Sample Paper – 3
Mathematics
Class XI Session 2022-23

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A

(Multiple Choice Questions)
Each question carries 1 mark.

1. Value of cot 570° is:

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$ 1

2. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number is:

(a) $n\pi \pm \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{2}$

(c) $n\pi \pm \frac{\pi}{2}$ (d) $2n\pi \pm \frac{\pi}{4}$ 1

3. Let A = $\{x : x \in R, x > 6\}$ and B = $\{x \in R : x < 9\}$. Then, $A \cap B$ =

(a) (7, 8] (b) (7, 8)

(c) [7, 8) (d) [7, 8] 1

4. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$ 1

5. The solution set $2(2x + 3) - 10 < 6(x - 2)$ is:

(a) $(4, \infty)$ (b) $(-\infty, 4)$

(c) $[4, \infty]$ (d) $[-\infty, 4]$ 1

6. Latus rectum of the parabola $y^2 = 8x$ is:

(a) 2 (b) 4

(c) 6 (d) 8 1

7. If $f(x) = \frac{1}{2 - \sin 3x}$, then range ($f$) is equal to:

(a) [–1, 1] (b) $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(c) $\left[\frac{1}{3}, 1\right]$ (d) $\left[-1, \frac{-1}{3}\right]$ 1

8. The angle in the radian through which a pendulum swings its length is 80 cm and tip describes an arc of length 20 cm is:

(a) $\frac{1}{4}$ (b) $\frac{2}{25}$

(c) $\frac{3}{25}$ (d) $\frac{4}{25}$ 1

9. The derivative of $\left(\frac{x}{2} + \frac{2}{x}\right)$ is:

(a) $\frac{1}{2} + \frac{2}{x^2}$ (b) $\frac{1}{2} - \frac{2}{x^2}$

(c) $\frac{x}{2} - \frac{2}{x^2}$ (d) $\frac{1}{2} - \frac{2}{x}$ 1

10. Find sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time is:

(a) 432 (b) 108

(c) 36 (d) 18 1

11. The mean deviation about the mean of the distribution is:

Size	20	21	22	23	24
Frequency	6	4	5	1	4

(a) 1.25 (b) 1

(c) 1.50 (d) 2 1

12. Let $f(x) = |x - 2|$. Then,

(a) $f(x^2) = [f(x)]^2$ (b) $f(x + y) = f(x)\, f(y)$

(c) $f(|x|) = |f(x)|$ (d) None of these 1

13. The probability that when a hand of 7 cards are drawn from the well-shuffled deck of 52 cards, it contains all kings is:

(a) $\frac{2}{7735}$ (b) $\frac{1}{7735}$

(c) $\frac{3}{7753}$ (d) $\frac{1}{7753}$ 1

14. The number of terms in the expansion of $(4 + 4x + x^2)^{20}$, when expanded in descending powers of x, is:

(a) 20 (b) 21

(c) 40 (d) 41 1

15. The value of $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$ is equal to:

(a) 10 (b) 11

(c) 12 (d) 13 1

16. Four geometric means between 3 and 96 are:

(a) 6, 12, 24, 48 (b) 6, 10, 24, 48

(c) 6, 10, 40, 48 (d) 48, 24, 10, 5 1

17. Let A, B, C be the feet of the perpendicular segments drawn from a point P(1, 2, 5) on the xy, yz and zx-planes, respectively. The distance of the points A, B, C from the point P (in units) respectively are:

(a) 5, 2, 4 (b) 3, 4, 5

(c) 5, 1, 4 (d) 3, 5, 4 1

18. Mean and standard deviation of 100 items are 50 and 4, respectively. The sum of the squares of the items.

(a) 25000 (b) 251600

(c) 26000 (a) None of these 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of the reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): The set $\{x : x$ is a month of a year not having 30 days$\}$ in roster form is {January, February, March, May, July, August, October, December}.

Reason (R): A collection of objects is called set. 1

20. Assertion (A): If 5th and 8th term of a G.P be 48 and 384 respectively, then the common ratio of G.P is 2.

Reason (R): If 18, x, 14 are in A.P, then $x = 16$. 1

SECTION - B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. If $m \sin \theta = n \sin (\theta + 2a)$, then prove that $\tan (\theta + a) \cot a = \frac{m+n}{m-n}$.

OR

Solve $\tan 4x = -\cot\left[x + \frac{\pi}{4}\right]$. 2

22. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, then find the range of pH value for the third reading that will result in the acidity level being normal. 2

23. Given that N = {1, 2, 3,. . . 100}, then:

(A) Write the subset A of N, whose elements are odd numbers.

(B) Write the subset B of N, whose elements are represented by $x + 2$, where $x \in N$. 2

24. Using Binomial theorem, find the value of $(0.98)^{14}$ upto 4 places of decimal.

OR

Expand the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$. 2

25. Define a relation R on the set N of natural numbers by

$R = \{(x, y) ; y = x + 3, x$ is a prime number less than $8 : x, y \in N\}$.

Depict this relationship using a roaster form. Write down the domain and the range. 2

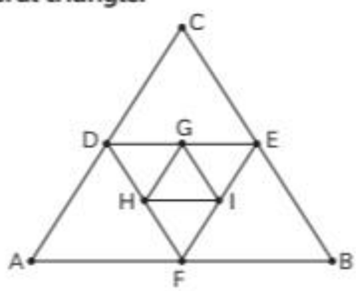
SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Find the value of the expression:

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right].$$ 3

27. A side of an equilateral triangle is 20 cm long. A second equilateral triangle is inscribed in it by joining the mid-points of the sides of the first triangle. The process is continued as shown in the accompanying diagram. Find the perimeter of the sixth inscribed equilateral triangle. 3



28. Evaluate: $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$.

OR

Differentiate $\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$ 3

29. Using binomial theorem, expand $(x + y)^6 - (x - y)^6$. Hence, find the value of $\left(\sqrt{3}+1\right)^6-\left(\sqrt{3}-1\right)^6$.

