OSCILLATION

11th Standard CBSE

Date : 27-Nov-22

Physics

Reg.No. :

Exam Time : 01:00:00 Hrs

Total Marks : 100

95 x 2 = 190

1) A simple harmonic motion given by $x = 6.0 \quad cos\left(100t + \frac{\pi}{4}\right)$ where x is in cm and t in second. What is the (i) displacement amplitude, (ii) frequency?

2) A simple harmonic motion given by $x = 6.0 \quad cos\left(100t + \frac{\pi}{4}\right)$ where x is in cm and t in second. What is the displacement amplitude

3) The maximum acceleration in a simple harmonic motion is a_m and the maximum velocity is v_o. What is the displacement amplitude of simple harminic motion?

4) When a particle oscillates simple harmonically, its potential energy varies periodically. If v is the frequency of oscillation of the particle, then what is the frequency of variation of potential energy?

5) In case of an oscillating simple pendulum what will be the direction of acceleration of the bob at
(a) the mean position,
(b) the end points?

6) Justify the following statement
(i) The motion of an artificial satellite around the earth cannot be taken as SHM.
(ii) The time period of a simple pendulum will get doubled if its length is increased four times.

7) Justify the following statements.
The time period of a simple pendulumwill get doubled if its length is increased four times.

8) A body of mass 12 kg is suspended by coil spring of natural length 50 cm and force constant 2.0 x $10^3 Nm^{-1}$. What is the streched length of the spring?If the bosy is pulled down further streching the spring to a length of 5.9 cm and then released,then what is the frequencyof oscillation of the suspended mass?

9) A spring compressed by 0.1 m develops a restoring force 10 N. A body of mass 4 kg placed on it . Deduce
(i) the force constant of the spring
(ii) the depression of the spring under the weight of the body (take g=10 N/kg)
(iii) the period of oscillation, the body is distributed and
(iv) the frequency of oscillation

10) A spring compressed by 0.1 m develops a restoring force 10 N. A body of mass 4 kg placed on it. Deduce the depression of the spring under the weight of the body (take r = 10 N/kg)

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12) A spring compressed by 0.1 m develops a restoring force 10 N. A body of mass 4 kg placed on it . Deduce frequency of oscillation

13) A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillation is found to be 1.5 s. The radius of the disc is 15 cm.Determine the torsional spring constant of the wire.
This is a question based on torsion pendulum for which $T = 2\pi\sqrt{\frac{I}{\alpha}}$ where I = moment of inertia of the disc about axis of rotation,α = torsion constant which is restoring couple per unit twist.

14) Define the restoring force and it characterstic in case of an oscillating body.

15) A particle excutes SHM of period 8 s. After what time of its passing through the mean position will be energy be half kinetic and half potential?

16) Which of the following examples represent periodic motion?
(i) A swimmer completing one (return) trip from one bank of a river to the other and back
(ii) A freely suspended bar magnet displaced from its N-S direction and released.
(iii) A hydrogen molecule rotating about its centre of mass.
(iv) An arrow released from a bow.

17) Which of the following examples represent periodic motion?
A freely suspended bar magnet displaced from its N-S direction and released.

18) Which of the following examples represent periodic motion?
A hydrogen molecule rotating about its centre of mass.

19) Which of the following examples represent periodic motion?
An arrow released from a bow.

20) Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
(i) The rotation of earth about its axis
(ii) Motion of an oscillating mercury column in a U-tube.
(iii) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lowermost point.
(iv) General vibrations of a polyatomic molecule about its equilibrium position.

21) Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

Motion of an oscillating mercury column in a U-tube.

22) Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lowermost point.

23) Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
General vibrations of a polyatomic molecule about its equilibrium position.

24) Every SHM is periodic motion, but every periodic motion need not to be a simple harmonic motion. Do you agree? Give an example to justify your statement.

25) Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?
(i) $a = 0.7x$
(ii) $a = -200x^2$
(iii) $a = -10x$
(iv) $a = 100x^3$

26) Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?
$a = - 200x^2$

27) Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?
$a = -10x$

28) Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?
$a = 100x^3$

29) The maximum acceleration of a simple harmonic oscillator is a_0 and the maximum velocity is v_0 . What is the displacement amplitude?

30) What happens to the time period of a simple pendulum if its length is doubled?

31) At what points is the energy entirely kinetic and potential in SHM?

32) How would the period of spring mass system change, when it is made to oscillate horizontally and then vertically?

