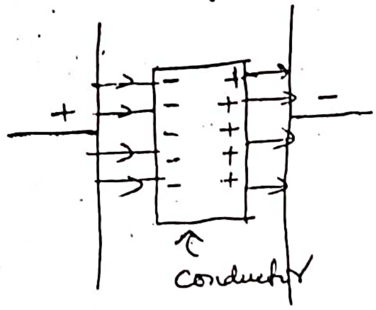
when $\theta = 180°$; $U = -pE \cos 180° = -(-pE)$
$= +PE$

P.E of a dipole is max. when its dipole moment $\vec{p}$ is antiparallel to the ext. field. This position of unstable equilibrium.

Electrostatics of Conductors

(i) Net electrostatic field is zero in the interior of a conductor

When a conductor is placed in an ext. $\vec{E}$, its free $\bar{e}$s tend to move in the opp. direction of $\vec{E}$. negative charges are induced on the left end & positive charges induced on the right end of the conductor, this process continues till the electric field $\vec{E}_{ind}$ set up by the induced charges become equal & opp. to the ext $\vec{E}$



∴ net field $(\vec{E} - \vec{E}_{in}) = 0$.

(ii) Just outside the surface of a charged conductor, electric field is normal to the surface.

If the electric field is not normal to the surface, it will have a component tangential to the surface which will cause charges to flow under static conditions which is not possible. Hence $\vec{E}$ is normal to the surface at every point.

(iii) The net charge in the interior of a conductor is zero and any excess charge resides at its surface

According to Gauss's theorem,

$$\phi_E = \oint \vec{E}\cdot\vec{ds} = q/\varepsilon_0$$

$E = 0$; $q = 0$.

Hence the entire charge resides just near

16 surface of the conductor.

(iv) Potential is constant within and on the surface of a conductor

Inside a conductor $E = 0$

$$\vec{E} = -\frac{dV}{dr} \qquad \therefore \frac{dV}{dr} = 0$$

or V = constant.

ie Electric potential is constant throughout the volume of a conductor & has const. value.
∴ the surface of a conductor is an equipotential surface.

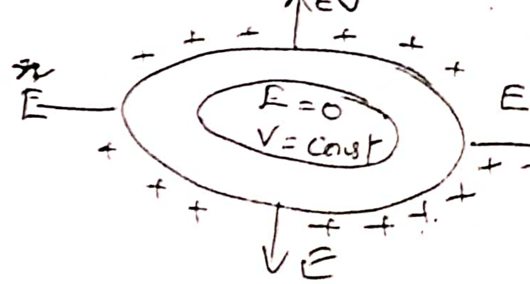
(v) Electric field at the surface of a charged conductor is proportional to the surface charge density

For a conductor of any shape, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

where $\sigma = q/\Delta s$

(vi) Electric field is zero in the cavity of a hollow charged conductor

By Gauss's law, $\vec{E}$ is zero inside a hollow conductor as $q = 0$. The entire excess charge lies on its surface



* The electric field inside a cavity made of a conductor will be zero irrespective of its size and shape.

* Such a cavity inside the conductor which is shielded from ~~ex~~ external electric field is called electro static shielding.

The phenomenon of making a region free from any electric field is called electrostatic shielding.

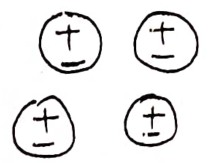
Application

* An earthed hollow conductor can act as a screen by allowing the induced charge to flow to earth
eg: outer case of a van de Graaff generator, communication cables etc.
* In a thunderstorm, it is safest to sit inside a car rather than under a tree or open ground. The metallic body of the car becomes an electrostatic shielding from the lightning.
* Sensitive components of electronic devices are protected or shielded from external electric field by placing metal shields around them.