OR

Evaluate $(102)^4$. 3

30. There are 230 students. 80 play football, 42 play soccer and 12 play rugby. 32 play exactly 2 sports and 4 play all three. How many students play none? 3

31. If $y=\cos x\cdot e^{\sin x^2}$, then find $\frac{dy}{dx}$.

OR

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constant m and n are integers).

(A) $\frac{4x+5\sin x}{3x+7\cos x}$ (B) $(ax+b)^n$ 3

SECTION - D

(This section comprises of long answer-type questions (LA) of 5 marks each.)

32. Find the value of cot 105° and cot 15°. 5

33. The mean and standard deviation of some data for the time taken to complete a test are calculated with the following results:

Number of observation = 25, mean = 18.2 seconds, standard deviation =3.25 seconds.

Further, another set of 15 observation x_1, x_2, ..., x_{15} also in seconds, is now available and we have

$$\sum_{i=1}^{15} x_i = 279 \quad \text{and} \quad \sum_{i=1}^{15} x_i^2 = 5524$$

Calculate the standard derivation based on all 40 observations. 5

34. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find ${}^rC_2$.

[Hint: From equation using ${}^nC_r / {}^nC_{r+1}$ and ${}^nC_r / {}^nC_{r-1}$ to find the value of r].

OR

Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements. 5

35. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $49y2 - 16x2 = 784$.

OR

Find the coordinates of the focii, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. 5

SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (A), (B), (C) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. Case-Study 1:

One card is drawn from a well shuffled deck of 52 cards. Each outcome is equally likely.

(A) Find the probability that the card will be a heart. 1

(B) Find the probability that the card will be a black card. 1

(C) Find the probability that the card will be an ace of spade.

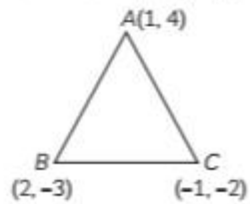
OR

If E and F are events such that $P(E)=\frac{7}{15}$, $P(F)=\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$,,

Find P(E or F). 2

37. Case-Study 2:

Tross bridges are formed with a structure of connected elements that form triangular structure to make up the bridge. Trusses are the triangles that connect to the top and bottom cord and two endposts. Consider the ΔABC with vertices $A(1, 4)$, $B(2, -3)$ and $C(-1, -2)$.



(A) Find the slope of BC. 1

(B) Find the slope of AC. 1

(C) Find the distance between A and C.

OR

Find the distance of the point (4, –6) from the line $4x - 5y - 32 = 0$. 2

38. Case-Study 3:

A quadratic equation can be defined as an equation of degree 2. This means that the highest exponent of the polynomial in it is 2. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a ,b and c are real numbers and $a \neq 0$. Every quadratic equation has two roots depending on the nature of its discriminant, $D = b^2 - 4ac$.

(A) Find the roots of a quadratic equation $3x^2 + x + 2 = 0$. 2

(B) Find the roots of a quadratic equation $25x^2 - 3x + 11 = 0$. 2

SOLUTION

SECTION - A

1. (a) $\sqrt{3}$

Explanation: Value of cot 570° is

$$= \cot(3\pi + 30°)$$
$$= \cot 30°$$
$$= \sqrt{3}$$

2. (c) $n\pi \pm \frac{\pi}{2}$

Explanation: Let $z = \frac{1+i\cos\theta}{1-2i\cos\theta}$

$$= \frac{(1+i\cos\theta)(1+2i\cos\theta)}{(1-2i\cos\theta)(1+2i\cos\theta)}$$

$$= \frac{1-2\cos^2\theta + i3\cos\theta}{1+4\cos^2\theta}$$

$$= \frac{1-2\cos^2\theta}{1+4\cos^2\theta} + i\left(\frac{3\cos\theta}{1+4\cos^2\theta}\right)$$

purely real, then

$Im(z) = 0$

$3\cos\theta = 0$

$$\theta = \frac{(2n\pm1)\pi}{2} \text{ where } n \in \text{N}$$

$$= n\pi \pm \frac{\pi}{2}$$

3. (b) (7, 8)

Explanation:

$A = \{x : x \in R, x > 6\} = A = \{7, 8, 9, \ldots\}$,

$B = \{x \in R: x < 9\} = \{8, 7, 6, 5, \ldots\}$

$A \cap B = \{x: x \in R, x > 6\} \cap \{x \in R: x < 9\}$

$= \{x: x \in R, x > 6 \text{ and } x < 9\} =$

$\{x : x \in R, 6 < x < 9\}$

(it shows a closed interval.)

$= (7, 8)$

4. (d) $\frac{4}{3}$

Explanation: Slope of given line $3x + y = 3$ is -3.

$\therefore$ Slope of perpendicular line $= \frac{1}{3}$

Thus, the equation of the required line is:

$$y - 2 = \frac{1}{3}(x-2)$$

$\Rightarrow \quad x - 3y + 4 = 0$

For y-intercept, put $x = 0$.

$0 - 3y + 4 = 0$

$\Rightarrow \quad y = \frac{4}{3}$,

which is y-intercept.

