GRAVITATION

11th Standard CBSE

Date : 27-Nov-22

Physics

Reg.No. :

Exam Time : 01:00:00 Hrs

Total Marks : 100

109 x 2 = 218

1) Imagine what would happen if the value of G becomes
(i) 100 times if its present value|
(ii) 1/100 times of its present value

2) Imagine what would happen if the value of G becomes 1/100 times of its present value

3) Calculate the ratio of the kinetic energy required to be given to a satellite so that it escape the gravitational field of earth to the kinetic energy required to put the satellite in a circular orbit just above the free surface of the earth.

4) Two particular of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. What is the speed of each particle?

5) An artificial satellite moving in a circular orbit around the earth has a total energy E_0 . What is its potential energy?

6) Two satellites have their masses in the ratio of 3:1. The radii of their circular orbits are in the ratio of 1:4. What is the ratio of total mechanical energy of A and B?

7) The time period of a satellite of earth is 5h. If the separation between the earth and the satellite is increased to four times the satellite is increased to four times the previous value, then what will be the new time period?

8) A plant moving along an elliptical orbit is closet to the Sun at a distance r_1 and farthest away at a distance of r_2 . If V_1 and V_2 are the linear velocities at these points respectively, then find the ratio v_1/v_2

9) A plant moving along an elliptical orbit is closet to the Sun at a distance r_1 and farthest away at a distance of r_2 . If V_1 and V_2 are the linear velocities at these points respectively, then find the ratio v_1/v_2

10) The gravitational force between two spheres os x when the distance between their centre is y. What will be the new force, if the separation is made 3y?

11) A mass M is broken into two parts, m and (M-m). How is m related to M so that the gravitational force between two parts is maximum?

12) A mass of 1 g is separated from another mass of 1 g by a distance of 1 cm. How many g-wt of force exists between them?

13) If the earth be at one half of its present distance from the sun, them how many days will be there in a year?

14) Calculate the force of attraction between two bodies, each of mass 100 kg 1 m apart on the surface of the earth.

15) Show that the orbital velocity of a satellite revolvimg the earth is 7.92 kms^{-1}

16) The orbiting velocity of an earth-satellite is 8 kms^{-1} .What will be the escape velocity?

17) A satellite does not need any fuel to circle around the earth.Why?

18) If the kinetic energy of a satellite revolving around the earthin any orbit is doubled,then what will happen to it?

19) On what factor does the escape speed from a surface depend?

20) An astronaut,by mistake,drops his food packet from an artificial satellite orbiting around the earth.Will it reach the surface of the earth?

21) If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, then what will happen to the satellite?

22) Which of the following symptoms is likely to affect an astronaut in space

(i) swollen feet

(ii) swollen face

(iii) headache

(iv) orientational problem

23) Which of the following symptoms is likely to affect an astronaut in space

swollen face

24) Why a tennis ball bounces higher on hills than on plains?

25) The acceleration due to gravity on a planet is $1.96 ms^{-2}$. If it is safe to jump from a height of 2m on the earth, then what will be the corresponding safe height on the planet?

26) Does the concentration of the earth's mass near its centre change the variation of g with height compared with a homogeneous sphere, how?

27) Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh 3/5th as much as at present. Take the equatorial radius as 6400km.

28) Which of the following symptoms is likely to affect an astronaut in space
headache

29) Which of the following symptoms is likely to affect an astronaut in space
orientational problem

30) The escape speed on the earth is 11.2 km/s.What is its value for a planet having double the radius and eight times the mass of the earth?

31) What is the gravitational potential energy of a body at height h from the earth surface?

32) A spherical planet has mass $M_P$and diameter D_P.A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to whom?

33) The earth is acted upon by the gravitational attraction of the sun.Why don't the earth fall into the sun?

34) The asteroid pallas has an orbital period of 4.62 yr. Find the semi-major axis of its orbit.Given $G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$ mass of the sum = $1.99 \times 10^{30} kg \; and \; 1 \; yr \;\; = 3.156 \times 10^7 s$

35) Does the change in gravitational potential energy of a body between two given points depends upon the nature of path followed, why?

36) How far away from the surface of the earth does the acceleration due to gravity become 4% of its value on the surface of the earth?

37) Find the percentage decrease in the weight of a body when taken 10 km below the surface of the earth.

38) How much below the surface of the earth does acceleration due to gravity become 1% of the value at the earth's surface?

39) Determine the speed of which the earth would have to rotate on its axis so that a person on the equator would weight 3/5 as much as the present.

40) The mount everest is 8848 m above sea level. Estimate the acceleration due to gravity at this height, given that mean g on the surface of the earth is 9.8m/s^2

41) What will be the value of g at the bottom of sea 7km deep?Diameter of the earth is 12800 km and g on the surface of the earth is $9.8 m/s^2$

42) If the earth was a perfect sphere of radius $6.37 \times 10^6 m$, rotating about its axis with a period of 1 day, then how much would the acceleration due to gravity (g) differ from the poles to the equator?

43) An astronaut inside a small spaceship orbiting around the Earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity?

44) For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii), e, (iii) f, (iv) g.

45) What is the gravitational potential at infinity?