Dielectrics and polarization

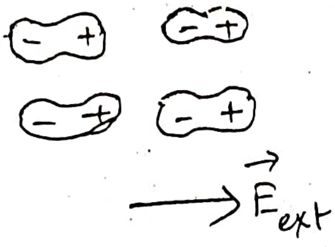
Dielectrics are essentially insulators which have bound charges which can get slightly displaced under the influence of electric field or in other words dielectrics get polarized under the influence of electric field Eg: Glass, mica, Transformer oil, air, H_2, Paraffin wax, etc.

Non-Polar dielectric — have molecules in which the centres of positive charges coincides with the centres of negative charges. Eg: H_2, N_2, CH_4, CO_2 etc.
* Non polar molecules have symmetrical shapes
* They have zero dipole moment in the absence of any electric field as shown in the fig.



$E = 0$

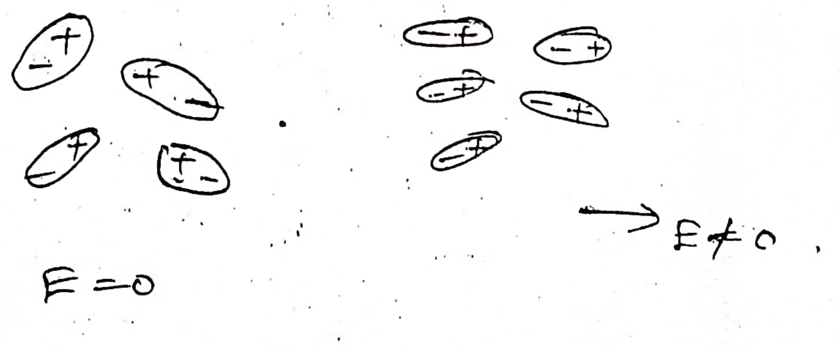
18 In the presence of an external field $\vec{E}$, the centres of +ve charges are displaced in the dir[ection] of E while centres of -ve charges app. to $\vec{E}$ as shown below. Due to the charge separation, a net dipole moment is induced in the direction of the external field.



<u>Polar dielectric</u> are those in which the centre of positive charges does not coincide wise the centre of negative charges in their molecules.

Eg: HCl, NH_3, CO, H_2O etc.

* have unsymmetrical shapes
* they have permanent dipole moment
* In the absence of an external field, the dipole moments of different molecules are randomly oriented due to their thermal energie & hence the net dipole moment is zero.
* when an ext. field $\vec{E}$ is applied, the dipole moments align with the field. As a result, a net dipole moment is developed in the direction of the external $\vec{E}$.



The extent of polarization depends on ~~relative~~ two factors

(i) The P.E of the dipole in the external $\vec{E}$ which tends to align the dipole with the field.

(ii) Thermal energy of the molecules which tends to disrupt the alignment.

* Both polar & non polar molecules develop a net dipole moment in the presence of an external electric field which is known as polarisation of the dielectric.

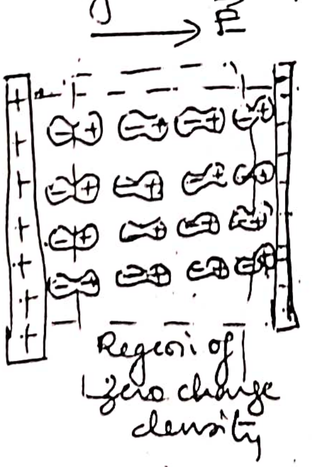
* Polarisation : is defined as the dipole moment per unit volume of the dielectric.

$$P = \chi_e E$$

where χ_e is a constant characteristic of the dielectric referred as electric susceptibility of the dielectric medium.

* Reduction of electric field by the polarisation of a dielectric

Consider a rectangular dielectric slab placed in a uniform electric field $\vec{E}$ as shown in fig



* its molecules align in the direction of E

* the vertical positive charges are cancelled by adjacent negative charges.