5. (A) $(4, \infty)$

Explanation: We have,

$2(2x+3) - 10 < 6(x-2)$

$\Rightarrow \quad 4x + 6 - 10 < 6x - 12$

$\Rightarrow \quad 4x - 4 < 6x - 12$

Transposing the term $6x$ to LHS and (-4) to RHS,

$\Rightarrow \quad 4x - 6x < -12 + 4$

$\Rightarrow \quad -2x < -8$

Dividing both sides by -2,

$\Rightarrow \quad \frac{-2x}{-2} > \frac{-8}{-2}$

$\Rightarrow \quad x > 4$

$\therefore$ Solution set $= (4, \infty)$.

6. (d) 8

Explanation: Equation of parabola is $y^2 = 8x$

Comparing with standard form

$y^2 = 4ax$ or, $4a = 8$

We know that length of latus rectum $= 4a = 8$

7. (c) $\left[\frac{1}{3}, 1\right]$

Explanation: We know that,

$$-1 \le -\sin 3x \le 1$$
$$-1 + 2 \le 2 - \sin 3x \le 1 + 2$$

$\Rightarrow \quad 1 \le 2 - \sin 3x \le 3$

$\Rightarrow \quad \frac{1}{3} \le \frac{1}{2-\sin 3x} \le 1$

$\Rightarrow \quad \frac{1}{3} \le f(x) \le 1$

$\therefore \quad$ Range $(f) = \left[\frac{1}{3}, 1\right]$

8. (a) $\frac{1}{4}$

Explanation: Given, length of pendulum = 20 cm

Radius (r) = length of pendulum = 80 cm

Length of arc (l) = 20 cm

Now, $\theta = \frac{l}{r} = \frac{20}{80} = \frac{1}{4}$ radian

9. (b) $\frac{1}{2} - \frac{2}{x^2}$

Explanation: $\frac{d}{dx}\left(\frac{x}{2} + \frac{2}{x}\right)$

$= \frac{d\left(\frac{x}{2}\right)}{dx} + \frac{d\left(\frac{2}{x}\right)}{dx}$

$= \frac{1}{2} - \frac{2}{x^2}$

10. (b) 108

Explanation: The sum of the digits in unit place of all the numbers formed with the help of 3, 4, 5 and 6 taken all at a time

$= (3 + 4 + 5 + 6)3!$
$= 108$

11. (a) 1.25

Given data distribution.

Now, we have to find the mean deviation about the mean of the distribution construct a table of the given data.

Size (x_i)	Frequency (f_i)	f_ix_i
20	6	$20 \times 6 = 120$
21	4	$21 \times 4 = 84$
22	5	$22 \times 5 = 110$
23	1	$23 \times 1 = 23$
24	4	$24 \times 4 = 96$
Total	20	433

We know that mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{433}{20} = 21.65$

To find mean deviation, we have to construct another table.

Size (x_i)	Frequency (f_i)	f_ix_i	$d_i = \lvert x_i - \text{mean} \rvert$	f_id_i
20	6	120	1.65	9.90
21	4	84	0.65	2.60
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
Total	20	433		25.00

Hence, Mean Deviation becomes,

M.D. $= \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{25}{20} = 1.25$

Therefore, the mean deviation about the mean of the distribution is 1.25.

12. (d) None of these

Explanation:

$$f(x) = |x - 2|$$

Here,

$$f(x^2) \neq [f(x)]^2$$
$$f(x + y) \neq f(x).\, f(y)$$

and

$$f(|x|) \neq |f(x)|$$

13. (b) $\frac{1}{7735}$

Explanation: Total cards to be drawn = 7

So, sample space contains = ${}^{52}C_7$

$$P(S) = \frac{52!}{7! \times 45!}$$

There are only 4 kings, so 3 cards come from remaining ones.

So, $P(A) = {}^{48}C_3$

$$= \frac{48!}{3! \times 45!}$$

Hence probability $= \frac{P(A)}{P(S)}$

$$= \frac{48!}{3! \times 45!} \times \frac{7! \times 45!}{52!}$$

$$= \frac{1}{7735}$$

14. (d) 41

Explanation: We have

$$(4 + 4x + x^2)^{20} = [(2 + x)^2]^{20}$$
$$= (2 + x)^{40}.$$

Therefore, there are 41 terms in the expansion of $(4 + 4x + x^2)^{20}$.

15. (c) 12

Explanation:

We have, $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

then $\dfrac{x^3-2^3}{x-2}=\dfrac{(x-2)(x^2+2^2+2x)}{x-2}$

$$= x^2 + 4 + 2x$$
$$= 2^2 + 4 + 4$$
$$= 12$$

16. *(a) 6, 12, 24, 48*

Explanation: Let G_1, G_2, G_3 and G_4 be the required GM's.

Then, 3 , G_1, G_2, G_3, G_4, 96 are in G.P.

Let r be the common ratio. Here, 96 is the 6^{th} term.

$\therefore \quad 96 = ar^{6-1} = 3r^5$

$\Rightarrow \quad 32 = r^5$

$\Rightarrow \quad (2)^5 = r^5$

$\Rightarrow \quad r = 2$

$\therefore \quad G_1 = ar = 3.2 = 6$

$G_2 = ar^2 = 3.2^2 = 12$

$G_3 = ar^3 = 3.2^3 = 24$

And $\quad G_4 = ar^4 = 3.2^4 = 48$

17. *(c) 5, 1, 4*

Explanation: We have, coordinates of A = (1, 2, 0), coordinates of B = (0, 2, 5), coordinates of C = (1, 0, 5)

Now, P = (1, 2, 5)

$\therefore \quad PA = \sqrt{(1-1)^2+(2-2)^2+(5-0)^2} = 5$ units

$PB = \sqrt{(1-0)^2+(2-2)^2+(5-5)^2} = 1$ units

$PC = \sqrt{(1-1)^2+(2-0)^2+(5-5)^2} = 4$ units

18. *(b) 251600*

Explanation: Given mean and standard deviation of 100 items are 50 and 4, respectively

Now, we have to find the sum of the squares of the items.

