

CBSE SAMPLE PAPER - 06

Class 11 - Mathematics

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x : x \leq 1 \end{cases}$ [1]

Then $f(-1) + f(2) + f(4)$ is

- | | |
|------------------|-------|
| a) 5 | b) 9 |
| c) none of these | d) 14 |
-
2. The value of $(1 + i)^4 + (1 - i)^4$ is [1]
a) -4 b) 8
c) 4 d) -8

3. Solutions of the inequalities comprising a system in variable x are represented on number lines as given below, [1]
then



- | | |
|--------------------|---|
| a) $x \in [-3, 1]$ | b) $x \in (-\infty, -4) \cup [3, \infty)$ |
| c) $x \in [-4, 3]$ | d) $x \in (-\infty, -4] \cup [3, \infty)$ |
-
4. Two cards are drawn one by one without replacement from a deck of 52 cards. The probability that the second card is higher in rank than the first card, is: [1]
(Ranks in increasing order can be taken from Ace to King.)
- | | |
|-------------------|--------------------|
| a) $\frac{8}{17}$ | b) $\frac{16}{17}$ |
| c) $\frac{9}{17}$ | d) $\frac{1}{17}$ |

5. Which of the following sets is a finite set? [1]
- a) $B = \{x : x \in Z \text{ and } x^2 \text{ is even}\}$ b) All of these
- c) $A = \{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\}$ d) $D = \{x : x \in Z \text{ and } x > -10\}$
6. $\lim_{x \rightarrow \pi/3} \frac{\sin(\frac{\pi}{3} - x)}{2 \cos x - 1}$ is equal to [1]
- a) $\sqrt{3}$ b) $\frac{1}{2}$
- c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{5}$
7. $\sum_{r=0}^n 4^r \cdot {}^n C_r$ is equal to [1]
- a) 6^n b) 5^n
- c) 4^n d) 5^n
8. The standard deviation of the observations 5, 5, 5, 5, 5 is: [1]
- a) 20 b) 5
- c) 0 d) 25
9. Sum of three numbers in a G.P. is 21. If 2 is subtracted from first term and 1 is subtracted from third term, the new numbers are in A.P. The largest of the original numbers is: [1]
- a) 18 b) 15
- c) 9 d) 12
10. In a $\triangle ABC$, it is given that $\angle A : \angle B : \angle C = 3 : 4 : 5$, then $a : c = ?$ [1]
- a) 3 : 5 b) $\sqrt{3} : \sqrt{5}$
- c) $3 : (\sqrt{2} + 1)$ d) $2 : (\sqrt{3} + 1)$
11. The area of triangle formed by the lines $y = x$, $y = 2x$ and $y = 3x + 4$ is [1]
- a) 8 b) 7
- c) 9 d) 4
12. $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{(\pi/2 - \theta) \cos \theta}$ is equal to [1]
- a) -1 b) 1
- c) $-\frac{1}{2}$ d) $\frac{1}{2}$
13. If $\frac{(a^2+1)^2}{2a-i} = x + iy$, then $x^2 + y^2$ is equal to [1]
- a) $\frac{(a^2+1)^4}{4a^2+1}$ b) $\frac{(a^2-1)^2}{(4a^2-1)^2}$
- c) None of these d) $\frac{(a+1)^2}{4a^2+1}$
14. If $f(x) = \frac{4x(x^2+1)}{x^2+(x^2+1)^2}$, $x \geq 0$ then range of $f(x)$ is: [1]
- a) $\left(0, \frac{8}{5}\right]$ b) $\left[0, \frac{8}{5}\right]$
- c) $\left(2, \frac{8}{5}\right]$ d) $\left[\frac{8}{5}, \infty\right) \cup \{0\}$
15. The number of lines that are parallel to $2x + 6y - 7 = 0$ and have an intercept 10 units between the coordinate [1]

axis is :

a) 3

b) 2

c) 4

d) 1

16. If $a_n = \sum_{r=0}^n \frac{1}{{}^nC_r}$, then $\sum_{r=0}^n \frac{r}{{}^nC_r}$ equals [1]

a) $\frac{1}{2}na_n$

b) $(n-1)a_n$

c) None of these

d) $n a_n$

17. $\lim_{x \rightarrow 0} \frac{(1-\cos 2x) \sin 5x}{x^2 \sin 3x}$ [1]

a) $\frac{5}{6}$

b) $\frac{3}{10}$

c) $\frac{6}{5}$

d) $\frac{10}{3}$

18. The solution set for $(x+3)+4 > -2x+5$: [1]

a) none of these

b) $(-\frac{2}{3}, \infty)$

c) $(-\infty, -2)$

d) $(2, \infty)$

19. **Assertion (A):** If $\sin x = -\frac{1}{3}$, then $\cos x = \frac{2\sqrt{2}}{3}$ [1]

Reason (R): If the value of $\cos x$ is -ve and $\sin x$ is -ve then $x \in [\frac{3\pi}{2}, 2\pi]$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The Total no. words with letters of the word civilization (all taken at a time) is 19958393. [1]

Reason (R): The number of permutation of n distincts objects (r taken at a time) is nP_r .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Prove that: $\cos A + \cos 3A + \cos 5A + \cos 7A = 4 \cos A \cos 2A \cos 4A$. [2]

22. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements ; how many elements does $X \cap Y$ have? [2]

23. If the letters of the word **MISSISSIPPI** are written down at random in a row, what is the probability that four S's come together. [2]

OR

A die is thrown twice. Each time the number appearing on it is recorded. Describe the following events:

i. A = Both numbers are odd.

ii. B = Both numbers are even.

iii. C = Sum of the numbers is less than 6

Also, find $A \cup B$, $A \cap B$, $A \cup C$, $A \cap C$. Which pairs of events are mutually exclusive?

24. Find the obtuse angle between the lines $x - 2y + 3 = 0$ and $3x + y - 1 = 0$. [2]

25. A and B are two sets given in such a way that $(A \times B)$ contains 6 elements. If three elements of $(A \times B)$ are [2]

(1, 3), (2, 5) and (3, 3), find its remaining elements.

Section C

26. If $(x + iy)^{1/3} = a + ib$, where $x, y, a, b \in \mathbb{R}$, then show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$. [3]
27. Solve $5x - 3 < 3x + 1$ when [3]
- x is a real number
 - x is an integer
 - x is a natural number

OR

Solve $\frac{8x^2 + 16x - 51}{2x^2 + 5x - 12} > 3$

28. Using binomial theorem, expand: $(x^2 - \frac{2}{x})^7$. [3]

OR

Evaluate: $(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$

29. Find the point in yz -plane which is equidistant from the points $A(3, 2, -1)$, $B(1, -1, 0)$ and $C(2, 1, 2)$. [3]

OR

Find the coordinates of a point on y -axis which are at a distance of $5\sqrt{2}$ from the point $P(3, -2, 5)$.