* However, at the surfaces of the dielectric there are net negative & positive charges (at the left & right surfaces)

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* The unbalanced charges are the induced charges due to the external field. Thus polarised dielectric is equivalent to two charged surfaces with induced surface charge densities σ_p and $-\sigma_p$ resp.
* The induced charges sets up an electric field $\vec{E}_p$ inside the dielectric in a direction opp. to that of ext field.
* ∴ the resultant field $\vec{E}_{net}$ in the dielectric will be $\vec{E} - \vec{E}_p$ in the direction of $\vec{E}$.
* The ratio of original field $\vec{E}$ to the reduced field $\vec{E} - \vec{E}_0$ in the dielectric is called dielectric Const (K) or relative permittivity (ε

$$K = \varepsilon_r = \frac{\vec{E}}{\vec{E} - \vec{E}_0} = \frac{\varepsilon}{\varepsilon_0}.$$

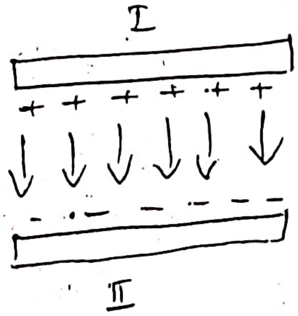
<u>Dielectric strength</u> : The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called dielectric strength of a medium.

[when a dielectric is placed in a very high electric field, the outer ēs may get detached from the atoms & it behaves like a conductor — Called dielectric breakdown]

* Dielectric strength for air is about 3×10^6 V/m
* For a capacitor to store a large amount of charge without leakage, its capacitance should be high enough so that the potential diff. and hence the electric field do not

Parallel plate capacitor

A parallel plate capacitor consists of two ~~small~~ large plane parallel conducting plates separated by a distance. Consider the intervening medium b/w the plates to be vacuum. Let A be the area of each plate and d the distance of separation. The two plates have charges +Q and −Q resp. & their surface charge densities be $+\sigma$ and $-\sigma$



The electric field region I & II will be

$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0 .$$

In the region b/w the plates 1 & 2, the electric fields due to the two charged plates will be

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} .$$

Potential difference, $V = Ed = \frac{1}{\epsilon_0} \cdot \frac{Qd}{A}$

$$C = \frac{Q}{V}$$

$$\therefore \boxed{C = \frac{\epsilon_0 A}{d}}$$

Combination of capacitors in series and in parallel

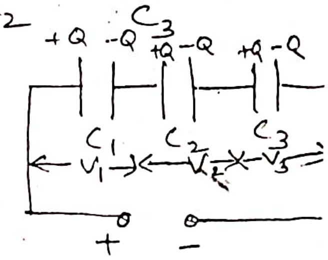
Fig shows three capacitors of capacitances C_1, C_2 and C_3 connected in series. A potential diff. is applied across the combination. The potential diff. across the various capacitors are

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3} .$$

(As same charge Q is induced across each capacitor during the charging process).

For series circuit, sum of these potential difference must be equal to the applied pot. diff.

$$\therefore V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$



$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If C_s is the equivalent capacitance of the combination, then $C_s = \frac{Q}{V}$ or $\frac{1}{C_s} = \frac{V}{Q}$.

$$\frac{1}{C_s} = \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$\therefore$ for a series combination,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

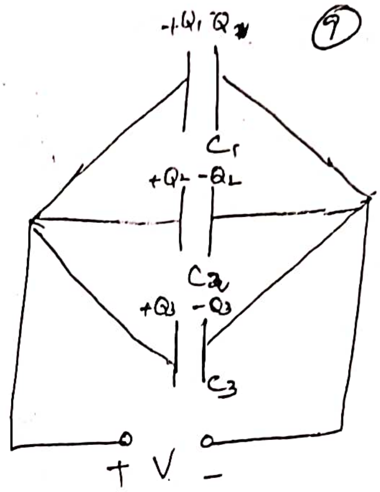
* the equivalent capacitance is smaller than the smallest individual capacitance.
* reciprocal of equivalent capacitance is equal to sum of the reciprocals of individual capacitances

Parallel Combination

Fig shows three capacitors of capacitances C_1, C_2 and C_3 connected in parallel. A potential diff. V is applied across the combination. ~~All the capaci~~