As per given criteria,

Number of items, $n = 100$

Mean of the given items, $\bar{x} = 50$

But we know,

$$\bar{x} = \frac{\Sigma x_i}{n}$$

Substituting the corresponding values, we get

$$50 = \frac{\Sigma x_i}{100}$$

$\Rightarrow \quad \Sigma x_i = 50 \times 100 = 5000$

Hence the sum of all the 100 items = 5000.

Also, given the standard deviation of the 100 items is 4.

i.e., $\quad \sigma = 4$

But we know

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n}-\left(\frac{\Sigma x_i}{n}\right)^2}$$

Substituting the corresponding values, we get

$$4 = \sqrt{\frac{\Sigma x_i^2}{100}-\left(\frac{5000}{100}\right)^2}$$

Now taking square on both sides, we get

$$4^2 = \frac{\Sigma x_i^2}{100}-(50)^2$$

$\Rightarrow \quad 16 = \dfrac{\Sigma x_i^2}{100}-2500$

$\Rightarrow \quad 16 + 2500 = \dfrac{\Sigma x_i^2}{100}$

On rearranging we get,

$\Rightarrow \quad \dfrac{\Sigma x_i^2}{100} = 2516$

$\Rightarrow \quad \Sigma x_i^2 = 2516 \times 100$

$\Rightarrow \quad \Sigma x_i^2 = 251600$

The sum of the squares of all the 100 items is 251600.

19. *(c) A is true but R is false.*

Explanation: The months not containing 30 days are January, February, March, May, July, August, October, and December.

So, the roster form of a given set = {January, February, March, May, July, August, October, December}, which is a well-defined collection of months.

R is wrong as mere collection of objects is not a set. The collection should be well-defined.

20. *(b) Both A and R are true but R is not the correct explanation of A.*

Explanation: $\quad T_5 = 48$

$ar^4 = 48$

$T_8 = 384$

$ar^7 = 384$

So, $\quad r^3 = 8$

$r = 2$

18, n, 14 are in A.P

So, $\quad n - 18 = 14 - n$

$\Rightarrow \quad 2n = 32$

$\Rightarrow \quad n = 16$

SECTION - B

21. Given, $m \sin\theta = n \sin(\theta + 2\alpha)$

$\therefore \quad \frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{m}{n}$

Applying componendo and dividendo, we get

$$\frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{2\sin\left(\frac{\theta+2\alpha+\theta}{2}\right)\cdot\cos\left(\frac{\theta+2\alpha-\theta}{2}\right)}{2\cos\left(\frac{\theta+2\alpha+\theta}{2}\right)\cdot\sin\left(\frac{\theta+2\alpha-\theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\left[\begin{array}{l}\therefore \quad \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2} \text{ and} \\ \qquad \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\end{array}\right]$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cdot\cos\alpha}{\cos(\theta+\alpha)\cdot\sin\alpha} = \frac{m+n}{m-n}$$

$$\Rightarrow \tan(\theta+\alpha)\cdot\cot\alpha = \frac{m+n}{m-n}$$

Hence, proved.

OR

$\tan\theta = \cot[90° - \theta]$

$$\Rightarrow \quad \tan 4x = -\cot\left[x+\frac{n}{4}\right]$$

$$\Rightarrow \quad \tan 4x = \tan\left[\frac{\pi}{2}+x+\frac{\pi}{4}\right]$$

$$\Rightarrow \quad \tan 4x = \tan\left[x+\frac{3\pi}{4}\right]$$

$$\Rightarrow \quad 4x = \left[x+\frac{3\pi}{4}\right]$$

$$4x = n\pi + x + \frac{3\pi}{4}$$

Where $n \in Z$,

$$3x = n\pi + \frac{3\pi}{4}$$

22. Given, first pH value = 8.48
and second pH value = 8.35

Let third pH value be x.

Since, it is given that average pH value lies between 8.2 and 8.5.

$$\therefore \quad 8.2 < \frac{8.48+8.35+x}{3} < 8.5$$

$$\Rightarrow \quad 8.2 < \frac{16.83+x}{3} < 8.5$$

$$\Rightarrow \quad 3 \times 8.2 < 16.83 + x < 8.5 \times 3$$

$$\Rightarrow \quad 24.6 < 16.83 + x < 25.5$$

$$\Rightarrow \quad 24.6 - 16.83 < x < 25.5 - 16.83$$

$$\Rightarrow \quad 7.77 < x < 8.67$$

Thus, third pH value lies between 7.77 and 8.67.