30. A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are defective. If a person takes 2 at random, what is the probability that either both are apples or both are good? [3]
31. Let $f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = 2^x$. Find [3]
- range (f)
 - $\{x : f(x) = 1\}$.
 - Find out whether $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$.

Section D

32. While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance. [5]
33. In a survey of 100 students, the number of students studying the various languages were found to be: English only 18, English but not Hindi 23, English and German 8, English 26, German 48, German and Hindi 8, no language 24. Find the number of students who were studying (i) Hindi (ii) English and Hindi (iii) English, Hindi and German. [5]

OR

A school awarded 58 medals in three sports, namely 38 in football; 15 in basketball and 20 in cricket. If 3 students got medals in all the three sports, how many received medals in exactly two sports?

34. Find the equation of the hyperbola whose vertices are at (0 ± 7) and foci at $(0, \pm \frac{28}{3})$. [5]

OR

Show that the equation $x^2 - 2y^2 - 2x + 8y - 1 = 0$ represents a hyperbola. Find the coordinates of the centre, lengths of the axes, eccentricity, latusrectum, coordinates of foci and vertices and equations of directrices of the hyperbola.

35. Differentiate If $y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$ show that $\frac{dy}{dx} = (\sec x \tan x + \sec x)$ [5]

Section E

36. **Read the text carefully and answer the questions:** [4]
- A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant.

Rahul being a plant lover decides to open a nursery and he bought few plants with pots. He wants to place pots in such a way that number of pots in first row is 2 , in second row is 4 and in third row is 8 and so on



- (i) If Rahul wants to place 510 pots in total, then the total number of rows formed in this arrangement is
- | | |
|------|------|
| a) 5 | b) 7 |
| c) 9 | d) 8 |
- (ii) Total number of pots upto 10th row is
- | | |
|---------|---------|
| a) 2046 | b) 1024 |
| c) 1046 | d) 1023 |
- (iii) The difference in number of pots placed in 7th row and 5th row is
- | | |
|-------|-------|
| a) 50 | b) 96 |
| c) 90 | d) 86 |

OR

The number of pots in 8th row is

- | | |
|--------|--------|
| a) 300 | b) 156 |
| c) 456 | d) 256 |

37. **Read the text carefully and answer the questions:**

[4]

A state cricket authority has to choose a team of 11 members, to do it so the authority asks 2 coaches of a government academy to select the team members that have experience as well as the best performers in last 15 matches. They can make up a team of 11 cricketers amongst 15 possible candidates. In how many ways can the final eleven be selected from 15 cricket players if:



- (i) One of them, who is in bad form, must always be excluded
- | | |
|---------|--------|
| a) 480 | b) 364 |
| c) 1365 | d) 640 |
- (ii) Two of them being leg spinners, one and only one leg spinner must be included?
- | | |
|--------------------------------------|-----------------------------------|
| a) ${}^2C_1 \times {}^{13}C_{10}$ | b) ${}^1C_2 \times {}^{13}C_{10}$ |
| c) ${}^2C_{10} \times {}^{13}C_{10}$ | d) ${}^2C_1 \times {}^{10}C_{13}$ |
- (iii) If there are 6 bowlers, 3 wicket-keepers, and 11 batsmen in all. The number of ways in which a

A team of 4 bowlers, 2 wicket-keepers, and 5 batsmen can be chosen.

a) ${}^6C_2 \times {}^3C_5 \times {}^{11}C_4$

b) ${}^6C_2 \times {}^3C_4 \times {}^{11}C_5$

c) ${}^6C_2 \times {}^3C_1 \times {}^{11}C_5$

d) ${}^6C_2 \times {}^3C_4 \times {}^{11}C_5$

OR

There is no restriction:

a) 2365

b) 1375

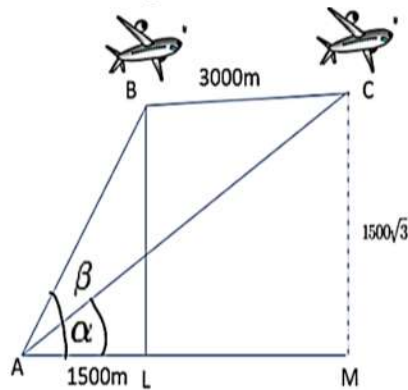
c) 1365

d) 1465

38. **Read the text carefully and answer the questions:**

[4]

The angle of elevation of an airplane from a point A on the ground is α . After a flight of 15 seconds, the angle of elevation is β . The airplane is flying at a constant height of $1500\sqrt{3}$ m. Distance between two positions of plane is 3000m. Again, distance AL = 1500m.



(i) Find the value of $\sin \beta$.

(ii) Find the value of $\sin \alpha$.

Solution

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Class 11 - Mathematics

Section A

1. (b) 9

Explanation: Given that,

$$f(x) = \begin{cases} 2x & : x > 3 \\ x^2 & : 1 < x \leq 3 \\ 3x^2 & : x \leq 1 \end{cases}$$

Now,

$$f(-1) = 3(-1) = -3 \text{ [since } -1 < 1 \text{ and } f(x) = 3x \text{ for } x \leq 1]$$

$$f(2) = 2^2 = 4 \text{ [since } 2 < 3 \text{ and } f(x) = x^2 \text{ for } 1 < x \leq 3]$$

$$f(4) = 2(4) = 8 \text{ [since } 4 > 3 \text{ and } f(x) = 2x \text{ for } x > 3]$$

$$\therefore f(-1) + f(2) + f(4) = -3 + 4 + 8 = 9$$

2. (d) -8

Explanation: -8

$$\text{Using } a^4 + b^4 = (a^2 + b^2) - 2a^2b^2$$

$$(1+i)^4 + (1-i)^4$$

$$= ((1+i)^2 + (1-i)^2)^2 - 2(1+i)^2(1-i)^2$$

$$= (1+i^2+2i+1+i^2-2i)^2 - 2(1+i^2+2i)(1+i^2-2i)$$

$$= (1-1+2i+1-1-2i)^2 - 2(1-1+2i)(1-1-2i)$$

$$= (0) - 2(2i)(-2i) \text{ (}\because i^2 = -1\text{)}$$

$$= 8i^2$$

$$= -8$$

3. (d) $x \in (-\infty, -4] \cup [3, \infty)$

Explanation: Common solution of the inequalities is from $-\infty$ to -4 and 3 to ∞

$$\{(-\infty, -4] \cup [3, \infty)\} \cap \{(-\infty, -3] \cup [1, \infty)\} = (-\infty, -4] \cup [3, \infty)$$