33) Can a simple pendulum vibrate at the centre of Earth?

34) Is oscillation of a mass suspended by a spring simple harmonic in nature?

35) Why does the time period of a swing not change when two persons sit on it instead of one?

36) The amplitude of a harmonic oscillator is doubled. How does its energy change?

37) Two exactly similar simple pendula are vibrating with amplitudes 1 cm and 3 cm. What is the ratio of their energies of vibration?

38) At what point the velocity and acceleration are zero in S.H.M?

39) Can the motion of an artificial satellite around earth be taken as S.H.M?

40) What provides restoring force in the following cases?
(i) a spring compressed and then left free to vibrate.
(ii) Water disturbed in Ll-tube.
(iii) Pendulum disturbed from its mean position

41) Sometimes, when an automobile picks up speed, its body begins to rattle. Why?

42) On what factors does the energy of a harmonic oscillator depend?

43) What determines the natural frequency of a body?

44) What is a second's pendulum?

45) What fraction of the total energy is kinetic energy when the displacement is one-half of amplitude?

46) What will be the change in the time period of a loaded spring when taken to moon?

47) Is simple harmonic motion always linear?

48) Define force constant.

49) How many times in one vibration, K.E. and P.E. become maximum?

50) Is the damping force constant on a system executing SHM?

51) Two springs of force constants k_1 and k_2 are joined in series. What is the force constant of the combination?

52) What will be the time period of oscillation, if the length of a second pendulum is one third?

53) A driver wearing an electronic digital watch goes down into sea water with terminal velocity v. How will the time in the water proof watch be affected?

54) Two springs of force constant k_1 and k_2 are joined in parallel. What is the force constant of tile combination?

55) Why should the amplitude of the vibrating pendulum be small?

56) What is the total energy of a simple harmonic oscillator?

57) When a pendulum clock gains time, what adjustment should be made?

58) The number of harmonic components in the oscillations are represented by, y = 4 $\cos^2$ at sin 4t. What are their corresponding angular frequencies?

59) Define S.H.M. what are its characteristics? At what distance from the mean position in S. H. M. of amplitude 'r', the energy is half kinetic and half potential?

60) Estimate the time taken by the oscillating pendulum to shift from x = 0 to x = A/2 where A is the amplitude.

61) Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period. Does it depend on the mass of the bob?

62) Why can't we use a pendulum to work as a clock in a satellite?

63) We know, $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$. Does T depend on the displacement? Give reason.

64) A spring attached with a mass m oscillates with a frequency f. What will be the frequency when the same is taken to the moon? Why?

65) If $y = \frac{1}{\sqrt{a}}\sin\omega t - \frac{1}{\sqrt{b}}\cos\omega t$, find the amplitude of motion.

66) What are isochronous vibrations?

67) The bob of a vibrating simple pendulum is made ofice. How will the period of swing change when the ice starts melting?

68) Time period of a particle in SHM depends on theforce constant k and mass m of the particle as-follows:

$$\mathbf{T} = 2\pi\sqrt{\frac{m}{k}}$$

A simple pendulum executes SHM approximately, why then is the time period of a simple pendulum independent of the mass of the pendulum?

69) The frequencies of two tuning forks A and Bare 250 Hz and 255 Hz respectively. Both are sounded together. How many beats will be heard in 5 seconds?

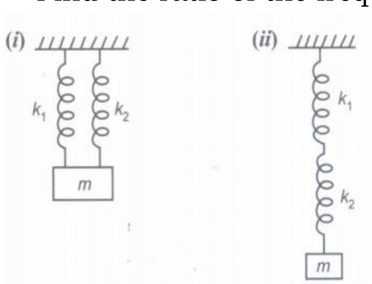
70) The bob of a simple pendulum is a ball full of water. If a fine hole is made in the bottom of the ball, what will be its effect on the time period of the pendulum?

71) Two exactly identical pendulums are oscillating with amplitudes 2 cm and 6 cm. Calculate the ratio of their energies of oscillation.

72) A man with a wrist watch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

73) A simple pendulum performs S.H.M. about x = 0 with an amplitude a and time period T. What is the speed of the pendulum at x = A/2?

74) Find the ratio of the frequencies in the cases given below:

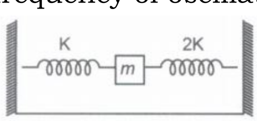


75) A particle executes S.H.M. of period 8 sec. After what time of its passing through the mean position will the energy be half kinetic and half potential?