23. Given, N = {1, 2, 3, . . . , 100} = {$x : x = n$ and $n \in$ N}

(A) A = {$x \mid x \in$ N and x is odd}= {1, 3, 5, 7, . . . 99}

(B) B = {$y \mid y = x + 2, x \in$ N}

The set whose elements are represented by $x + 2$ where $x \in$ N is obtained by putting x = 1, 2, 3 and so on in $y = x + 2$ and we get

$y = x + 2 = 1 + 2 = 3$

$y = x + 2 = 2 + 2 = 4$

$y = x + 2 = 3 + 2 = 5$

$y = x + 2 = 4 + 2 = 6. . . .$

$y = x + 2 = 100 + 2 = 102$

So, the required set will be A = {3, 4, 5, 6,.. 102}

24. $(0.98)^{14} = (1 - 0.02)^{14}$

$= 1 + {}^{14}C_1(-0.02)^1 + {}^{14}C_2(-0.02)^2 + {}^{14}C_3(-0.02)^3$

[Neglecting higher powers of (0. 01)]

$= 1 - 14(0.02) + 91(0.0004) - 364(0.000008)$

$= 1 - 0.28 + 0.0364 - 0.002912 = 0.753488.$

OR

By using Binomial Theorem, the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ can be expanded as

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2 - {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5$$

$$= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32}$$

$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$$

25. Given N = Set of all natural numbers and

$R = \{(x, y) : y = x + 3, x$ is a prime number less than $8 ; x, y \in N\}$

$= \{(x, y) : y = x + 3, x \in \{2, 3, 5, 7\} ; x, y \in N\}$.

The given relation in roaster form can be written as

$R = \{(2, 5), (3, 6), (5, 8) (7, 10)\}$.

Hence, domain of $R = \{2, 3, 5, 8\}$ and

range of $R = \{5, 6, 8, 10\}$.

SECTION - C

26. Given,

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right] -2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

$= 3[\cos^4 \alpha + \sin^4 (\pi + \alpha)]-2[\cos^6 \alpha + \sin^6 (\pi - \alpha)]$

$= 3[\cos^4 \alpha + \sin^4 \alpha] - 2[\cos^6 \alpha + \sin^6 \alpha]$

$= 3[(\cos^4 \alpha + \sin^4 \alpha) + 2 \sin^2 \alpha \cos^2 \alpha - 2 \sin^2 \alpha \cos^2 \alpha] - 2[(\cos^2 \alpha + \sin^2 \alpha)^3 - 3 \cos^2 \alpha \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha)]$

$= 3[(\cos^2 \alpha + \sin^2 \alpha)^2 - 2\sin^2 \alpha \cos^2 \alpha] -2 [1 - 3 \cos^2 \alpha \sin^2 \alpha]$

$= 3[1 - 2 \sin^2 \alpha \cos^2 \alpha] - 2 + 6 \cos^2 \alpha \sin^2 \alpha$

$= 3 - 6 \sin^2 \alpha \cos^2 \alpha - 2 + 6 \sin^2 \alpha \cos^2 \alpha$

$= 1$

27. The side of the first equilateral $\Delta ABC = 20$ cm

By joining the mid-points of the sides of this triangle, we get the second equilateral triangle which each side $= \frac{20}{2} = 10$ cm.

[$\because$ The line joining the mid-points of two sides of a triangle is 1/2 and parallel to the third side of the triangle].

Similarly, each side of the third equilateral triangle $= \frac{10}{2} = 5$ cm

$\therefore$ Perimeter of first triangle $= 20 \times 3 = 60$ cm

Perimeter of the second triangle

$= 10 \times 3 = 30$ cm

And the perimeter of the third triangle

$= 5 \times 3 = 15$ cm

Therefore, the series will be 60, 30, 15, . . .

Which is G.P. in which $a = 60$, and $r = \frac{30}{60} = \frac{1}{2}$

Now, we have to find the perimeter of the sixth inscribed equilateral triangle

$\therefore \quad a_6 = ar^{6-1}$

$= 60\times\left(\frac{1}{2}\right)^5$

$= 60\times\frac{1}{32}$

$= \frac{15}{8}$ cm.

Hence, the required perimeter $= \frac{15}{8}$ cm

28. Given that $\lim\limits_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

$$= \lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}\times\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\frac{2-(1+\cos x)}{\sin^2 x\left[\sqrt{2}+\sqrt{1+\cos x}\right]}$$

$$= \lim_{x\to 0}\frac{1-\cos x}{\sin^2 x\left[\sqrt{2}+\sqrt{1+\cos x}\right]}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x/2}{(2\sin x/2\cos x/2)^2}\times\frac{1}{\left[\sqrt{2}+\sqrt{1+\cos x}\right]}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x/2}{(4\sin^2 x/2\cos^2 x/2)^2}\times\frac{1}{\left[\sqrt{2}+\sqrt{1+\cos x}\right]}$$

$$= \lim_{x\to 0}\frac{2}{4\cos^2\frac{x}{2}}\times\frac{1}{\left[\sqrt{2}+\sqrt{1+\cos x}\right]}$$

Taking limit, we get

$$= \frac{2}{4\cos^2 0}\times\frac{1}{(\sqrt{2}+\sqrt{2})} = \frac{1}{2}\times\frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

Hence, the required answer is $\frac{1}{4\sqrt{2}}$.

OR

$$\frac{d}{dx}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$$

$$= \frac{(\sin x-\cos x)\cdot\frac{d}{dx}(\sin x+\cos x) -(\sin x+\cos x)\cdot\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$$

$$= \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{-\left[(\sin x - \cos x)^2 + (\sin x + \cos x)^2\right]}{(\sin x - \cos x)^2}$$

$$= \frac{-2(\sin^2 x + \cos^2 x)}{(\sin^2 x + \cos^2 x - 2\sin x \cos x)}$$

$$= \frac{-2}{(1 - \sin 2x)}$$

29. $(x + y)^6 - (x - y)^6 = {}^6C_0x^6 + {}^6C_1x^5y + {}^6C_2x^4y^2 + {}^6C_3x^3y^3 + {}^6C_4x^2y^4 + {}^6C_5xy^5 + {}^6C_6x^0y^6 - [{}^6C_0x^6 + {}^6C_1x^5(-y)^1 + {}^6C_2x^4(-y)^2 + {}^6C_3x^3(-y)^3 + {}^6C_4x^2(-y)^4 + {}^6C_5x(-y)^5 + {}^6C_6x^0(-y)^6]$