4. (a) $\frac{8}{17}$

Explanation: $\frac{8}{17}$

5. (c) $A = \{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\}$

Explanation: $A = \{x : x \in Z \text{ and } x^2 - 5x + 6 = 0\}$

6. (c) $\frac{1}{\sqrt{3}}$

Explanation: $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(\frac{\pi}{3}-x)}{2 \cos x - 1}$

$$= \lim_{h \rightarrow 0} \frac{\sin \frac{\pi}{3} - \left(\frac{\pi}{3} - h\right)}{2 \cos\left(\frac{\pi}{3} - h\right) - 1}$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{2 \left[\cos \frac{\pi}{3} \cos h + \sin \frac{\pi}{3} \sin h \right] - 1}$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{2 \left[\frac{1}{2} \cos h + \frac{\sqrt{3}}{2} \sin h \right] - 1}$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{\cos h + \sqrt{3} \sin h - 1}$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{-2 \sin^2 \frac{h}{2} + \sqrt{3} \sin h}$$

Dividing N^r and D^r by h

$$= \lim_{h \rightarrow 0} \frac{\frac{\sin h}{h}}{-\left(2 \times \frac{h}{4}\right) \left(\frac{\sin^2 \frac{h}{2}}{\frac{h}{4}}\right) + \frac{\sqrt{3} \sin h}{h}}$$

$$= \frac{1}{\sqrt{3}}$$

7. (d) 5^n

Explanation: $\sum_{r=0}^n 4^r \cdot {}^n C_r = 4^0 \cdot {}^n C_0 + 4^1 \cdot {}^n C_1 + 4^2 \cdot {}^n C_2 + \dots + 4^n \cdot {}^n C_n$
 $= 1 + 4 \cdot {}^n C_1 + 4^2 \cdot {}^n C_2 + \dots + 4^n \cdot {}^n C_n$
 $= (1 + 4)^n = 5^n$

8. (c) 0

Explanation: Given observation are 5, 5, 5, 5 and 5.

$$\therefore \text{Mean } (\bar{x}) = \frac{5+5+5+5+5}{5} = \frac{25}{5} = 5$$

$$\text{Now, SD} = \sqrt{\frac{\sum_{i=1}^5 (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{(5-5)^2 + (5-5)^2 + (5-5)^2 + (5-5)^2 + (5-5)^2}{5}}$$

$$= 0$$

9. (d) 12

Explanation: Let a, ar, ar² be the three numbers. Then, a + ar + ar² = 21

$$\Rightarrow a + ar^2 = 21 - ar \dots (i)$$

Also, a - 2, ar, ar² - 1 are in A.P.

$$\Rightarrow 2ar = a - 2 + ar^2 - 1$$

$$\Rightarrow a + ar^2 = 2ar + 3 \dots (ii)$$

From (i) and (ii), we get 2ar + 3 = 21 - ar

$$\Rightarrow ar = 6 \Rightarrow r = \frac{6}{a}$$

Substituting $r = \frac{6}{a}$ in (i), we get $a + \frac{36}{a} = 21 - 6$

$$\Rightarrow a^2 - 15a + 36 = 0$$

$$\Rightarrow a = 12, 3$$

$$\Rightarrow r = \frac{1}{2}, 2 \dots [\because r = \frac{6}{a}]$$

\therefore the numbers are 12, 6, 3 or 3, 6, 12

\therefore the largest number is 12.

10. (d) $2 : (\sqrt{3} + 1)$

Explanation: Let the angles be $3x^\circ$, $4x^\circ$ and $5x^\circ$. Then,

$$3x + 4x + 5x = 180 \Rightarrow 12x = 180 \Rightarrow x = 15,$$

$\therefore \angle A = 45^\circ$, $\angle B = 60^\circ$ and $\angle C = 75^\circ$.

$$\therefore a : c = \sin 45^\circ : \sin 75^\circ = \frac{1}{\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}} = 2 : (\sqrt{3} + 1).$$

11. (d) 4

Explanation: Area of the triangle formed by the coordinates (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by

$$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

On solving the lines 1 and 2, the point of intersection is (0, 0)

On solving the lines 2 and 3, the point of intersection is (4, 8)

On solving the line 3 and 1, the point of intersection is (-2, -2)

Now substituting the values to find the area of the triangle,

$$\text{Area} = \frac{1}{2} | [0 + 4(-2 - 0) + (-2)(0 - 8)] |$$

$$= 4 \text{ sq units}$$

12. (d) $\frac{1}{2}$

Explanation: $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{\left(\frac{\pi}{2} - \theta\right) \cos \theta}$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos h}{\left(\frac{\pi}{2} - \left(\frac{\pi}{2} - h\right)\right) \sin h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{h \sin h} \\
&= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{\frac{4h^2}{4}} \\
&= \frac{2}{4} \\
&= \frac{1}{2}
\end{aligned}$$

13. (a) $\frac{(a^2+1)^4}{4a^2+1}$

Explanation: $\frac{(a^2+1)^4}{4a^2+1}$

$$x + iy = \frac{(a^2+1)}{2a-i}$$

Taking modulus on both the sides, we get:

$$\sqrt{x^2 + y^2} = \frac{(a^2+1)^2}{\sqrt{4a^2+1}}$$

$$x^2 + y^2 = \frac{(a^2+1)^4}{4a^2+1}$$

14. (b) $\left[0, \frac{8}{5}\right]$

Explanation: $y = \frac{4x(x^2+1)}{x^2+(x^2+1)^2} = \frac{4\left(x+\frac{1}{x}\right)}{1+\left(x+\frac{1}{x}\right)^2}$

Let $x + \frac{1}{x} = t$

For $x > 0, t \geq 2$

$$y = \frac{4t}{1+t^2} = \frac{4}{\frac{1}{t}+t}$$

For $t \geq 2, t + \frac{1}{t} \geq \frac{5}{2}$

$0 < y \leq \frac{8}{5}$ and $y = 0$ for $x = 0$

\therefore Range of $f(x)$ is $\left[0, \frac{8}{5}\right]$

15. (b) 2

Explanation: The slope of the given line $2x + 6y = 7$ is $-\frac{1}{3}$

Hence the line which is parallel to the above line is

$$y = \left(-\frac{1}{3}\right)x + c$$

That is the y-intercept is $(0, c)$ and the x-intercept is $(3c, 0)$

Using the distance formula

$$d^2 = (0 - 3c)^2 + (3c - 0)^2$$

$$= 10c^2$$

Since the distance is given as 10, then

$$100 = 10c^2$$

Therefore $c = \pm 10$

Since two values are possible, two lines can be drawn.