46) The masses of two bodies are doubled and the distance is halved, how will the gravitational force change?

47) A satellite revolving around earth loses height. How will its time period be changed?

48) Give two uses of polar satellites.

49) Distance between two bodies is increased to three times its original value. What is the effect on the gravitational force between them?

50) From where does a satellite revolving around a planet get the required centripetal force?

51) Is gravitational potential a scalar or a vector? Why?

52) Two planets are at distances R_1 and R_2 from the Sun. What will be the ratio of the squares of their periods?

53) Where does a body weigh more at the surface of the earth or in a mine?

54) Two artificial satellites, one of mass 400 kg and another of mass 2500 kg, are set in the same orbit around a planet. What is the ratio of their (i) orbital velocities, (ii) time periods?

55) Can we determine the gravitational mass of a body inside on artificial satellite?

56) why are space rockets usually launched from west to east in the equatorial plane?

57) The gravitational potential energy of a body at a distance r from the centre of earth is U. What is the weight of the body at that point?

58) A mass thrown up returns to the surface of earth. What is the nature of total energy possessed?

59) Should the value Of escape velocity be less or more on the surface of Moon as compared to Earth?

60) What is the relation between the orbital and escape velocity?

61) What would happen to an artificial satellite, if its orbital velocity is slightly decreased due to some defects in it?

62) What is the time period of revolution of polar satellite of Earth?

63) What do you mean by a parking orbit of a satellite?

64) Two satellites of masses 3m and m orbit the earth in circular orbits of radii r and 3r respectively. What is the ratio of their orbital speeds?

65) Name one factor on which the period of revolution of a planet around the Sun depends.

66) Give the S.l. units of 'g' and 'G'.

67) Give the dimensional formula of 'g' and 'G'.

68) A satellite is orbiting around the Earth with a speed v. To make the satellite escape, what is the minimum percentage increase in its speed?

69) Is it possible to put a satellite into its orbit by firing it from a huge sized gun?

70) If a satellite is revolving around a planet of density ρ, show that the entity $\rho\ T^2$ is a universal constant. When a satellite is orbiting close to earth, then

71) State Newton's law of gravitation in vector form.

72) A planet reduces its radius by 1% with its mass remaining same. How acceleration due to gravity varies?

73) What is the gravitational force on a body inside a spherical shell? Why is it so?

74) Planet Mars has two moons-phobos and deimos. Phobos has a period of 7 hours 39 minutes and orbital radius of 9.4×10^3 km. Calculate the mass of Mars.(G = 6.67×10^{-11} $Nm^2\ kg^2$)

75) What is the angular velocity at any point on the equator so that the body feels weightlessness?

76) What is gravitational potential energy at a point? How much of work is done in shifting a mass from the surface to a height equal to its radius?

77) What is escape velocity. Derive an expression for the same.

78) Show graphically how gravitational field strength varies with distance from the centre of earth, outwards. Give the relation also.

79) The value of acceleration due to gravity at the moon is $\frac{1}{6}$ th of the value of g at the surface of the earth, and the diameter of the moon is $\frac{1}{4}th$ f the diameter of the earth. Compare the ratio ofthe escape velocities.

80) The change in the value of g at a height h above the earth is same as at a depth d below it. If hand dare small as compared to the radius of the earth, what is the ratio (h/d)?

81) The radius of the earth is reduced by 4%. The mass of the earth remains unchanged. What will be the change in escape velocity?

82) If a planet existed whose mass and radius were both half of those of the earth, what would be the value of the acceleration due to gravity on its surface as compared to what it is on the earth's surface?

83) Objects at rest on the earth's surface move in circular paths with a period of 24 hours. Are they in orbit in the same sense that an earth's satellite is in orbit? Why not? What would the length of the day have to be to put such objects in a true orbit?

84) While approaching a planet circling a distant star, a space traveller determines the planet's radius to be half that of the earth. After landing on the surface, he finds the acceleration due to gravity to be twice that on the surface of the earth. Find the ratio of the mass of the planet to that of the earth.

85) The mass and diameter of a planet are twice of those of the earth. What will be the period of oscillation of a pendulum on this planet, ifit is a second's pendulum on the earth?

86) Satellite A is in a certain circular orbit about a planet, while satellite B is in a larger circular orbit. Which satellite has
(i) the longer period and
(ii) the greater speed?

87) State Kepler's laws of planetary motion and deduce Newton's Law of gravitation from them.

88) Discuss the variation of 'g' with depth. What happens to 'g' at the centre of earth?

89) Time period of a planet around the Sun is 11.6 years. How far is the planet from the Sun? The distance between the Sun and the earth is 1.5 x 10^8 km.

90) If the earth has a mass nine times and radius twice that of the planet Mars, calculate the maximum velocity required by a rocket to pull out of the gravitational force of Mars. Given escape velocity on the surface of earth is 11.2 km/sec.

91) Explain gravitational potential at a point in gravitational field. Give relation between gravitational field intensity and gravitational potential.

92) Discuss the variation of 'g'
(i) due to height
(ii) due to depth.

93) Draw the graph showing the variation of acceleration due to gravity with
(a) height above the earth's surface.
(b) depth below the earth's surface.