$= 2(6x^5y + 20x^3y^3 + 6xy^5)$

$= 4xy(3x^4 + 10x^2y^2 + 3y^4)$

On substituting $x = \sqrt{3}$ and $y = 1$, we get

$$= 4 \times \sqrt{3} \times 1\left(3(\sqrt{3})^4 + 10(\sqrt{3})^2(1)^2 + 3(1)^4\right)$$

$$= 4\sqrt{3}(3 \times 9 + 10 \times 3 + 3)$$

$$= 4\sqrt{3}(27 + 30 + 3)$$

$$= 4\sqrt{3}(60)$$

$$= 240\sqrt{3}$$

OR

Given: $(102)^4$.

Here, 102 can be written as the sum or the difference of two number, such that the binomial theorem can be applied.

Therefore $\quad 102 = 100 + 2$

Hence, $\quad (102)^4 = (100 + 2)^4$

Now, by applying binomial theorem, we get:

$(102)^4 = (100 + 2)^4 = {}^4C_0(100)^4 + {}^4C_1(100)^3(2) + {}^4C_2(100)^2(2)^2 + {}^4C_3(100)^1(2)^3 + {}^4C_4(2)^4$

$= (100)^4 + 8(100)^3 + 24(100)^2 + 32(100) + 16$

$= 100000000 + 8000000 + 240000 + 3200 + 16$

$= 108243216$

30. We will calculate the number of students who play none of the sports by the formula which is given below:

Total students = students play football + students play soccer + students play rugby − students who play exactly 2 sports − 2× (students who play all three sports) + students who play none

Putting the values in the above formula, we get,

$230 = 80 + 42 + 12 - 32 - 2 \times 4$ + students who play none

Students who play none $= 230 - 80 - 42 - 12 + 32 + 8$

Students who play none = 136

Hence, the number of students who play none of the sports is 136.

31. $y = \cos x \,.\, e^{\sin x^2}$

Using product rule.

$$= \cos x \frac{d}{dx}(e^{\sin x^2}) + (e^{\sin x^2})\frac{d}{dx}(\cos x)$$

$$\frac{dy}{dx} = \cos x \cdot e^{\sin x^2} \cdot \cos x^2 \cdot 2x - e^{\sin x^2} \cdot \sin x$$

$$= 2x \cdot \cos x e^{\sin x^2} \cdot \cos x^2 - e^{\sin x^2} \cdot \sin x$$

So, required solution is

$$\frac{dy}{dx} = e^{\sin x^2}(2x \cos x \cdot \cos x^2 - \sin x)$$

OR

(A) Let $(x) = \dfrac{4x + 5\sin x}{3x + 7\cos x}$

By quotient rule,

$$f'(x) = \frac{(3x + 7\cos x)\frac{d}{dx}(4x + 5\sin x) - (4x + 5\sin x)\frac{d}{dx}(3x + 7\cos x)}{(3x + 7\cos x)^2}$$

$$= \frac{(3x + 7\cos x)\left[4\frac{d}{dx}(x) + 5\frac{d}{dx}(\sin x)\right] - (4x + 5\sin x)\left[3\frac{d}{dx}(x) + 7\frac{d}{dx}(\cos x)\right]}{(3x + 7\cos x)^2}$$

$$= \frac{(3x + 7\cos x)[4 + 5\cos x] - (4x + 5\sin x)[3 - 7\sin x]}{(3x + 7\cos x)^2}$$

$$= \frac{12x + 15x\cos x + 28\cos x + 35\cos^2 x - 12x + 28x\sin x - 15\sin x + 35\sin^2 x}{(3x + 7\cos x)^2}$$

$$= \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35(\cos^2 x + \sin^2 x)}{(3x + 7\cos x)^2}$$

$$= \frac{35 + 15x\cos x + 28\cos x + 28x\sin x - 15\sin x}{(3x + 7\cos x)^2}$$

(B) Let $f(x) = (ax+b)^n$.

Accordingly,

$$f(x+h) = \{a(x+h)+b\}^n$$
$$= (ax+ah+b)^n$$

By first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(ax+ah+b)^n-(ax+b)^n}{h}$$

$$= \lim_{h\to 0} \frac{(ax+b)^n\left(1+\frac{ah}{ax+b}\right)^n-(ax+b)^n}{h}$$

$$= (ax+b)^n \lim_{h\to 0}\frac{1}{h}\left[\left\{1+\left(\frac{ah}{ax+b}\right)+\frac{n(n-1)}{2}\left(\frac{ab}{ax+b}\right)^2+\ldots\right\}-1\right]$$