16. (a) $\frac{1}{2}na_n$

Explanation: Let $b = \sum_{r=0}^n \frac{r}{{}^nC_r} = \sum_{r=0}^n \frac{n-(n-r)}{{}^nC_r}$

$$= n \sum_{r=0}^n \frac{1}{{}^nC_r} - \sum_{r=0}^n \frac{n-r}{{}^nC_r}$$

$$= na_n - \sum_{r=0}^n \frac{n-r}{{}^nC_{n-r}} \quad [\because {}^nC_r = {}^nC_{n-r}]$$

$$= na_n - b \Rightarrow 2b = na_n \Rightarrow b = \frac{n}{2}a_n$$

17. (d) $\frac{10}{3}$

Explanation: $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x) \sin 5x}{x^2 \sin 3x}$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \frac{2 \sin^2 x \sin 5x}{x^2 \sin 3x} \\
&= \lim_{x \rightarrow 0} \frac{\frac{2 \sin^2 x}{x^2} \left(\frac{\sin 5x}{5x} \right) \times 5}{\left(\frac{\sin 3x}{3x} \right)^3} \\
&= \frac{10 \times 1^2 \times 1}{1 \times 3} \\
&= \frac{10}{3}
\end{aligned}$$

18. (b) $\left(\frac{-2}{3}, \infty\right)$

Explanation: $(x + 3) + 4 > -2x + 5$

$$\Rightarrow x + 7 > -2x + 5$$

$$\Rightarrow x + 7 + 2x > -2x + 5 + 2x$$

$$\Rightarrow 3x + 7 > 5$$

$$\Rightarrow 3x + 7 - 7 > 5 - 7$$

$$\Rightarrow 3x > -2$$

$$\Rightarrow x > \frac{-2}{3}$$

$$\Rightarrow x \in \left(\frac{-2}{3}, \infty\right)$$

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Assertion

$$\sin x = \frac{-1}{3}$$

So, $x \in$ 3rd or 4th quadrant.

$$\cos x = \pm \sqrt{1 - \sin^2 x} \quad [\because \sin^2 x + \cos^2 x = 1]$$

$$= \pm \sqrt{1 - \left(\frac{-1}{3}\right)^2}$$

$$= \pm \sqrt{1 - \frac{1}{9}}$$

$$= \pm \sqrt{\frac{8}{9}}$$

$$= \cos x = \pm \frac{2\sqrt{2}}{3}$$

$\sin x = -ve$, $\cos x = +ve$, possible only when $x \in \pi$ quad. i.e. $x \in \left(\frac{3\pi}{2}, 2\pi\right)$

20. (d) A is false but R is true.

Explanation: Civilization: Letter \Rightarrow 12

i repeats 4 times

$$\text{Total no. of rearrangement} = \frac{12!}{4!}$$

$$= 19958400$$

Section B

21. To prove: $\cos A + \cos 3A + \cos 5A + \cos 7A = 4 \cos A \cos 2A \cos 4A$

Consider LHS:

$$\cos A + \cos 3A + \cos 5A + \cos 7A$$

$$= 2 \cos \left(\frac{A+3A}{2}\right) \cos \left(\frac{A-3A}{2}\right) + 2 \cos \left(\frac{5A+7A}{2}\right) \cos \left(\frac{5A-7A}{2}\right) \quad \left\{ \because \cos A + \cos B = 2 \cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right) \right\}$$

$$= 2 \cos 2A \cos (-A) + 2 \cos 6A \cos (-A)$$

$$= 2 \cos 4A \cos A + 2 \cos 6A \cos (-4A)$$

$$= 2 \cos A \{ \cos 2A + \cos 6A \}$$

$$= 2 \cos A \times \left\{ 2 \cos \left(\frac{2A+6A}{2}\right) \cos \left(\frac{2A-6A}{2}\right) \right\}$$

$$= 4 \cos A \cos 4A \cos (-2A)$$

$$= 4 \cos A \cos 2A \cos 4A$$

$$= \text{RHS}$$

Hence proved.

22. Here,

$$n(X) = 8, n(Y) = 15, \text{ and } n(X \cup Y) = 18$$

We know that-

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$\Rightarrow 18 = 8 + 15 - n(X \cap Y)$$

$$\Rightarrow 18 = 23 - n(X \cap Y)$$

$$\Rightarrow n(X \cap Y) = 23 - 18$$

Therefore,

$$n(X \cap Y) = 5$$

23. We need to find the probability that four S's come together.

There are 11 letters in the word 'MISSISSIPPI' which can be arranged in 11! ways.

Number of the letter S = 4

Let us consider the four S's in the given word as one letter.

So, when the four letters are clubbed together, we have (SSSS) MIIIPPI. We can arrange eight letters in a row in 8! ways.

Also, the four S's can be arranged in 4! ways.

Hence, required probability = $8! \times 4! / 11! = 8! \times 4 \times 3 \times 2 \times 1 \times 10 \times 9 \times 8! = 4 \times 3 \times 2 \times 1 \times 10 \times 9 = 4165$.

OR

We have given that

A dice is thrown twice. And each time number appearing on it is recorded.

We have to find:

i. A = Both numbers are odd.

ii. B = Both numbers are even.

iii. C = Sum of the numbers is less than 6

Explanation: when the dice is thrown twice then the number of sample spaces are $6^2 = 36$

Now,

The possibility both odd numbers are:

$$A = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$$

Since, Possibility of both even numbers are:

$$B = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$$

And, Possible outcome of sum of the numbers is less than 6

$$C = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$$

Therefore,

$$(A \cup B) = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5), (2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$$

$$(A \cap B) = \{\Phi\}$$

$$(A \cup C) = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5), (1, 2), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$$

$$(A \cap C) = \{(1, 1), (1, 3), (3, 1)\}$$

Hence, $(A \cap B) = \emptyset$ and $(A \cap C) \neq \emptyset$, A and B are mutually exclusive, but A and C are not.

24. Let m_1 and m_2 be the slopes of the straight lines $x - 2y + 3 = 0$ and $3x + y - 1 = 0$.

Then,

$$m_1 = -\frac{1}{-2} = \frac{1}{2} \text{ and } m_2 = -\frac{3}{1} = -3$$

Let θ be the angle between the given lines. Then,

$$\text{We know that, } \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{\frac{1}{2} + 3}{1 - \frac{3}{2}} \right| = 0$$

$$\Rightarrow \theta = \tan^{-1}(0)$$

Thus, the acute angle between the lines is $\tan^{-1}(0)$ and the obtuse angle is $\tan^{-1}(0)$.

25. Here it is given that, (1, 3), (2, 5) and (3, 3) are in $(A \times B)$, it follows that 1, 2, 3 are elements of A and 3, 5 are elements of B.

$$\therefore (A \times B) = \{1, 2, 3\} \times \{3, 5\}$$

$$= \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}.$$

Hence, the remaining elements of $(A \times B)$ are (1, 5), (2, 3) and (3, 5).