76) The amplitude of an oscillating simple Pendulum is doubled. What will be its effect on the
(i) Periodic time;
(ii) Total energy;
(iii) Maximum velocity?

77) Show that the function of time y = $(\sin \omega t - \cos \omega t)$ represents simple harmonic motion.

78) Two springs of force constant and 2K are connected to a block of m as shown below. What is the frequency of oscillation of this block?



79) The frequency of oscillations of a mass m suspended by a spring is 'V_1'. If the length of spring is cut to one half, the same mass oscillates with frequency 'v_2 '. Determine the value of v_2/v_1.

80) With the help of examples differentiate between free oscillations and forced oscillations.

81) List any two characteristics of simple harmonic motion.

82) If the acceleration due to gravity on moon is one sixth that on the earth what will be the length and time period of a second's pendulum there? ($g = 9.8\ ms^2$)

83) A cylindrical wooden block of cross-section 15.0 cm^2 and 230 grams is floated over water with an extra weight 50 grams attached to its bottom. The cylinder floats vertically. From the state of equilibrium, it is slightly depressed and released. If the specific gravity of wood is 0.3 and g = 9.8 ms^{-1}, deduce the frequency of oscillation of the block.

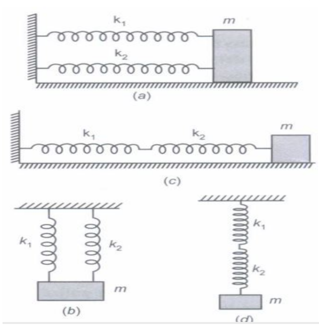
84) A particle of mass 10 g is placed in a potential field given by $U = 50x^2 + 100$ erg/gm. Calculate the frequency of oscillation.

85) A vertical U-tube of uniform cross-section contains water upto a height of 20 cm. Calculate the timeperiod of the oscillation of water when it is disturbed.

86) The kinetic energy of a particle vibrating in S.H.M. is 4 J when it passes the mean position. If the mass of the body is 2 kg and the amplitude is 1m, calculate its time period.

87) A simple pendulum of a length I and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

88) Figure below shows four different spring arrangements. If the mass in each arrangement is displaced from its equilibrium position and released, what is the resulting frequency of vibration in each case? neglect the mass of the spring.[Fig. (a) and (b) represent an arrangement of springs in parallel, and (c) and (d) represent springs in series]



89) What is a second's pendulum? How much is its length on the surface of moon?

90) Derive an expression for the potential energy of an elastic stretched spring.

91) A girl swinging suddenly stands up on the swing. What is the influence on the time period and frequency?

92) What provides the restoring force for simple harmonic oscillations in the following cases?
(i) Simple pendulum
(ii) Spring
(iii) Column of mercury in U-tube.

93) A simple pendulum of length 1 suspended from a roof of a trolley which moves in a horizontal direction with an acceleration a. Find its time period of oscillation.

94) The angular velocity and amplitude of a simple pendulum is co and r respectively. At a displacement x from the mean position, if its kinetic energy is T and potential energy is V, find the ratio of T to V.

95) Two simple harmonic motions are represented by

$x_1 = 10\sin\left(4\pi t + \frac{\pi}{4}\right)$

$x_2 = 5(\sin 4\pi t + \sqrt{3}\cos 4\pi t)$

What is the ratio of the amplitudes?

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1) (i) - 6.0cm

(ii) 16 Hz

2) 16 Hz

3) $\frac{v_0^2}{a_0}$

4) 2v

5) The direction of acceleration of the bobat its mean position is radial i.e.towards the point of suspensiion.

At extreme points however, the acceleration is tagential towards the mean position.

6)

(i) The motion of an artificial satellite around the earth is periodic as it repeats after a regular interval of time. But it cannot be taken as SHM because it is not a to and fro motion about any fixed point that is, mean position.

(ii) Time period of simple pendulum,

$T = 2\pi\sqrt{\frac{l}{g}}$ i.e.,$T\alpha\sqrt{l}$

Clearly, if the length is increased four times, the time period gets doubles.