94) The radii of two planets are Rand 2R respectively and their densities ρ and $\rho/2$ respectively. What is the ratio of acceleration due to gravity at their surfaces?

95) How far above the earth's surface does the value of g becomes 20% of its value on the surface?

96) What is the acceleration due to gravity at the bottom of a sea 30 km deep taking radius of the earth as 6.3 x 10^6 km?

97) Earth's radius is about 6370 km. A mass of 20 kg is taken to a height of 160 km above the earth's surface.
(a) What is the mass of the objects at that height?
(b) How much does the object weigh at this height?

98) Give one example each of central force and noncentral force.

99) Draw areal velocity versus time graph for mars.

100) How is the gravitational force between two point masses affected when they are dipped in water keeping the separation between then the same?

101) Is it possible for a body to have inertia but no weight?

102) Out of aphelion and perihelion, where is the speed of the Earth more and why?

103) How can you find the mass of earth, starting from the law of gravitation?

104) Is it possible to place a satellite, so that it is always over New Delhi? Why?

105) Define gravitational field strength. What is the field at a point distance r from a mass M?

106) A high jumper can jump 1.5 m on earth. With the same effort, how high will he be able to jump on a planet whose density is one-third and radius is one-fourth of that of the earth?

107) Calculate the gravitational force which Sun exerts on Jupiter

Given: G = 6.67 x 10^{-11} Nm^2/kg^2

M_J = 1.9 x 1.0^{27} kg

M_s = 1.99 x 10^{30} kg

Mean distance = 7.8 x 10^{+11} m

108) What is the percentage increase in velocity for moon to escape from the gravitational pull of the earth?

109) What is the period of revolution of Neptune around the sun, given that the diameter of its orbit is 30 times the diameter of the earth's orbit around the sun, both orbits being assumed to be circular?

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109 x 2 = 218

1) (i) Earth's attraction would be so large that you would be crushed to the earth.

(ii) Earth's attraction would be so less that we can easily jump from the top of a multi-storey building.

2) Earth's attraction would be so less that we can easily jump from the top of a multi-storey building.

3) 2

4) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$

5) $U = 2E_0$

6) 12:1

7) $T_2 = T_1\left[\frac{R_2}{R_1}\right]^{3/2} = T_1[4]^{3/2} = 8T_1 = 40h$

8) From the law of conservation of angular momentum

$mr_1v_1 = mr_2v_2 \quad \Rightarrow \quad r_1v_1 = r_2v_2 \ \ or \ \ \frac{v_1}{v_2} = \frac{r_2}{r_1}$

9) From the law of conservation of angular momentum

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10) $F\alpha\frac{1}{r^2}$ So, if r is increased by a factor of 3, F will be reduced by a factor of 9. Thus, the new force will be x/9.

11)
$Let \quad m_1 = m, \quad m_2 = M - m$
$F = G\frac{m(M-m)}{r^2} = \frac{G}{r}(Mm - m^2)$
$Differentiating\ w.r.t\ m, \frac{df}{dm} = \frac{G}{r^2}(M - 2m)$
$for\ F\ to\ be\ maximum,\ \frac{df}{dm} = 0$
$\frac{G}{r^2}(M - 2m = 0$
$M = 2m\ or\ m = \frac{M}{2}$
$m_1 = m_2 = \frac{M}{2}$

12)
$F = G\frac{m_1 m_2}{r^2}$
$= 6.67 \times 10^{-8}) \left(\frac{1 \times 1}{1^2}\right) dyne$
$= 6.67 \times 10^{-8} \quad dyne = \frac{6.67 \times 10^{-8}}{980}$
$= 7 \times 10^{-11} g - wt$

13)
129 days

14)
6.67 x 10^{-7} N

15)
Orbital velocity ,
$v_o = \sqrt{gR}$
$= \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 kms^{-1}$

16)
Escape velocity,
$v_e = \sqrt{2} v_0$
$v_e = \sqrt{2} \times 8 = 11.31 kms^{-1}$

17)
The gravitation force between satellite and the earth provides the centripetal force required by the satellite to move in a circular orbit.The satellite orbits around earth at such a higher height where air friction is neglible.

18)
The total energy of a satellite in any orbit,E = -K, where K is its KE in that orbit.
If it kinetic energy is doubled,i.e an addition kinetic energy (K) is given to it $E = -K + K = 0$ and the satellite will leave its orbit and go to infinity.

19)
Value of escape speed at the surface of a planet is given by the relation
$v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$
Thus,the value of escape speed from the surface of planet depends upon(i) value of acceleration due to gravity g at the surface and (ii) the size(i.e. radius) R of the planet only.It is independent of all other factors. e.g The mass and size of the body to be projected,angle of projection,etc

20)
The food packet will not fall on the earth.As the satellite as well as astronaut were in a state of weightlesseeness,hence,the food packet.when dropped by mistake,will also start moving with the same velocity as that of satellite and will continue to move along with the satellite in the same orbit.

21)

If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero,satellite will be not able to rvolve around the earth.Instead the satellite will start moving along a straight line tagentially at the point on its orbit,where it is at the time of gravitational force becoming zero.