(using binomial theorem)

$$= (ax+b)^n \lim_{h\to 0}\frac{1}{h}\left[n\left(\frac{ah}{ax+b}\right)+\frac{n(n-1)a^2h^2}{2(ax+b)^2}+\ldots \text{(terms containing higher degree of } h)\right]$$

$$= (ax+b)^n \lim_{h\to 0}\left[\frac{na}{(ax+b)}+\frac{n(n-a)a^2h^2}{2(ax+b)^2}+\ldots\right]$$

$$= (ax+b)^n\left[\frac{na}{(ax+b)}+0\right]$$

$$= na\frac{(ax+b)^n}{ax+b}$$

$$= na(ax+b)^{(n-1)}$$

SECTION - D

32. $\therefore \cot(105)^\circ = \cot(60^\circ+45^\circ)$

We know $\cot(A+B) = \dfrac{\cot A\cot B-1}{\cot B+\cot A}$

So, Applying the formula in $\cot(60^\circ+45^\circ)$

We get, $\cot(60^\circ+45^\circ) = \dfrac{\cot 60^\circ\cot 45^\circ-1}{\cot 60^\circ+\cot 45^\circ}$

$$= \frac{\frac{1}{\sqrt{3}}\times 1-1}{\frac{1}{\sqrt{3}}+1} \quad \left\{\cot 60^\circ=\frac{1}{\sqrt{3}}, \cot 45^\circ = 1\right\}$$

$$= \frac{\frac{1}{\sqrt{3}}-1}{\frac{1+\sqrt{3}}{\sqrt{3}}} = \frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{1+\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{(1-\sqrt{3})^2}{1-3} \quad \text{[On Rationalisation]}$$

$$= \frac{1+3-2\sqrt{3}}{-2} = \frac{4-2\sqrt{3}}{-2}$$

$$= -2+\sqrt{3} = \sqrt{3}-2$$

Now, $\cot 15^\circ$

We have, $\cot(A-B) = \dfrac{\cot A\cdot\cot B+1}{\cot B-\cot A}$

$$\Rightarrow (\cot 60^\circ-45^\circ) = \frac{\cot 60^\circ\cdot\cot 45^\circ+1}{\cot 45^\circ-\cot 60^\circ}$$

$$\Rightarrow \cot(15^\circ) = \frac{\frac{1}{\sqrt{3}}\times 1+1}{1-\frac{1}{\sqrt{3}}}$$

$$\Rightarrow \cot(15^\circ) = \frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}$$

$$\Rightarrow \cot(15^\circ) = \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$\Rightarrow \cot(15^\circ) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\Rightarrow \cot(15^\circ) = \frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-1^2} \quad \text{[On Rationalisation]}$$

$$\Rightarrow \cot(15^\circ) = \frac{3+1+2\sqrt{3}}{3-1}$$

$$\Rightarrow \cot(15^\circ) = \frac{4+2\sqrt{3}}{2} = \frac{2\left(2+\sqrt{3}\right)}{2}$$

$$\therefore \cot 15^\circ = 2+\sqrt{3}$$

33. Given number of observation = 25, mean = 18.2 seconds, standard deviation = 3.25 seconds. Another set of 15 observation $x_1, x_2, ..., x_{15}$ also in seconds, is $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

Now, we have to find the standard derivation based on all 40 observation.

As per the given criteria,

In first set,

Number of observation, $n_1 = 25$

Mean, $\bar{x}_1 = 18.2$

And standard deviation, $\sigma_1 = 3.25$

And

In second set,

Number of observation, $n_2 = 15$

$$\sum_{i=1}^{15} x_i = 279 \text{ and } \sum_{i=1}^{15} x_i^2 = 5524$$

For the first set we have

$$\bar{x}_1 = 18.2 = \frac{\Sigma x_i}{25}$$

$$\Sigma x_i = 25 \times 18.2 = 455$$

Therefore the standard deviation becomes,

$$\sigma_1^2 = \frac{\Sigma x_i^2}{25} - (18.2)^2$$

Substituting the values, we get

$$(3.25)^2 = \frac{\Sigma x_1^2}{25} - 331.24$$

$$\Rightarrow 10.5625 + 331.24 = \frac{\Sigma x_i^2}{25}$$

Rearranging we get

$$\Rightarrow \quad \frac{\Sigma x_i^2}{25} = 341.8025$$

On cross multiplication we get

$\Rightarrow \Sigma x_1^2 = 25 \times 341.8025 = 8545.06$

For the combined standard deviation of the 40 observation, $n = 40$

And

$\Rightarrow \Sigma x_i^2 = 8545.06 + 5524 = 14069.06$

$\Rightarrow \Sigma x_i = 455 + 279 = 734$

Therefore the standard deviation can be written as,

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

Substituting the values, we get

Therefore the standard deviation can be written as,

$$\sigma = \sqrt{\frac{14069.06}{40} - \left(\frac{734}{40}\right)^2}$$

On simplifying we get

$$\sigma = \sqrt{351.7265 - (18.35)^2}$$

$$\sigma = \sqrt{351.7265 - 336.7225}$$

$$\sigma = \sqrt{15.004}$$

$$\sigma = 3.87$$

Hence, the mean standard deviation based on all 40 observations is 3.87.

34. We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

According to the question,

$$^nC_{r-1} = 36,$$
$$^nC_r = 84,$$
$$^nC_{r+1} = 126$$

$$\frac{^nC_r}{^nC_{r+1}} = \frac{84}{126}$$

$$\Rightarrow \quad \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{84}{126} = \frac{2}{3}$$

$$2n - 2r = 3r + 3$$

$$\Rightarrow \quad 2n - 3 = 5r \quad ...(i)$$

$$\frac{^nC_r}{^nC_{r-1}} = \frac{84}{36}$$

$$\Rightarrow \quad \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{7}{3}$$

$$= 3n - 3r + 3 = 7r$$

$$3n + 3 = 10r \quad ...(ii)$$

From eqs (i) and (ii),

We get

$$2(2n - 3) = 3n + 3$$
$$4n - 3n - 6 - 3 = 0$$
$$n = 9$$

And $r = 3$

Now $^rC_2 = {}^3C_2 = 3!\ /\ 2!$

$= 3$

OR

We know that,

$$^nP_r = \frac{n!}{(n-r)!}$$

According to the question,

W_1 can occupy chairs marked 1 to 4 in 4 different ways.