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8)
Given m = 12Kg, original length l = 50 cm

K = 2.0 x 10^3 Nm^{-1}

F = ky

$y = \frac{F}{k} = \frac{mg}{k} = \frac{12 \times 9.8}{2 \times 10^3} = 5.9 \times 10^{-2} m = 5.9 \quad cm$

Stretched length of the spring = l + y = 50 + 55.9 cm

= 105.9 cm

Frequency of oscillations, $v = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= \frac{1}{2 \times 3.14}\sqrt{\frac{2 \times 10^3}{12}} = 2.06 s^{-1}$

9)
(i) Here F = 10 N,$\triangle l = 0.1m, m = 4kg$

$k = \frac{F}{\triangle l} = \frac{10}{0.1} = 100 Nm^{-1}$

(ii) Here F = 10 N,$\triangle l = 0.1m, m = 4kg$

$y = \frac{mg}{k} = \frac{4 \times 10}{100} = 0.4m$

(iii) Here F = 10 N,Δl = 0.1m,m = 4kg

$T = 2\pi\sqrt{\frac{m}{k}} = 2 \times \frac{22}{7}\sqrt{\frac{4}{100}} = 1.26s$

(iv) Here F = 10 N,Δl = 0.1m,m = 4kg

Frequency, $v = \frac{1}{T} = \frac{1}{1.26} = 0.8 \quad Hz$

10)
Here F = 10 N,$\triangle l = 0.1m, m = 4kg$

$y = \frac{mg}{k} = \frac{4 \times 10}{100} = 0.4m$

11)
Here F = 10 N,Δl = 0.1m,m = 4kg

$T = 2\pi\sqrt{\frac{m}{k}} = 2 \times \frac{22}{7}\sqrt{\frac{4}{100}} = 1.26s$

12)
Here F = 10 N,Δl = 0.1m,m = 4kg

Frequency, $v = \frac{1}{T} = \frac{1}{1.26} = 0.8 \quad Hz$

13)

Given mass of the disc m = 10 kg

Radius of the disc r = 15 cm = 0.15 m

T = 1,5 s

I is the moment of inertia of the disc about the axis of rotation which is perpendicular to the plane of the disc and passing through its centre.

$I = \frac{1}{2}mr^2 = \frac{1}{2} \times (10) \times (0.15)^2$

= 0.1125 kg-m^2

Time period, $T = 2\pi\sqrt{\frac{1}{\alpha}}$

$\alpha = \frac{4\pi^2 I}{T^2} = \frac{4 \times (3.14)^2 \times 0.1125}{(1.5)^2}$

$= 1.972 \ Nm/rad$

14)

A force which takes the body towards the mean postion in oscillation is called restoring force.

Characterstic of Restoring Force

The restoring force is aleays directed towards the mean positin and its magnitude of any instant is directly. Proportional to the displacement of the particle froom its mean postion of that instance.

15) Given PE = KE

i.e $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

$x^2 = A^2 - x^2 \Rightarrow x = \frac{A}{\sqrt{2}}$

$Now\ x = A\ sin\ \omega t = A\ sin\left(\frac{2\pi}{T}\right)t$

$So,\ \frac{A}{\sqrt{2}} = A\ sin 2\pi \frac{t}{8}$

$or\ sin\frac{\pi t}{4} = \frac{A}{\sqrt{2}} = sin\frac{\pi}{4}$

$or\ \frac{\pi t}{4} = \frac{\pi}{4}\ or\ t = 1s$

16)

(i) There is no repetition of the motion as the swimmer just completes one trip, hence not periodic.

(ii) The motion is repeated after a certain interval of time, hence periodic. In fact, the bar magnet oscillates about its mean position with a definite period of time.

(iii) Rotatory motion is periodic as repeating after fixed time-interval.

(iv) There is no repetition, hence not periodic.

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The motion is repeated after a certain interval of time, hence periodic. In fact, the bar magnet oscillates about its mean position with a definite period of time.

18) Rorarary motion is periodic as repeating after fixed time-interval.

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20)

(i) There is no to and fro motion which is a must for a periodic motion to be SHM. Hence, rotation of earth about its axis is not SHM.

(ii) This is a periodic motion and as it follows F=-kx (about mean position, to and fro motion) hence SHM

(iii) A periodic motion, oscillatory in nature about lower most point as mean position following SHM force law, hence, it is SHM.

(iv) A polyatomic molecule has a number of natural frequencies. So, in general, its vibration is a superposition of SHMs of a number of different frequencies. Thus, superposition is periodic but not necessarily SHM.

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A polyatomic molecule has a number of natural frequencies. So, in general, its vibration is superposition of SHMs of a number of different frequencies. Thus, superposition is periodic but not necessarily SHM.

24)

Yes, every periodic motion need to be SHM. e.g. the motion of the earth round the sun is a periodic motion, but not simple harmonic motion as the back and forth motion is not taking place.