22)

(i) We know that the legs carry the weight of the body in the normal position due to gravity pull. The astronaut in space is in weightlessness state. Hence, swollen feet may not affect his working.
(ii) In the conditions of weightlessness, the face of the astronaut is expected to get more supply. Due to it, the astronaut may develop swollen face. As eyes ears, nose, mouth etc.are all embedded in the face,hences swollen face may affect to great extent the seeing/hearing/smelling/eating capabilities of the astronaut in the space.
(iii) Headache is due to mental strain. It will persist whether a person in an astronaut in space or he is on earth. It means headache will have the same effect on the astronaut in space as on a person on earth.
(iv) Space also an orientation. We also have the frames of reference in space. Hence, orientational problem will affect the astronaut in space.

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24)

As the acceleration due to gravity on hills is less than that on the surface of the earth (effect of height), therefore, a tennis ball bounces higher on hills than on plains.

25)

The safety of a person depends upon the momentum with which the person hits the planet. Since, the mass of the person is constant, therefore the maximum velocity v is the limiting factor.

26)

Any change in the distribution of the earth's mass will not affect the variation of acceleration due to gravity with height. This is because for a point outside the earth, the whole mass of the earth is effective and the earth behaves as a homogenous sphere.

27) Acceleration due to gravity at the equator is

$g_e = g - R\omega^2$

$mg_e = mg - mR\omega^2 \ or$

$\frac{3}{5}mg = mg - mR\omega^2 \ \left[\because mg_e = \frac{3}{5}mg\right]$

$\therefore \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2\times9.8}{5\times6400\times10^3}} = 7.8\times10^{-4} rad/s$

28)

Headache is due to mental strain.It will persist whether a person in an astronaut in space or he is on earth.It means headache will have the same effect on the astronaut in space as on a person on earth.

29)

Space also an orientation.We also have the frames of reference in space.Hence, orientational problem will affect the astronaut in space .

30)
v_p (escape speed on a planet) = $\sqrt{\frac{GM_p}{R_p}}$

v_e (escape speed on the earth) = $\sqrt{\frac{GM_e}{R_e}}$

Clearly , $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{8 \times \frac{1}{2}} = 2$

$v_p = 2v_e = 22.4km/s$

31)
Gravitational potential energy, i.e

$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h}\left[where, g = \frac{GM}{R^2}\right]$

$= -\frac{gR^2m}{R\left(1+\frac{h}{R}\right)} = -\frac{mgR}{1+\frac{h}{R}}$

32)
Force is given by

$F = \frac{GM_em}{R^2} = \frac{GM_pm}{(D_p/2)^2} = \frac{4GM_pm}{D_p^2}$

$\frac{F}{m} = \frac{4GM_p}{D_p^2}$

33)

The earth is orbiting round the sun in a stable orbit(nearly circular) such that the gravitational attraction of the sun just provides the required centripetal force to the earth for its orbital motion.So, net force on the earth is zero and consequently, the earth does not fall into the sun.

34)
$T^2 = \frac{4\pi^2a^2}{GM_5}$

or Substituting values, we get a = $4.15 \times 10^{11}m$

35)

The change in gravitational potential energy of a body between two given points depends only upon the position of the given points and is independent of the path followed.is due to the fact that the gravitational force is a conservative force and work done by a conservative force depends only on the position of initial and final poits and is independent of path followed.

36)
25600 km

37)
0.25

38)
6.32×10^3km

39)
5 kms^{-1}

40)
9.77m/s^2

41)
9.789m/s^2

42) $3.37 \times 10^{-2} m/s^2$

43)

Yes. If the size of the spaceship is extremely large, then the gravitational effect of the spaceship may become measurable. The variation in g can also be detected.

44)

Using the explanation given in the solution of the previous problem, the direction of the gravitational field intensity at P will be along e. So, option (ii) is correct.

45)

Zero.

46)

Since $F \propto \frac{m_1 m_2}{r^2}$

Thus gravitational force will be 8 times more.

47)

Time period of satellite is, given by $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

Therefore, T will decrease if h decreases.

48)

They are used for (i) ground water survey, (ii) detecting the areas under forest.

49)

Since $F \propto \frac{1}{r^2}$ hence the force will remain only $\frac{1}{9}$th part of its original value when the distance between two bodies is increased to three times.

50)

From gravitational attraction of planet exerted on satellite.

51)

Gravitational potential at a point is a scalar quantity because it is defined as work done per unit mass and the work done is a scalar quantity.

52)

$\frac{R_1^3}{R_2^3}$

53)

Since value of g in a mine is lesser than at the surface of the Earth, so weight of body in a mine is lesser than the weight of the body on the surface of the earth.

54)

$\frac{v_1}{v_2} = \frac{T_1}{T_2} = 1$ because both satellites are revolving in same orbit and for a given orbit the orbital velocity, as well as time period, is independent of the mass of satellite.

55)

No, artificial satellite is like a freely falling body and the.weightof. the body inside the satellite is zero.

56)

Space rockets are usually launched from west to east in the equatorial plane, so the linear velocity of Earth's rotation about its own axis is added with the launching velocity of rocket and linear velocity of Earth's rotation is maximum in the equatorial plane.