Chair	1	2	3	4	5	6	7	8
People	W_1, W_2	W_1, W_2	W_1, W_2	W_1, W_2				

W_2 can occupy 3 chairs marked 1 to 4 in 3 different ways.

So, total no. of ways in which women can occupy the chairs,

$$^4P_2 = \frac{4!}{(4-2)!}$$

$$= \frac{(4\times3\times2\times1)}{(2\times1)}$$

$$^4P_2 = 12$$

Now, 6 chairs will be remaining.

Chair	1	2	3	4	5	6	7	8
People	W_1	W_2						

M_1 can occupy any of the 6 chairs in 6 different ways,

M_2 can occupy any of the remaining 5 chairs in 5 different ways.

M_3 can occupy any of the remaining 4 chairs in 4 different ways.

So, total no. of ways in which men can occupy the chairs.

$$^6P_3 = \frac{6!}{(6-3)!}$$

$$= 120$$

Hence, total number of ways in which men and women can be seated

$$^4P_2 \times {}^6P_3 = 120 \times 12$$

$$= 1440$$

35. The given eq is $49y^2 - 16x^2 = 784$

It can be written as

$$49y^2 - 16x^2 = 784$$

Or, $$\frac{y^2}{16} - \frac{x^2}{49} = 1$$

Or, $$\frac{y^2}{4^2} - \frac{x^2}{7^2} = 1 \quad ...(i)$$

On comparing eq (i) with the standard equation of hyperbola, *i.e.*, $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = 4$ and $b = 7$.

We know that $a^2 + b^2 = c^2$

$\therefore \quad c^2 = 16 + 49 = 65$

$\Rightarrow \quad c = \sqrt{65}$

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{65})$.

The coordinates of the vertices are $(0, \pm 4)$.

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{65}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times49}{4} = \frac{49}{2}$

OR

The given equation is $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis. On comparing the given equation with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36-16} = \sqrt{20} = 2\sqrt{5}$

We obtain $a = 6$ and $b = 4$.

Therefore,

The coordinates of the foci are $(2\sqrt{5}, 0)$ and 8 $(-2\sqrt{5}, 0)$

The coordinates of the vertices are (6, 0) and (–6, 0)

Length of major axis = $2a = 12$

Length of minor axis = $2b = 8$

Eccentricity, $e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2\times16}{6} = \frac{16}{3}$

SECTION - E

36. (A) Total number of possible outcomes = 52

Probability of drawing a heart card $= \frac{13}{52} = \frac{1}{4}$

(B) Probability of drawing a black card $= \frac{26}{52} = \frac{1}{2}$

(C) Probability of drawing an ace of spade

$$= \frac{1}{52}.$$

OR

$$P(\text{E and F}) = P(E \cap F) = \frac{1}{8}$$

We need to find

P(E or F) = P $(E \cup F)$

We know that

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Putting values

$$P(E \cup F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$= \frac{2+4-1}{8}$$

$$= \frac{6-1}{8}$$

$$= \frac{5}{8}$$

37. (A) Here, B(2, – 3) and C(– 1, – 2)

So,

Slope of BC is $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2+3}{-1-2} = \frac{1}{-3} = \frac{-1}{3}$$

(B) Here, A(1, 4) and C(–1, –2).

So,

Slope of AC is $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2-4}{-1-1} = \frac{-6}{-2} = 3$$

(C) Here, A(1, 4) and C(–1, –2).

So,

$$AC = \sqrt{(-1-1)^2 + (-2-4)^2}$$

$$= \sqrt{4+36}$$

$$= \sqrt{40}$$

$$= 2\sqrt{10}$$

OR

Given line is $4x - 3y - 32 = 0$

Here, $A = 4, B = -5y - 32 = 0$

Given point is (4, – 6_)

So, $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$

$$= \left|\frac{4 \times 4 + (-5)(-6) + (-32)}{\sqrt{16+25}}\right|$$

$$= \left|\frac{16+30+32}{41}\right|$$

$$= \frac{14}{\sqrt{41}}$$

38. (A) Given quadratic equation is $3x^2 + x + 2 = 0$

Here $a = 3, b = 1, c = 2$

So, $D = 1^2 - 4 \times 3 \times 2$

$\Rightarrow$ $= 1 - 24 = -23$

So, $x = \frac{-b \pm \sqrt{D}}{2a}$

$$= \frac{-1 \pm \sqrt{-23}}{2 \times 3}$$

$$= \frac{-1 \pm \sqrt{23}i}{6} \quad [\because \sqrt{-1} = i]$$

Hence, the roots are $\frac{-1+\sqrt{23}i}{6}$ and $\frac{-1-\sqrt{23}i}{6}$.

(B) Given, $25x^2 - 30x + 11 = 0$

On comparing equation with $ax^2 + bx + c = 0$

We get,

$$a = 25, b = -30, c = 11$$

$\because$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \frac{30 \pm \sqrt{(-30)^2 + 4 \times 25 \times 11}}{2 \times 25}$

$\Rightarrow$ $x = \frac{30 \pm \sqrt{900 - 1100}}{50}$

$\Rightarrow$ $x = \frac{30 \pm \sqrt{-200}}{50}$

$\Rightarrow$ $x = \frac{30 \pm 10i\sqrt{2}}{50}$

$\Rightarrow$ $x = \frac{3}{5} \pm \frac{\sqrt{2}}{5}i$

Hence, the roots are $\frac{3}{5} + \frac{\sqrt{2}}{5}i$ and $\frac{3}{5} - \frac{\sqrt{2}}{5}$.