25)

(i) No negative sign on RHS, hence, not SHM

(ii) Displacement on RHS is squared, hence not SHM

(iii) a = -10x follows the condition of SHM, accelerationα -displacement hence, SHM.

(iv) No negative sign on RHS and displacement appears as cubed, hence, not SHM

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29)

Let A be the displacement and ω be the angular frequency of the simple harmonic oscillator.

$Then,\ \ a_0 = \omega^2 A\ and\ v_0 = \omega A$

$on\ dividing,\ \frac{v_0^2}{a_0} = \frac{\omega^2 A^2}{\omega^2 A} = A\ or\ A = \frac{v_0^2}{a_0}$

30)

The time period is increased by a factor of $\sqrt{2}$

31)

At mean position, the energy is entirely K.E. At extreme positions, the energy is entirely P.E.

32)

The time period remains same in both the cases.

33)

No. This is because of zero value of g at the centre of Earth.

34)

Yes, it is if the spring is perfectly elastic.

35)

$T = 2\pi\sqrt{\frac{l}{g}}$ so it does not depend upon the mass.

36)

As E ∝ A^2, the energy of harmonic oscillator will became 4 times, its original value when its amplitude is doubled.

37)

$\frac{E_1}{E_2} = \frac{a_1^2}{a_2^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

38)

The velocity is zero at the extreme point of motion and acceleration is zero at the mean position of motion.

39)

No, it is a circular and periodic motion but not to and fro about a mean position which is essential for SHM

40)

(i) Elasticity of the material of the spring

(ii) Weight of water

(iii) Weight of pendulum.

41) This is because of resonant vibrations.

42) Energy of a harmonic oscillator depends on the mass, frequency and amplitude of oscillation.

43) Natural frequency of a body depends upon
(i) elastic properties of the material of the body and
(ii) dimensions of the body.

44) A pendulum, whose time period is 2 seconds is called a second's pendulum.

45) $\frac{K.E}{Total\ energy} = \frac{\frac{1}{2}m\omega^2\left(a^2 - \frac{a^2}{4}\right)}{\frac{1}{2}m\omega^2 a^2} = \frac{3}{4}$

46) No change, since $T = 2\pi\sqrt{\frac{m}{k}}$

47) No, it is not essential. Simple harmonic motion may be either a linear simple harmonic motion or an angular SHM

48) Force constant is defined as the restoring force developed in a body per unit displacement

49) Two times.

50)

No, because damping force depends upon velocity and is more when the system moves fast and is less when the system moves slow

51) The force constant k of series combination is given by
$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} or\ k = \frac{k_1 k_2}{k_1 + k_2}$

52) $\frac{T_2^2}{T_1^2} = \frac{l_2}{l_1} = \frac{\left(\frac{l}{3}\right)}{l} = \frac{1}{3} \quad or \quad \frac{T_2^2}{T_1^2} = \frac{(2)^2}{3} or\ T_2 = \frac{2}{\sqrt{3}} s$

53) It will not be affected as its action is independent of gravity and buoyant force

54)

Force constant k of parallel combination is given by k = k_1+ k_2. Thus, force constant of the parallel combination is equal to the sum of individual force constants of two springs.

55)

When amplitude of the vibrating pendulum is small, then angular displacement of the bob used in simple pendulum is small. Here the restoring force F = mg sin θ = mgθ = mgx/l, where x is the displacement of the bob and I is the length of pendulum. Hence F ex: x. Since F is directed towards mean position, therefore the motion of the bob of simple pendulum will be S.H.M. if θ is small

56) $\frac{1}{2}$m$\omega^2$$r^2$ where r = amplitude, ω = angular frequency, m = mass of the oscillator.

57)

When a pendulum clock gains time, it means it has gone fast i.e., it makes more vibrations per day than required. This shows that the time period of oscillation has decreased. Therefore, to correct it, the length of pendulum should be properly increased.

58)

y = 4 $\cos^2$ 2t sin 4t = 2(cos 4t + 1) sin 4t [$\because$ 2 $\cos^2\theta$ = cos 2θ + 1]

= 2 sin 4t cos 4t + 2 sin 4t = sin 8t + 2 sin 4t

= 2 sin 4t + sin 8t

Thus the resulting harmonic oscillation is a combination of two harmonic motions of angular frequencies 4 rad/ s and 8 rad/ s.