For parallel combination, V is common but charges across each capacitor is different.

$$Q_1 = C_1 V, \quad Q_2 = C_2 V, \quad Q_3 = C_3 V$$



∴ Total charge stored in the combination is

$Q = Q_1 + Q_2 + Q_3$

$= (C_1 + C_2 + C_3)V$

If C_p is the equivalent capacitance, $Q = C_p V$

$Q = C_p V = (C_1 + C_2 + C_3)V$

∴ $\boxed{C_p = C_1 + C_2 + C_3}$.

* eq. capacitance is larger than the largest individual capacitance.
* eq. cap. is equal to sum of the individual capacitances.
* pot. diff. across each capacitor is same.

Energy stored in a parallel plate capacitor

[when a capacitor is charged by a battery, work is done by the battery at the expense of its chemical energy & this W.D is stored as electrostatic P.E)

Consider a capacitor of capacitance C. Let a initial charge & p.d. on the capacitor is zero. Let a charge Q be given to it bit by bit. When charge is given to it, the p.d. b/w its plates increases. Let at any instant, when charge on capacitor be q, the potential diff. is $V = q/C$

Small W.D. in giving additional infinitesimal charge dq to capacitor,

$dW = V dq = \frac{q}{C} dq$

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The total W.D. in giving charge from 0 to Q (
will be equal to

$$W = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

Subs. $Q = CV$

$$W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This W.D is stored as the electrostatic P.E. of capacitor denoted by U_E

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

<u>Energy density</u>

Energy stored in a capacitor, $U = \frac{Q^2}{2C}$

Subs $Q = \sigma A = \varepsilon_0 E A$

$C = \varepsilon_0 A/d$

$$U = \frac{(\varepsilon_0 E A)^2}{2(\varepsilon_0 A/d)} = \frac{1}{2}\varepsilon_0 E^2 A d$$

But Ad – Vol. of space b/w the plates.

∴ Electrostatic energy stored per unit vol. or energy density

$$u_e = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2 \quad \left[\text{for any medium } u_e = \frac{1}{2}K\varepsilon_0 E^2\right]$$

for air or free space $K = 1$

$$\therefore u_e = \frac{1}{2}\varepsilon_0 E^2$$

Capacitor and Capacitance.

When a capacitor is given some charge, it is raised to a certain potential. If a charge Q is given to a conducter, its potential raises by V, then it is found that

$$Q \propto V$$

$$Q = CV$$ where C is the constant of proportionality → Capacitance of the Conductor

$$C = \frac{Q}{V}$$

It is defined as the ratio of charge given to the conductor to the rise in potential. It may also be defined as the measure of its ability or capacity of a conductor to store electric charge or energy. SI unit of Capacitance is Farad or F

$$1 \text{ farad } (F) = \frac{1 \text{ Coulomb}}{1 \text{ Volt}} = 1\, C V^{-1}$$

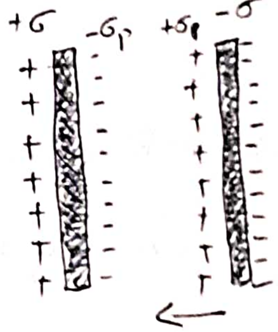
A conductor is said to have a capacitance of one farad, when a charge of one Coulomb raises its potential by one Volt.

* The Capacitance C, is independent of Q or V. It depends on the geometrical configuration (Shape, size and separation of the system).