57) $U = \frac{GMm}{r} = (\frac{GM}{r^2})rm = grm$

$\Rightarrow$ U = mgr or $mg = \frac{U}{r}$ (weight of the body).

58) Total energy has to be negative since potential energy dominates over kinetic energy.

59)

Since, escape velocity, $v_e = \sqrt{2gR}$

The value of 'g' and' R' is less on Moon than the corresponding values on Earth, therefore, value of escape velocity at Moon surface is less.

60) If v_e is escape and V_o is orbital velocity, then

$v_e = \sqrt{2}v_0$

61) It will fall onto the Earth.

62) About 100 minutes.

63)

The orbit of a satellite which is concentric and coplanar with the equatorial plane of Earth and having a revolution period of 24 hours is called a parking orbit.

64) Since $v = \sqrt{\frac{GM}{r}}$ i.e., $v = \propto \frac{1}{\sqrt{r}}$

Thus $\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}} = \sqrt{3}$

65)

Period of revolution of a planet around the Sun depends upon the distance of the planet from the Sun (i.e., orbital radius of the planet).

66) The S.I.unit of 'g' is ms^{-2} and the S.I.unit of 'G' is Nm^2/kg^2 .

67) Dimensional formula of 'g' $\rightarrow$ $[LT^{-2}]$

Dimensional formula of 'G' $\rightarrow$ $[M^{-1}L^3T^{-2}]$

68) Percentage increase in speed $= \frac{v_e - v_0}{v_0} \times 100 = (\frac{v_e}{v_0} - 1) \times 100$

$= (\sqrt{2} - 1) \times 100 = 41.4\%$

69)

Theoretically, it is possible but practically it is not feasible on account of large air resistance and other technical difficulties.

70) $\frac{GMm}{R^2} = mR\omega^2 \; or \; \frac{G\frac{4}{3}\pi R^3\rho m}{R^2} = mR\frac{4\pi^2}{T^2} \; or \; \rho T^2 = \frac{3\pi}{G} = a \; constant.$

71) The force of attraction between a pair of masses m, and m_2 separated by a length 'r' is given by.

$\overrightarrow{\mathrm{F}_{12}} = -\frac{Gm_1m_2}{r^2}\hat{r}_{21}$

[$F_{12} \rightarrow$ force on 1 due to 2].

$\hat{r}_{21} \rightarrow$ Points from 2 to 1.

-ve sign shows that the force is attractive.

72) When mass is same $g \propto \frac{1}{R^2}$

$\therefore \; \frac{\Delta g}{g} = 2\frac{\Delta R}{R}$

% variation of g is 2%

73)

Inside a spherical shell, gravitational force is zero. Since there is no mass inside, the gravitational field and thereby the force is zero.

74) $M_{\text{mars}} = \frac{4\pi^2 r^3}{GT^2}$

r = 9.4 x 10^3 km = 9.4 x 10^6 m.

G = 6.67 x 10^{-11} Nm^2 kg^2

T = 7 hr. 39 min = 27540 sec.

$M_{\text{mars}} = \frac{4\times\left(\frac{22}{7}\right)^2\times\left(9.4\times10^6\right)^3}{6.67\times10^{-11}\times(27540)^2}$

= 6.49 x 10^{23} kg

75) At any point on the surface of earth, the acceleration due to gravity is,

$g' = g\left(1 - \frac{R\omega^2}{g}\cos^2\alpha\right)$

where α is the latitude.

For equator, α = 0. To feel weightlessness

g' = 0

$\therefore \; g\left(1 - \frac{R\omega^2}{g}\right) = 0$

$\therefore \; \omega = \sqrt{\frac{g}{R}}$

76)

Gravitational potential energy is the work done in shifting a mass m from one point to the other. At any distance x from the centre of earth (M), the gravitational force is $\frac{\mathrm{GMm}}{x^2}$

∴ Work done in shifting (m) from the surface to a height equal to the radius, then,

$\mathrm{W} = \int_{\mathrm{R}}^{2\mathrm{R}} \frac{\mathrm{GMm}}{x^2}dx$

$= \mathrm{GM}m\left[-\frac{1}{x}\right]_{\mathrm{R}}^{2\mathrm{R}} = -\frac{\mathrm{GM}m}{2\mathrm{R}}$

77)

The minimum velocity required to escape from the gravitational force of earth is called escape velocity. Total energy is the sum of P.E. and K.E.

$\mathrm{T.E.} = -\frac{\mathrm{GM}m}{\mathrm{R}} + \frac{1}{2}mv^2$

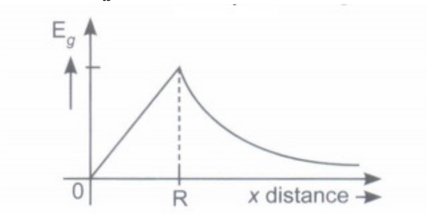
To escape K.E. should be greater than P.E., i.e.,

$\frac{1}{2}mv^2 \geq -\frac{\mathrm{GMm}}{\mathrm{R}}$

$v_e = \sqrt{2\frac{\mathrm{GM}}{\mathrm{R}}} = \sqrt{2g\mathrm{R}}$

78) For points inside the earth,

$\mathrm{E}_{gi} = \frac{\mathrm{G}\frac{4}{3}\pi x^3\rho}{x^2} = \frac{4\mathrm{G}}{3}\pi x\rho$