Section C

26. We have, $(x + iy)^{1/3} = a + ib$

$$\Rightarrow x + iy = (a + ib)^3 \text{ [cubing on both sides]}$$

$$\Rightarrow x + iy = a^3 + i^3 b^3 + 3iab(a + ib)$$

$$\Rightarrow x + iy = a^3 - ib^3 + i3a^2b - 3ab^2$$

$$\Rightarrow x + iy = a^3 - 3ab^2 + i(3a^2b - b^3)$$

On equating real and imaginary parts from both sides, we get

$$x = a^3 - 3ab^2 \text{ and } y = 3a^2b - b^3$$

$$\Rightarrow \frac{x}{a} = a^2 - 3b^2 \text{ and } \frac{y}{b} = 3a^2 - b^2$$

$$\text{Now, } \frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$$

$$= -2a^2 - 2b^2 = -2(a^2 + b^2)$$

Hence proved.

27. Here $5x - 3 < 3x + 1$

$$\Rightarrow 5x - 3x < 1 + 3$$

$$\Rightarrow 2x < 4$$

Dividing both sides by 2

$$\therefore \frac{2x}{2} < \frac{4}{2}$$

$$\Rightarrow x < 2$$

i. If x is a real number,

Thus solution set of the given inequation is $(-\infty, 2)$

ii. If x is an integer,

Thus solution set of the given inequation is $(-\infty, 1]$

iii. If x is a natural number,

Thus solution set of the given inequation is $[1]$.

OR

Solution We have,

$$\frac{8x^2+16x-51}{2x^2+5x-12} > 3$$

$$\Leftrightarrow \frac{8x^2+16x-51}{2x^2+5x-12} - 3 > 0$$

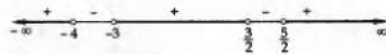
$$\Leftrightarrow \frac{8x^2+16x-51-6x^2-15x+36}{2x^2+5x-12} > 0$$

$$\Leftrightarrow \frac{2x^2+x-15}{2x^2+5x-12} > 0$$

$$\Leftrightarrow \frac{2x^2+6x-5x-15}{2x^2+8x-3x-12} > 0$$

$$\Leftrightarrow \frac{(x+3)(2x-5)}{(x+4)(2x-3)} > 0 \dots(i)$$

Sign of expression on number line



Hence

$$x \in (-\infty, -4) \cup (-3, 3/2) \cup (5/2, \infty)$$

28. To find: Expansion of $(x^2 - \frac{3x}{7})^7$

$$\text{Formula used: } {}^nC_r = \frac{n!}{(n-r)!(r)!}$$

$$\text{We know that } (a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$$

$$\text{Here We have, } (x^2 - \frac{3x}{7})^7$$

$$\Rightarrow [{}^7C_0(x^2)^{7-0}] + [{}^7C_1(x^2)^{7-1}(-\frac{3x}{7})^1] + [{}^7C_2(x^2)^{7-2}(-\frac{3x}{7})^2] + [{}^7C_3(x^2)^{7-3}(-\frac{3x}{7})^3] + [{}^7C_4(x^2)^{7-4}(-\frac{3x}{7})^4]$$

$$+ [{}^7C_5(x^2)^{7-5}(-\frac{3x}{7})^5] + [{}^7C_6(x^2)^{7-6}(-\frac{3x}{7})^6] + [{}^7C_7(-\frac{3x}{7})^7]$$

$$\Rightarrow \left[\frac{7!}{0!(7-0)!} (x^2)^7 \right] - \left[\frac{7!}{1!(7-1)!} (x^2)^6 \left(\frac{3x}{7} \right) \right] + \left[\frac{7!}{2!(7-2)!} (x^2)^5 \left(\frac{9x^2}{49} \right) \right] - \left[\frac{7!}{3!(7-3)!} (x^2)^4 \left(\frac{27x^3}{343} \right) \right]$$

$$+ \left[\frac{7!}{4!(7-4)!} (x^2)^3 \left(\frac{81x^4}{2401} \right) \right] - \left[\frac{7!}{5!(7-5)!} (x^2)^2 \left(\frac{243x^5}{16807} \right) \right] + \left[\frac{7!}{6!(7-6)!} (x^2)^1 \left(\frac{729x^6}{117649} \right) \right] - \left[\frac{7!}{7!(7-7)!} \left(\frac{2187x^7}{823543} \right) \right]$$

$$- \left[\frac{7!}{7!(7-7)!} \left(\frac{2187x^7}{823543} \right) \right] + \left[21(x^{10}) \left(\frac{9x^2}{49} \right) \right] - \left[35(x^8) \left(\frac{27x^3}{343} \right) \right]$$

$$+ \left[35(x^6) \left(\frac{81x^4}{2401} \right) \right] - \left[21(x^4) \left(\frac{243x^5}{16807} \right) \right] + \left[7(x^2) \left(\frac{729x^6}{117649} \right) \right] - \left[1 \left(\frac{2187x^7}{823543} \right) \right]$$

$$\Rightarrow x^{24} - 3x^{13} + \left(\frac{27}{7} \right) x^{12} - \left(\frac{135}{49} \right) x^{11} + \left(\frac{405}{343} \right) x^{10} - \left(\frac{729}{2401} \right) x^9 + \left(\frac{729}{16807} \right) x^8 - \left(\frac{2187}{823543} \right) x^7$$

$$x^{14} - 3x^{13} + \left(\frac{27}{7} \right) x^{12} - \left(\frac{135}{49} \right) x^{11} + \left(\frac{405}{343} \right) x^{10} - \left(\frac{729}{2401} \right) x^9 + \left(\frac{729}{16807} \right) x^8 - \left(\frac{2187}{823543} \right) x^7$$

OR

To find: Value of $(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$

Formula used: ${}^nC_r = \frac{n!}{(n-r)!r!}$

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$$

$$(a+1)^5 = {}^5C_0a^5 + {}^5C_1a^{5-1}1 + {}^5C_2a^{5-2}1^2 + {}^5C_3a^{5-3}1^3 + {}^5C_4a^{5-4}1^4 + {}^5C_51^5$$

$$= {}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5 \dots (i)$$

$$(a-1)^5 = [{}^5C_0a^5] + [{}^5C_1a^{5-1}(-1)^1] + [{}^5C_2a^{5-2}(-1)^2] + [{}^5C_3a^{5-3}(-1)^3] + [{}^5C_4a^{5-4}(-1)^4] + [{}^5C_5(-1)^5]$$

$$= {}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5 \dots (ii)$$

Subtracting (ii) from (i)

$$(a+1)^5 - (a-1)^5 = [{}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5] - [{}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5]$$

$$= 2[{}^5C_1a^4 + {}^5C_3a^2 + {}^5C_5]$$

$$= 2 \left[\left(\frac{5!}{1!(5-1)!} a^4 \right) + \left(\frac{5!}{3!(5-3)!} a^2 \right) + \left(\frac{5!}{5!(5-5)!} \right) \right]$$

$$= 2[(5)a^4 + (10)a^2 + (1)]$$

$$= 2[5a^4 + 10a^2 + 1] = (a+1)^5 - (a-1)^5$$

Putting the value of a, $= \sqrt{3}$ in the above equation we get..

$$(\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5 = 2 [5(\sqrt{3})^4 + 10(\sqrt{3})^2 + 1]$$

$$= 2[(5)(9) + (10)(3) + 1]$$

$$= 2[45 + 30 + 1]$$

$$= 152$$

29. The general point on yz plane is D(0, y, z).

Consider this point is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2).