59)

SHM is the projection of uniform circular motion on a diameter of a circle of a circle. Characteristics of SHM are:

(i) Motion is always directed towards a fixed point or equilibrium point.

(ii) Acceleration is directly proportional to negative of displacement.

Amplitude = r

Displacement from the mean position, where the energy is half kinetic and half potential.

$\frac{k}{2} = \frac{U}{2}$ or K.E. = P.E.

$\frac{1}{2}kx^2 = \frac{1}{2}k\left(A^2 - x^2\right) = \pm\frac{A}{\sqrt{2}}$

60)

Since initial position at t = 0 is x = 0.

We represent S.H.M.

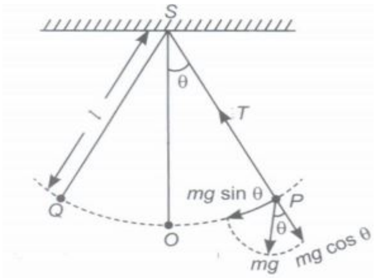
by x = A sin = ωt

When x = $= \frac{A}{2}, \frac{A}{2}$ = A sin = ωt

$\therefore \quad \omega t = \frac{\pi}{6}, t = \frac{\mathrm{T}}{12}$

61)

Expression for time period :



Let m = Mass of the bob

I = Length of the simple pendulum

OP=x

when the bob is displaced to point P through small 1 angle θ .

Two forces acting on the bob are

(i) Weight mg of the bob acting vertically downward.

(ii) Tension T in string along PS, resolving mg into two components

(a) mg cos θ opposite to tension T.

(b) mg sin θ directed towards O.

Tension in the string, T = mg cos θ

The force mg sin θ tends to bring back the bob to its mean position O.

$\therefore$ Restoring force acting on bob is F = -mg sin θ -ve sign shows force is directed towards mean position: If 8 is small, then

$\sin\theta = \theta\frac{(\text{arc}\, OP)}{l} = \frac{x}{l}$

F = $-mg\theta = -mg\frac{x}{l}$ ......(i)

F $\propto$ displacement (x) and F is directed towards mean position O.

In S.H.M., Restoring force

F = -kx ......(ii)

Comparing (i) and (ii)

$k = \frac{mg}{l}$

Inertia factor = Mass of bob = m

T = $2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

$= 2\pi\sqrt{\frac{m}{mg/l}} = 2\pi\sqrt{\frac{l}{g}}$

T = $2\pi\sqrt{\frac{l}{g}}$

No, T does not depend on the mass of the bob.

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62)

Time period of an oscillating pendulum changes with acceleration due to gravity. It is zero in a satellite. So, only clocks with spring can be used.

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63) No. Since acceleration is proportional to negative of displacement. Time period is independent of displacement.

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64)

Frequency of a mass attached to a spring is f = $= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. It is independent of acceleration due to gravity. So, the frequency is not affected on the surface of moon.

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65) Amplitude of the displacement = $\sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2}$ since phase difference between the two portions is $\pi/2$.

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66) When the time period is independent of amplitude, the oscillation is called isochronous.

67)

As ice melts the centre of gravity raises. So, the time period reduces. As complete ice melts, the centre of gravity retains its original position. So, time period decreases and increases back to the same value.

68) In simple pendulum, k = $\frac{mg}{l}$

So, $\mathrm{T} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}}$

Hence, T is independent of m.

69) f_1 = 250 Hz;.f_2 = 255 Hz.

Number of beats per second or beat frequency = 255 - 250 = 5.

Number of beats heard in 5 seconds = 5 x 5 = 250.

70)

As water drips out, the concentration of mass becomes more on the lower portion. The centre of mass shifts down as water completely drop out then it reaches back to the original point i.e., centre of ball. So, the length increases and decreases to the original value. Time period increases first and then decreases back to the original value.

71) Total energy = $\frac{1}{2}m\omega^2 A^2$, since amplitudes are 2 cm and 6 cm, the ratio is 1 :3.

$\therefore$ Ratio of total energy = 1:9.

72)

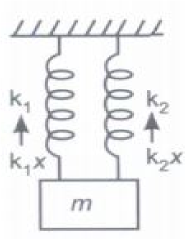
Since time period of the wrist watch working on oscillation 0'[spring is independent of acceleration due to gravity, it will give correct time during free fall also.