For points outside the earth,

$\mathrm{E}_{\mathrm{go}} = \frac{\mathrm{G}\frac{4}{3}\pi\mathrm{R}^3\rho}{x^2} = \frac{\mathrm{GM}}{x^2}$

79) $v_e = \sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}} = \sqrt{2g\mathrm{R}} = \sqrt{g\mathrm{D}}$

$\frac{(v_e)_{\mathrm{moon}}}{(v_e)_{\mathrm{earth}}} = \frac{\sqrt{(g\mathrm{D})_{\mathrm{moon}}}}{\sqrt{(g\mathrm{D})_{\mathrm{earth}}}}$

$\sqrt{\frac{1}{6} \times \frac{1}{4}} = \frac{1}{4.9}$

80) $g_h = \frac{g\mathrm{R}^2}{(\mathrm{R}+h)^2} = \frac{g\mathrm{R}^2}{\mathrm{R}^2\left(1+\frac{h}{\mathrm{R}}\right)^2}$

$= g\left(1 + \frac{h}{\mathrm{R}}\right)^{-2} = g\left(1 - \frac{2h}{\mathrm{R}}\right)$

$g_d = g\left(1 - \frac{d}{\mathrm{R}}\right)$ ....(i)

But gh = gd;

$\therefore \quad g\left(1 - \frac{2h}{\mathrm{R}}\right) = g\left(1 - \frac{d}{\mathrm{R}}\right)$

or $1 - \frac{2h}{R} = 1 - \frac{d}{R}$, or $\frac{h}{d} = \frac{1}{2}$

81) Escape velocity, say v, is given by

$v = \sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}$

or $v^2 = (2\ \mathrm{GM})\ \mathrm{R}^{-1}$

Differentiate w.r.t 'R', we get,

$2v\frac{dv}{dR} = -(2\mathrm{GM})\mathrm{R}^{-2}$

or $v\frac{dv}{d\mathrm{R}} = \frac{-\mathrm{GM}}{\mathrm{R}^2}$ ....(i)

also $v^2 = \frac{2GM}{R}$ ....(ii)

Dividing (i) by (ii), we get,

$\frac{1}{v}\frac{dv}{d\mathrm{R}} = \frac{-1}{2R}$

or $\frac{dv}{v} = \frac{-d\mathrm{R}}{2\mathrm{R}}$

or $\left|\frac{dv}{v}\right| = \left|\frac{-d\mathrm{R}}{2\mathrm{R}}\right| = \frac{4}{2}\% = 2\%$

Thus, the decrease in the radius by 4% will increase the escape velocity by 2%.

82) We know,

$g = \frac{GM}{R^2}$ ....(i)

$g' = \frac{GM'}{(R')^2}\left[M' = \frac{1}{2}M, R' = \frac{1}{2}R\right]$ ....(ii)

From (i) and (ii), we have

$\frac{g'}{g} = \frac{1}{2} \cdot 2^2 = 2$

∴ g' = 2g

83)

The objects on the earth's surface are not in orbital motion w.r.t. the earth. In order that an object has an orbital motion close to the earth's surface, its orbital velocity, and period of motion must be

$v_o = \sqrt{gR}$

$T = \frac{2\pi R}{v_o} = 2\pi\sqrt{\frac{R}{g}} = 1.4\text{hrs}$

Therefore, the length of the day has to be 1.4 hrs in case the objects on the earth's surface are in the true orbital motion like that of the earth satellite.

84) In case of the earth $\frac{GM_e m}{r_e^2} = mg_e$

In case of the planet, $\frac{GM_p m}{r_p^2} = mg_p$

Dividing these two equations, we get

$\left(\frac{M_p}{M_e}\right)\left(\frac{r_e^2}{r_p^2}\right) = \frac{g_p}{g_e}$

but $g_p = 2g_e$ and $r_p = \frac{r_e}{2}$

$\therefore \frac{M_p}{M_e} = \frac{2}{4} = \frac{1}{2}$

Thus the ratio of the mass of the planet to the mass of the earth is 1/2.

85) We know, $g = \frac{GM}{R^2}$

$\therefore g_e = \frac{GM_e}{R_e^2}$

and $g_p = \frac{GM_p}{R_p^2}$

Given: $M_p = 2M_e$ and $R_p = 2R_e$

$\therefore \frac{g_p}{g_e} = \frac{1}{2}$

The time period of a simple pendulum is given by

$T_e = 2\pi\sqrt{\frac{l}{g_e}}$

and $T_p = 2\pi\sqrt{\frac{l}{g_p}}$

for $T_e = 1\text{ s},\ T_p = \sqrt{2}\text{ s}$

86) (i) $T = 2\pi\sqrt{\frac{r}{GM}}$ ∴ larger -r, larger T.

∴ Satellite B has longer period.

(ii) $v_0 = \sqrt{\frac{GM}{r}}$ lesser r, more V_o

∴ Satellite A has greater speed.

87)

(i) The planets including earth, go around the sun in elliptical orbits.

(ii) The line joining the Sun and the planet sweeps equal areas in equal intervals of time.