$\therefore AD = BD$

$$\sqrt{(0-3)^2 + (y-2)^2 + (z+1)^2} = \sqrt{(0-1)^2 + (y+1)^2 + (z-0)^2}$$

Squaring both sides,

$$(0-3)^2 + (y-2)^2 + (z+1)^2 = (0-1)^2 + (y+1)^2 + (z-0)^2$$

$$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 1 + y^2 + 2y + 1 + z^2$$

$$-6y + 2z + 12 = 0 \dots(1)$$

Also, $AD = CD$

$$\sqrt{(0-3)^2 + (y-2)^2 + (z+1)^2} = \sqrt{(0-2)^2 + (y-1)^2 + (z-2)^2}$$

Squaring both sides,

$$(0-3)^2 + (y-2)^2 + (z+1)^2 = (0-2)^2 + (y-1)^2 + (z-2)^2$$

$$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 4 + y^2 - 2y + 1 + z^2 - 4z + 4$$

$$-2y + 6z + 5 = 0 \dots(2)$$

By solving equation (1) and (2) we get

$$y = \frac{31}{16} \quad z = \frac{-3}{16}$$

The point which is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2) is $(\frac{31}{16}, \frac{-3}{16})$.

OR

Let Q (0, y, 0) be any point on y-axis.

$$PQ = \sqrt{(0-3)^2 + (y+2)^2 + (0-5)^2}$$

$$= \sqrt{9 + y^2 + 4 + 4y + 25} = \sqrt{y^2 + 4y + 38}$$

$$\text{But } \sqrt{y^2 + 4y + 38} = 5\sqrt{2}$$

Squaring both sides, we have

$$y^2 + 4y + 38 = 50$$

$$\Rightarrow y^2 + 4y - 12 = 0$$

$$\Rightarrow (y-2)(y+6) = 0$$

$$\Rightarrow y = 2, -6$$

Thus coordinates of point Q are (0, 2, 0) and (0, -6, 0)

30. From 30 items, two can be selected in ${}^{30}C_2$ ways.

Therefore, total number of elementary events = ${}^{30}C_2$.

Consider the events: A = Getting two apples; B = Getting two good apples

Required probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (i)

There are 20 apples, out of which 2 can be drawn in ${}^{20}C_2$ ways. Therefore, we have,

$$P(A) = \frac{{}^{20}C_2}{{}^{30}C_2}$$

There are 8 defective pieces and the remaining 22 are good. Out of 22 good pieces, two can be selected in ${}^{22}C_2$ ways. Therefore, we have,

$$P(B) = \frac{{}^{22}C_2}{{}^{30}C_2}$$

Since there are 15 pieces which are good apples out of which 2 can be selected in ${}^{15}C_2$ ways.

$\therefore P(A \cap B) =$ Probability of getting 2 pieces which are good apples = $\frac{{}^{15}C_2}{{}^{30}C_2}$

putting the values of $P(A)$, $P(B)$ and $P(A \cap B)$ in (i), we get,

$$\text{Required probability} = \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{22}C_2}{{}^{30}C_2} - \frac{{}^{15}C_2}{{}^{30}C_2} = \frac{316}{435}$$

31. Given that, $f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = 2^x$

i. **To find:** Range of x Here, $f(x) = 2^x$ is a positive real number for every $x \in \mathbb{R}$ because 2^x is positive for every $x \in \mathbb{R}$

Moreover, for every positive real number x , $\exists \log_2 x \in \mathbb{R}$ such that

$$f(\log_2 x) = 2^{\log_2 x}$$

$$= x \quad [\because a^{\log_a x} = x]$$

Hence, the range of f is the set of all positive real numbers.

ii. To find: $\{x : f(x) = 1\}$

We have, $f(x) = 1$... (i)

and $f(x) = 2^x$... (ii)

From eq. (i) and (ii), we get

$$2^x = 1$$

$$\Rightarrow 2^x = 2^0 \quad [\because 2^0 = 1]$$

Comparing the powers of 2, we obtain

$$\Rightarrow x = 0$$

$$\therefore \{x : f(x) = 1\} = 0$$

iii. **To find:** $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$, We have,

$$f(x + y) = 2^{x+y}$$

$$= 2^x \cdot 2^y$$

[The exponent "product rule" tells us that, when multiplying two powers that have the same base, you can add the exponents or vice - versa]

$$= f(x) \cdot f(y) \quad [\because f(x) = 2^x]$$

$$\therefore f(x + y) = f(x) \cdot f(y) \text{ holds for all } x, y \in \mathbb{R}$$

Section D

32. To find: the correct mean and the variance.

As per given criteria,

Number of reading, $n=10$

Mean of the given readings before correction, $\bar{x} = 45$

But we know,

$$\bar{x} = \frac{\sum x_i}{n}$$

Substituting the corresponding values, we get

$$45 = \frac{\sum x_i}{10}$$

$$\Rightarrow \sum x_i = 45 \times 10 = 450$$

It is said one reading 25 was wrongly taken as 52,

$$\text{So } \sum x_i = 450 - 52 + 25 = 423$$

So the correct mean after correction is

$$\bar{x} = \frac{\sum x_i}{n} = \frac{423}{10} = 42.3$$

Also given the variance of the 10 readings is 16 before correction,

i.e., $\sigma^2=16$

But we know

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

Substituting the corresponding values, we get

$$16 = \frac{\sum x_i^2}{10} - (45)^2$$

$$\Rightarrow 16 = \frac{\sum x_i^2}{10} - 2025$$

$$\Rightarrow 16 + 2025 = \frac{\sum x_i^2}{10}$$

$$\Rightarrow \frac{\sum x_i^2}{10} = 2041$$

$$\Rightarrow \sum x_i^2 = 20410$$

It is said one reading 25 was wrongly taken as 52, so

$$\Rightarrow \sum x_i^2 = 20410 - (52)^2 + (25)^2$$

$$\Rightarrow \sum x_i^2 = 20410 - 2704 + 625$$

$$\Rightarrow \sum x_i^2 = 18331$$

So the correct variance after correction is

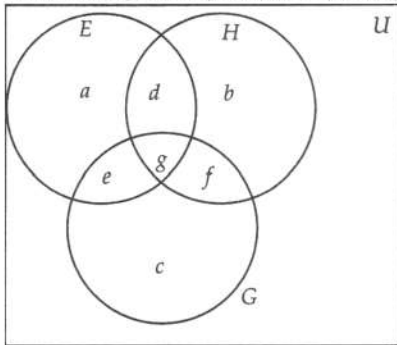
$$\sigma^2 = \frac{18331}{10} - \left(\frac{423}{10}\right)^2$$

$$\sigma^2 = 1833.1 - (42.3)^2 = 1833.1 - 1789.29$$

$$\sigma^2 = 43.81$$

Hence the corrected mean and variance is 42.3 and 43.81 respectively.