* Capacitance of a spherical Conductor of radius R is $C = 4\pi\epsilon_0 R$.

pacitance of a parallel plate Capacitor with dielectric medium

Consider a parallel plate Capacitor each of area A separated by a distance 'd'. The charge on the plates be $\pm Q$ and their surface charge densities be $\pm\sigma$. When there is vacuum b/w the plates, the electric field is



$$E_0 = \frac{\sigma}{\varepsilon_0}.$$

And the potential difference V_0 is

$$V_0 = E_0 d = \frac{\sigma d}{\varepsilon_0}$$

The Capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{——(1)}$$

Consider that a dielectric is inserted b/w the plates such that it fully occupies the space b/w them. Due to the effect of electric field, the dielectric gets polarised. The net effect is equivalent to two charged plane sheets of surface charge densities σ_p and $-\sigma_p$ resp. Then the total surface charge densities on the plates is $\pm(\sigma - \sigma_p)$

$\therefore$ The electric field will be

$$E = \frac{\sigma - \sigma_p}{\varepsilon_0}$$

The potential diff. across the plates will be,

$$V = Ed = \frac{(\sigma - \sigma_p)}{\varepsilon_0} \cdot d \quad \text{——(2)}$$

In case of linear dielectrics, σ_p is proportional to E_0 or in other words σ. That is, $\sigma - \sigma_p$ is proportional to σ.

or $\quad \sigma \propto \sigma - \sigma_p$

$$\sigma = k(\sigma - \sigma_p)$$

$$\sigma - \sigma_p = \frac{\sigma}{k} \quad \text{where } k \text{ is the dielectric const.} \quad \text{--- (3)}$$

Substituting (3) in eqn (2)

$$V = \frac{\sigma d}{\varepsilon_0 k}.$$

Also $Q = \sigma A$.

$\therefore$ the capacitance is given by

$$C = \frac{Q}{V} = \frac{\cancel{\sigma} A \varepsilon_0 k}{\cancel{\sigma} d} = \frac{k \varepsilon_0 A}{d}.$$

But $\varepsilon_0 A/d = C_0$

$$\therefore \quad C = k C_0.$$

Also $k = \frac{C}{C_0}$ which is the ratio of capacitance of the capacitor with dielectric to that without dielectric or in free space.

"extra points"

Sharing of charges by Capacitors

Consider 2 capacitors having capacitance C_1, C_2 charges q_1, q_2 and potentials V_1, V_2 resp.

Therefore total charge before sharing

$q_1 = C_1 V_1$ and $q_2 = C_2 V_2$

$\therefore$ Total charge on capacitors $= q_1 + q_2 = C_1V_1 + C_2V_2$

Common potential

Let these capacitors be connected in parallel. Now the charge will flow from a capacitor at higher potential to lower potential, till the potential become equal. This is called common potential.

If q_1' and q_2' be the charges on capacitors C_1 and C_2 resp. after sharing then

$$q_1' = C_1 V \quad \text{and} \quad q_2' = C_2 V$$

$$\therefore \frac{q_1'}{q_2'} = \frac{C_1}{C_2}.$$

Now total charge on the capacitors

$$q = q_1' + q_2' = C_1V + C_2V = (C_1 + C_2)V$$

According to law of conservation of charge

$$(C_1 + C_2)V = C_1V_1 + C_2V_2$$

$$\therefore V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Potential Energies

P.E of 2 capacitors before sharing

$$U = U_1 + U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

After sharing, the P.E will be

$$U' = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}(C_1+C_2)V^2$$

But $V = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$

Substituting

$$U' = \frac{1}{2}(C_1+C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1+C_2}\right]^2$$

$$= \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1+C_2}$$

<u>Loss of energy</u>

On sharing of charges, some energy is dissipated in the form of heat.

ie $U - U' = \frac{1}{2}\left[C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{(C_1V + C_2V_2)^2}{C_1+C_2}\right.$

On simplifying

$$U - U' = \frac{C_1C_2(V_1-V_2)^2}{2(C_1+C_2)}$$

$U - U' > 0$ or $U > U'$

(1) If a charged capacitor is connected parallel to an uncharged capacitor

$$(C_1+C_2)V = C_1V_1 + C_2\times 0 \quad (V_2 = 0)$$

Common potential $V = \frac{C_1V_1}{C_1+C_2}$

New P.E, $U = U_1 + U_2 = \frac{1}{2}(C_1+C_2)V^2$