(iii) The square of the time period of revolution is directly proportional to the cube of the semi-major axis of the elliptical orbit.

Since $T^2 \propto r^3$ we have,

$\left(\frac{2\pi r}{v}\right)^2 \propto r^3$

$v^2 = 4\pi^2 \frac{r^2}{r^3} = \frac{4\pi^2}{r}$

$\frac{mv^2}{r} = \frac{4m\pi^2}{r^2}$

The centripetal force $\frac{mv^2}{r}$ is caused by M - earth on the planet of mass m.

Thus $\mathrm{F} \propto \frac{\mathrm{Mm}}{r^2}$

It is the Newton's Universal Law of Gravitation.

88) Let the planet earth be made of material of density ρ with radius R.

At a depth d, the gravitational force is due to the mass distributed in the sphere of radius (R - d).

∴ The acceleration due to gravity at a depth d

$g' = \frac{\mathrm{GM}'}{(\mathrm{R}-d)^2} = \mathrm{G}\frac{4}{3}\frac{\pi(\mathrm{R}-d)^3\rho}{(\mathrm{R}-d)^2}$

$= \mathrm{G}\frac{4}{3}\pi(\mathrm{R}-d)r$

i.e., $g' = \mathrm{G}\frac{4}{3}\pi\frac{\mathrm{R}^3\rho}{\mathrm{R}^2}\frac{(\mathrm{R}-d)}{\mathrm{R}}$

$= g\frac{(\mathrm{R}-d)}{\mathrm{R}} = g\left(1 - \frac{d}{\mathrm{R}}\right)$

∴ g reduces as we move from surface inwards and is zero at the centre.

89) T_p = 11.6 years, r_p = ?, T_e = 1 year, r_e = 1.5 x 10^{11} m

$\frac{T_P^2}{T_e^2} = \frac{r_P^3}{r_e^3}$

$\Rightarrow r_P = r_e\left(\frac{T_P}{T_e}\right)^{2/3}$

$r_P = 1.5 \times 10^{11} \times \left(\frac{11.6}{1}\right)^{2/3}$

r_p = 7.68 x 10^{11} m

90) Escape velocity = $\sqrt{\frac{2\mathrm{GM_P}}{\mathrm{R_P^2}}}$

where M_p is the mass of the planet and R_p is the radius of the planet.

∴ Escape velocity on Mars

$= \sqrt{\frac{2\times\mathrm{G}\times4\mathrm{M}_e\mathrm{R}_e}{9\mathrm{R}_e^2}}$

$= \sqrt{\frac{4}{9}}\sqrt{\frac{2\mathrm{GM}}{\mathrm{R}_e^2}} = \sqrt{\frac{4}{9}}v_e$

$= \sqrt{\frac{4}{9}} \times 11.2 = 7.47\ \mathrm{km/sec}$

91)

Gravitational potential at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

Consider two points A and B, distance dr apart in a uniform gravitational field of intensity $\vec{I}$. Let the direction of $\vec{I}$ be along AB.

Gravitational force on the particle of mass m placed at B will be $\vec{F} = m\vec{I}$

Work done by gravitational force for the displacement of the particle from B to A (i.e. a displacement $d\vec{r}$) will be

$dW = \vec{F} \cdot d\vec{r} = m\vec{I} \cdot d\vec{r} = mIdr\cos 180^\circ$

Change in gravitational potential,

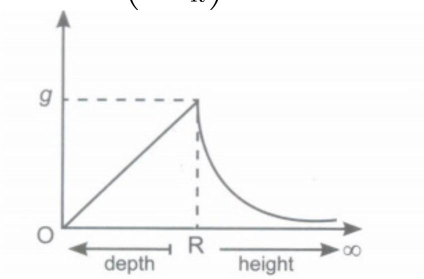
$dV = \frac{dW}{m} = -Idr \Rightarrow I = -\frac{dV}{dr}$

Here $\frac{dV}{dr}$ is ca e gravitational potential gradient.

92)

(i) $g = g\left(1 - \frac{2h}{\mathrm{R}}\right)$

(ii) $g' = g\left(1 - \frac{d}{R}\right)$



The value of g decreases both on moving up and on moving down from the surface of earth. This can be shown graphically as above.

93) (a) The variation of 'g' with height 'h' is related by relation $g \propto \frac{1}{r^2}$

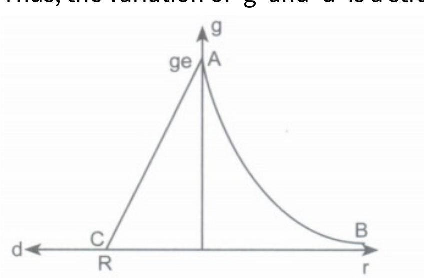
where r = R + h

Thus, the variation of g and r is parabolic curve, i.e. part AB of the graph as shown in figure alongside

(b) The variation of' g' with depth is related by equation g' = g(1 - d/R)

i.e. g' ∝ (R - d)

Thus, the variation of 'g' and 'd' is a straight line, i.e. part AC of the graph.