33. Let U be the set of total students surveyed i.e. the universal set. Let E, H and G be the sets of students studying English, Hindi and German respectively. In the Venn diagram shown, let a, b, c, d, e, f and g denote the number of students in the respective regions.



Clearly, $n(U) = 100$.

It is given that:

Number of students studying English only = 18 $\Rightarrow a = 18$

Number of students studying English but not Hindi = 23 $\Rightarrow a + e = 23 \Rightarrow e = 5$

Number of students studying English and German, $n(E \cap G) = 8 \Rightarrow e + g = 8 \Rightarrow g = 3$

Number of students studying English, $n(E) = 26 \Rightarrow a + d + e + g = 26 \Rightarrow d = 0$

Number of students studying German and Hindi, $n(G \cap H) = 8 \Rightarrow g + f = 8 \Rightarrow f = 5$

Number of students studying German, $n(G) = 48 \Rightarrow c + e + g + f = 48 \Rightarrow c = 35$

Also it is given that 24 students study no language.

Therefore, the number of students who study at least one language, $n(E \cup H \cup G) = n(U) - 24 = 100 - 24 = 76$

From Venn diagram,

$$\Rightarrow a + b + c + d + e + f + g = 76$$

$$\Rightarrow 18 + b + 35 + 0 + 5 + 5 + 3 = 76$$

$$\Rightarrow b = 10$$

i. The number of students studying Hindi, $n(H) = b + d + g + f = 10 + 0 + 3 + 5 = 18$

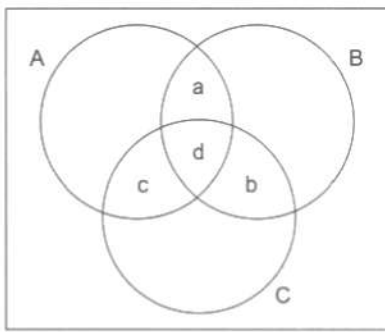
ii. The number of students studying English and Hindi, $n(E \cap H) = d + g = 0 + 3 = 3$

iii. The number of students studying English, Hindi and German, $n(E \cap H \cap G) = g = 3$

OR

Suppose A, B and C denote the sets of students who won medals in football, basketball and cricket respectively.

Then, we have $n(A) = 38$, $n(B) = 15$, $n(C) = 20$, $n(A \cap B \cap C) = 3$ and $n(A \cup B \cup C) = 58$.



We know that $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
 $\Rightarrow n(A \cap B) + n(B \cap C) + n(A \cap C)$
 $= n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C) = \{(38 + 15 + 20 + 3) - 58\} = (76 - 58) = 18.$

Suppose a, b, c and d denote respectively the number of students who won medals in football and basketball both; basketball and cricket both; football and cricket both and all the 3 sports.

Then, we have $n(A \cap B) + n(B \cap C) + n(A \cap C) = 18$

$\Rightarrow (a + d) + (b + d) + (c + d) = 18$

$\Rightarrow (a + b + c) + 3d = 18$

$\Rightarrow (a + b + c) + 3 \times 3 = 18$

$\Rightarrow a + b + c = 9$ [$\because d = 3$]

Therefore, 9 students received medals in exactly two sports.

34. Since, the vertices are on y-axis, so let the equation of the required hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \dots(i)$$

The coordinate of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$\therefore b = 7$ [\because vertices = $(0, \pm 7)$]

$\Rightarrow b^2 = 49$

and,

$be = \frac{28}{3}$ [\because Foci = $(0, \pm \frac{28}{3})$]

$\Rightarrow 7 \times e = \frac{28}{3}$

$\Rightarrow e = \frac{4}{3}$

$\Rightarrow e^2 = \frac{16}{9}$

Now,

$a^2 = b^2 (e^2 - 1)$

$\Rightarrow a^2 = 49 \left(\frac{16}{9} - 1 \right)$

$\Rightarrow a^2 = 49 \times \frac{7}{9}$

$\Rightarrow a^2 = \frac{343}{9}$

Putting $a^2 = \frac{343}{9}$ and $b^2 = 49$ in equation (i), we get

$$\frac{\frac{x^2}{\frac{343}{9}}}{\frac{343}{9}} - \frac{y^2}{49} = -1$$

$$\frac{9x^2}{343} - \frac{y^2}{49} = -1$$

This is the equation of the required hyperbola.

OR

We have,

$x^2 - 2y^2 - 2x + 8y - 1 = 0$

$\Rightarrow (x^2 - 2x) - 2(y^2 - 4y) = 1$

$\Rightarrow (x^2 - 2x + 1) - 2(y^2 - 4y + 4) = -6$

$\Rightarrow (1 - x)^2 - 2(y - 2)^2 = -6$

$\Rightarrow \frac{(x-1)^2}{(\sqrt{6})^2} - \frac{(y-2)^2}{(\sqrt{3})^2} = -1 \dots (i)$

Shifting the origin at $(1, 2)$ without rotating the coordinate axes and denoting the new coordinates with respect to these axes by X and Y, we obtain

$$x = X + 1 \text{ and } y = Y + 2 \dots \text{(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{(\sqrt{6})^2} - \frac{Y^2}{(\sqrt{3})^2} = -1 \dots \text{(iii)}$$

Comparing equation (iii) with standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, we get

$$a^2 = (\sqrt{6})^2 \text{ and } b^2 = (\sqrt{3})^2$$

$$a = (\sqrt{6}) \text{ and } b = (\sqrt{3})$$

Centre:

The coordinates of the centre with respect to the new axes are (X = 0, Y = 0).

So, the coordinates of the centre with respect to the old axes are

(1, 2) [Putting X = 0, Y = 0 in (ii)]

Lengths of the axes:

Since the transverse axis of the hyperbola is along new T-axis.