73) Velocity of oscillating body = v = $\omega\sqrt{\mathrm{A}^2 - x^2}$

x = $\frac{\mathrm{A}}{2}$

v = $\omega\sqrt{A^2 - \left(\frac{A^2}{4}\right)}$

74)
(i) Restoring force in the springs will be in the same direction and the displacement is same in both.

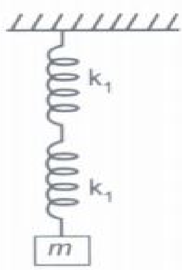


=-k_1x - k_2X

k_{eq} =k_1 + k_2

$\therefore \quad f=\frac{1}{2\pi}\sqrt{\frac{\kappa_1+k_2}{m}}$

(ii) Restoring force is in same direction I but the extensions are different even though the force is same.



$\therefore \quad x=x_1+x_2$

$1\frac{1}{k_{eq}}=\frac{1}{k_1}+\frac{1}{k_2}$

$\frac{1}{k_{eq}}=\frac{k_2+k_1}{k_1k_2}$

$f=\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{m(k_1+k_2)}}$

Amplitude of x_1 = 10

75)
Given P.E = K.E

$\frac{1}{2}m\omega^2y^2=\frac{1}{2}m\omega^2\left(a^2-y^2\right)$

$y^2=a^2-y^2$

i.e., $y=\frac{a}{\sqrt{2}}$

Now Y = a sin ωt

or $y=a\sin(2\pi/\mathrm{T})t$

$\frac{a}{\sqrt{2}}=a\sin\left(\frac{2\pi}{8}\right)t\ldots(\mathrm{T}=8\mathrm{sec.})$

$\sin\frac{\pi t}{4}=\frac{1}{\sqrt{2}}=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\frac{\pi t}{4}=\frac{\pi}{4}$ or $t=1\mathrm{sec}$

76)
(i) Time period is independent of amplitude. So no change in T with amplitude.

(ii) Total energy = $1/2m\omega^2A^2$ If A is doubled, total energy becomes four times.

(iii) Maximum velocity v = $\omega\mathrm{A}$. If A is doubled, maximum velocity becomes double.

77) We have y(t) = $\sin \omega t - \cos \omega t$

$= \sqrt{2}\left(\sin \omega t \times \frac{1}{\sqrt{2}} - \cos \omega t \times \frac{1}{\sqrt{2}}\right)$

$= \sqrt{2}\left(\sin \omega t \cos \frac{\pi}{4} - \cos \omega t, \sin \frac{\pi}{4}\right)$

$\begin{bmatrix} \because & \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \\ \text{and} & \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \end{bmatrix}$

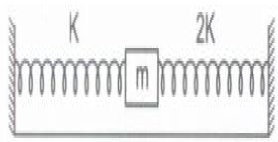
$= \sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right)$

$[\because \sin(A - B) = \sin A \cos B - \cos A \sin B]$

Moreover, $y\left(t + \frac{2\pi}{\omega}\right) = \sqrt{2}\left(\omega t + 2\pi - \frac{\pi}{4}\right) = y(t)$

Hence, it represents simple harmonic motion.

78) Force constants of two springs K and 2K are connected to a block of mass 'm'



Springs are in parallel combinations. So equivalent.

K' = K + 2K = 3K

In simple harmonic motion,

Frequency (f) = $(f) = \frac{1}{2\pi}\sqrt{\frac{K'}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$

Hence, the frequency of the oscillation of the block $= \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$

79)

Frequency of oscillation of mass m suspended by a spring of constant k is $v_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

If the length is cut into one half, the force constant will becomes 2k for each portion. The frequency with same mass m is,

$v_2 = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

$\because \quad \frac{v_2}{v_1} = \frac{\sqrt{2}}{1}$

80)

In the absence of air resistance, a pendulum oscillates freely but another pendulum dipped in a liquid oscillates only when external force exists. Oscillations which exist by the use of an external force overcoming any loss of energy are called forced oscillations.

81) (i) Motion is always directed towards a fixed point or equilibrium point.

(ii) Acceleration is directly proportional to negative of displacement.