94)

$g = \frac{\mathrm{GM}}{\mathrm{R}^2} = \frac{g\frac{4}{3}\pi\mathrm{R}^3\rho}{\mathrm{R}^2} = g\frac{4}{3}\pi\mathrm{R}\rho$

$\therefore \frac{g_1}{g_2} = \frac{\mathrm{R}}{2\mathrm{R}} \cdot \frac{\rho}{\rho/2} = 1$

∴ $g_1 : g_2 = 1 : 1$

95)

$\frac{g'}{g} = \frac{20}{100}, g' = g\frac{20}{100} = g\left(1 - \frac{2h}{R}\right)$

$\frac{20}{100} = \left(1 - \frac{2h}{R}\right), \frac{1}{5} = 1 - \frac{2h}{R}$

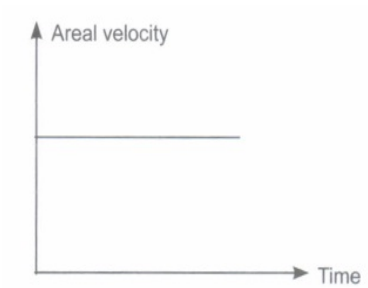
$\frac{2h}{R} = \frac{4}{5}, h = \frac{2}{5}R$

96) $g' = g\left(1 - \frac{d}{R}\right)$
$= 9.8\left(1 - \frac{30 \times 1000}{6.3 \times (1000)^2}\right)$
$= 9.8\left(1 - \frac{1}{210}\right)$
$= 9.8\left(\frac{209}{210}\right) = 9.75 \text{ ms}^{-2}$

97) (a) The mass of the object remain 20 kg at that height.
(b) $\mathrm{W} = \frac{\mathrm{GMm}}{r^2} \Rightarrow \frac{\mathrm{W}_2}{\mathrm{W}_1} = \frac{r_1^2}{r_2^2}$ since G M and m are constant.
$\therefore W_2 = 9.8 \times 20 \times \left(\frac{6370}{6370+160}\right)^2 = 186.5 \text{ N}$

98) Central forces:
(i) Gravitational force due to a point mass.
(ii) Electrostatic force due to a point charge.
Non-central forces:
(i) Spin-dependent nuclear forces.
(ii) Magnetic force between two current-earring loops.

99)
Since areal velocity (of mars) does not change with time (kepler's second law), its graph velocity versus time is a straight line parallel to time axis.



100)
The gravitational force does not depend on the medium separating the two point masses. This is due to the reason that when point masses are dipped in water, their apparent weights change, but their masses remain the same.

101)
Yes A body in a state of weightlessness (like in a satellite or at the centre of the Earth has no weight but it always possesses inertia, which is due to its mass.

102)
In order to keep the areal velocity constant (kepler's second law), the Earth has to cover a greater linear distance at the perihelion. Thus, the speed of the Earth is more at the perihelion than that at the aphelion.

103)
From law of gravitation $\mathrm{F} = \frac{GMm}{R^2}$ If m is a mass on the surface of earth of radius R. The acceleration of the mass m on the surface of earth is g.
∴ Force experienced F = mg
Using the two forces we have, M $= \frac{gR^2}{G}$

104)

No, generally the geo-stationary satellites will be in the equatorial plane. Since New Delhi does not lie in this plane, it is not possible.

105)

Gravitational field strength is defined as the force experienced by unit mass kept at a point.

At a distance r, the force is F $= \frac{\mathrm{GMm}}{r^2}$

The gravitational field = $\frac{\mathrm{F}}{m} = \frac{\mathrm{GMm}}{r^2 m} = \frac{\mathrm{GM}}{r^2}$

106)

Let h_e be the height in metre, the man jumps on the earth and h_p on the planet. If the effort is same, the P.E. gained is same. Therefore,

$Mg_p h_p = Mg_e h_e$

$h_p = \frac{g_e h_e}{g_p}$

we know $g_e = \frac{\mathrm{GM}_e}{\mathrm{R}_e} = \mathrm{G}\frac{4}{3}\pi \mathrm{R}_e \rho_e$

$\frac{g_e}{g_p} = \frac{\mathrm{R}_e \rho_e}{\mathrm{R}_p \rho_p} = 12$

∴ h_p 12 x h_e = 12 x 15. = 18 m

107)

$\mathrm{F}_g = \frac{\mathrm{GM_S M_J}}{r^2}$

$= \frac{6.67\times10^{-11}\times1.99\times10^{30}\times1.9\times10^{27}}{\left(7.8\times10^{11}\right)^2}$

= 4.145×10^{23} N

108)

Orbital velocity of moon

$v_\mathrm{o} = \sqrt{\frac{\mathrm{GM}}{r}}$

Escape Velocity

$v_e = \sqrt{\frac{2\mathrm{GM}}{r}}$

∴ % increase required $= \frac{v_e - v_0}{v_0} \times 100$

$= (\sqrt{2} - 1) \times 100 = 41.4\%$

109)

According to Kepler's law $T^2 \propto r^3$

$T_2^2 = T_1^2 \left(\frac{r_2}{r_1}\right)^3$

Taking T_1 = 1 year

$T_2^2 = (1)^2 (30)^3 = 27000$

or $\mathrm{T}_2 = \sqrt{27000} = 164.3$ years