82) On moon, g_m= g/6 = 9.8/6 ; T = 2 s

T = $2\pi\sqrt{\frac{l}{g_m}}$

or $l = \frac{T^2 g_m}{4\pi^2} = \frac{2^2 \times (9.8/6)}{4 \times (22/7)^2}$

= 0.165 m = 16.5 cm

83) mg = Vpg

(230 + 50) x 980 = 15.0 x l x 1.0 x 980

l = $\frac{280}{15\times1.0}$ = 18.66 cm

T = $2\pi\sqrt{\frac{18.66}{980}}$ = = 0.8 sec

84) P.E. of 10 gram particle is U

= 10(50 x^2 + 100) erg. The force acting on the particle is given by

F = $\frac{-dU}{dx} = \frac{-d}{dx}$ (500 x^2 + 1000)

= -1000 x

But F $m\frac{d^2x}{dt^2}$

$\therefore \quad m\frac{d^2x}{dt^2} = -1000x$

or $\frac{d^2x}{dt^2} = \frac{-1000x}{m} = \frac{-1000x}{10}$

= -100x

As $\frac{d^2x}{dt^2} = -\omega^2x;\ \text{so}\ \omega^2 = 100$

or $\omega = \sqrt{100}$ = 10

Frequency of oscillation,

$v = \frac{\omega}{2\pi} = \frac{\sqrt{100}}{2\pi} =$ 1.58 s^{-1}.

85) The length of the liquid column,

L = 2 x 20 = 40 cm

Time period of oscillation,

T = $2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{40}{2\times980}} = \frac{2\pi}{7}$

= 0.9 s

86) In its mean position, the kinetic energy of the particle is maximum,

E_{Kmax} = $\frac{1}{2}m\omega^2A^2$

Given: E_{Kmax} = 4 J ; m = 2 kg ; A = 1 m

4 J $\frac{1}{2}\times 2\times\omega^2\times 1$

or $\omega = 2,\ \mathrm{T} = \frac{2\pi}{\omega} = \pi\mathrm{s}$.

87) The acceleration associated with the bob, hung in a car moving on a circular track is,

$a_{\mathrm{N}} = \sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}$ since centripetal acceleration will be experienced besides the force of gravity.

$\therefore \mathrm{T} = 2\pi\sqrt{\frac{l}{a_{\mathrm{N}}}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2+\frac{v^4}{r^2}}}}$

88) (a), (b) In parallel equivalent value of k = $k_1 + k_2$

$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$

(c), (d) In series equivalent k = $\frac{k_1k_2}{k_1+k_2}$

$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)m}}$

89) A second's pendulum is one whose time period of oscillation is 2 seconds. On the surface of moon, $a=\frac{g}{6}$

$\therefore$ Using $\mathrm{T}=2\pi\sqrt{\frac{l}{g}}$,

We have $l=\frac{4g}{6\times4\pi^2}=\frac{1}{6}\mathrm{~m}$

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90) Consider a spring attached with a mass m stretching by a length y after t sec.

Restoring force

F = mass x acceleration

= - $m\omega^2 y=-ky$

where, k = spnng constant = $m\omega^2$

Work done for an additional displacement dy against restoring force is

dW =-Fdy

= - (- ky) dy = ky dy

Total work done

W = $\int_0^y kydy=\frac{1}{2}ky^2$

This work done appears as a P.E. 'U' of the particle.

$\mathrm{U}=\frac{1}{2}ky^2=\frac{1}{2}m\omega^2y^2$

= $\frac{1}{2}m\omega^2a^2\sin^2\omega t$

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91) Girl can be considered as an extended body. As the girl stands up on the swing so, the separation' d' between the point of suspension and the centre of gravity decreases. Since time period is inversely proportional to $\sqrt{d}$, time period increases and frequency decreases.

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92) (i) Part of the force of gravity.

(ii) Elastic restoring force.

(iii) Force due to difference in column of mercury or pressure difference between the levels on the two limbs.

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93) The trolley is accelerated horizontally by a. So, there will be two accelerations, g vertically down and horizontal acceleration a. The net acceleration is $\sqrt{g^2+a^2}$ .The time period

$\mathrm{T}=2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

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94) Kinetic energy at x is $\mathrm{T}=\frac{1}{2}\mathrm{~m}\omega^2\left(\mathrm{~A}^2-x^2\right)$

Potential energy at x is

$\mathrm{V}=\frac{1}{2}m\omega^2x^2$

$\frac{T}{V}=\frac{A^2-x^2}{x^2}=\left(\frac{A^2}{x^2}-1\right)$

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95) $x_2=5\sin4\pi t+5\sqrt{3}\cos4\pi t$

Amplitude of x_2 = $\sqrt{5^2+(5\sqrt{3})^2}=10$

Since the sin 4nt and cos 4nt functions are out of phase by $\pi/2$.

Amplitude of x_2 = 10

$\therefore$ Ratio of amplitudes is 1:1