∴ Transverse axis = 2b = 2√3 and, Conjugate axis = 2a = 2√6.

Eccentricity:

$$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{6}{3}} = \sqrt{3}$$

Latusrectum:

$$\text{Length of the latus rectum} = \frac{2a^2}{b} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

Foci:

The coordinates of foci with respect to the new axes are (X = 0, Y = ± be) i.e. (X = 0, y = ± 3). So, the coordinates of foci with respect to the old axes are

(1, 2 ± 3) i.e. (1, 5) and (1, -1) [Putting X = 0, y = ± 3 in (ii)]

Vertices:

The coordinates of the vertices with respect to the new axes are X = 0, Y = ± b) i.e. (x = 0, y = ±√3)

So, the coordinates of the vertices with respect to the old axes are

(1, 2 ±√3) i.e. (1, 2 + √3) and (1, 2 - √3) [Putting X = 0, Y = ±√3 in (ii)]

Directrices:

The equations of the directrices with respect to the new axes are Y = ± $\frac{b}{e}$ i.e. y = ± 1.

So, the equations of the directrices with respect to the old axes are

y = 2 ± 1 i.e. y = 1 and y = 3 [Putting Y = ± 2 in (ii)]

35. We have to show that $\frac{dy}{dx} = (\sec x \tan x + \sec x)$

where, it is given that

$$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$$

$$y = \sqrt{\frac{1 - \sin x}{\cos x}} + \frac{\sin x}{\cos x} = \sqrt{\frac{1 - \sin x}{1 + \sin x}}$$

$$u = 1 - \sin x, v = 1 + \sin x, x = \frac{1 - \sin x}{1 + \sin x}$$

$$\text{if } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 + \sin x) \times (-\cos x) - (1 - \sin x) \times (\cos x)}{(1 + \sin x)^2}$$

$$= \frac{-\cos x - \sin x \cos x - \cos x + \sin x \cos x}{(1 + \sin x)^2}$$

$$= \frac{-2 \cos x}{(1 + \sin x)^2}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[-\frac{\cos x}{1} \times \left(\frac{1 - \sin x}{1} \right)^{-\frac{1}{2}} \right] \times \left[\frac{1}{(1 + \sin x)^{2 - \frac{1}{2}}} \right]$$

$$= \left[\cos x \times (1 + \sin x)^{-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left(\frac{1 + \sin x}{1 + \sin x} \right)^{\frac{3}{2}}$$

Multiplying and dividing by $(1 + \sin x)^{\frac{3}{2}}$

$$\begin{aligned}
&= \left[\cos x \times (1 + \sin x)^{\frac{2}{2} - \frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{2}{2}} \times \left(\frac{1}{1 + \sin x} \right)^{\frac{3}{2}} \\
&= \left[\cos x \times (1 + \sin x)^{\frac{2}{2} - \frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{2}{2}} \times (1 + \sin x)^{-\frac{3}{2}} \\
&= [\cos x \times (1 + \sin x)^1] \times (1 - \sin^2 x)^{-\frac{3}{2}} \\
&= [\cos x \times (1 + \sin x)^1] \times (\cos^2 x)^{-\frac{3}{2}} \\
&= [\cos x \times (1 + \sin x)^1] \times (\cos x)^{-3} \\
&= [(1 + \sin x)^1] \times (\cos x)^{-3+1} \\
&= \frac{1 + \sin x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} \times \frac{1 + \sin x}{\cos^2 x} \\
&= \sec x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x} \right) \\
&= \sec x (\sec x + \tan x)
\end{aligned}$$

Hence proved

Section E

36. Read the text carefully and answer the questions:

A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant.

Rahul being a plant lover decides to open a nursery and he bought few plants with pots. He wants to place pots in such a way that number of pots in first row is 2, in second row is 4 and in third row is 8 and so on ...



- (i) (d) 8

Explanation: 8

- (ii) (a) 2046

Explanation: 2046

- (iii) (b) 96

Explanation: 96

OR

- (d) 256

Explanation: 256

37. Read the text carefully and answer the questions:

A state cricket authority has to choose a team of 11 members, to do it so the authority asks 2 coaches of a government academy to select the team members that have experience as well as the best performers in last 15 matches. They can make up a team of 11 cricketers amongst 15 possible candidates. In how many ways can the final eleven be selected from 15 cricket players if:



- (i) (b) 364

Explanation: 364

- (ii) (a) ${}^2C_1 \times {}^{13}C_{10}$

Explanation: ${}^2C_1 \times {}^{13}C_{10}$

(iii) (c) ${}^6C_2 \times {}^3C_1 \times {}^{11}C_5$

Explanation: ${}^6C_2 \times {}^3C_1 \times {}^{11}C_5$

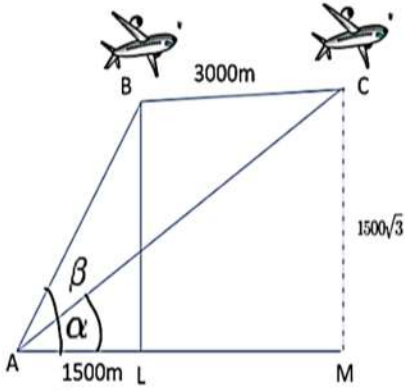
OR

(c) 1365

Explanation: 1365

38. Read the text carefully and answer the questions:

The angle of elevation of an airplane from a point A on the ground is α . After a flight of 15 seconds, the angle of elevation is β . The airplane is flying at a constant height of $1500\sqrt{3}$ m. Distance between two positions of plane is 3000m. Again, distance AL = 1500m.



(i) From $\triangle ABL$, $AB^2 = AL^2 + BL^2$

$$\Rightarrow AB^2 = (1500)^2 + (1500\sqrt{3})^2 = 4 \times (1500)^2$$

$$\Rightarrow AB = 2 \times 1500 = 3000 \text{ m}$$

$$\sin \beta = \frac{BL}{AB} = \frac{1500\sqrt{3}}{3000} = \frac{\sqrt{3}}{2}$$

(ii) From $\triangle ACM$, $AC^2 = AM^2 + CM^2$

$$\Rightarrow AC^2 = (4500)^2 + (1500\sqrt{3})^2 = 4500 \times 4500 + 1500 \times 4500$$

$$\Rightarrow AC^2 = 100^2(45)(45 + 15) = 100^2(9 \times 5)(4 \times 3 \times 5)$$

$$\Rightarrow AC = \sqrt{100^2 \times 2^2 \times 3^2 \times 5^2 \times 3} = 100 \times 2 \times 3 \times 5\sqrt{3}$$

$$\Rightarrow AC = 3000\sqrt{3}$$

$$\sin \alpha = \frac{CM}{AC} = \frac{1500\sqrt{3}}{3000\sqrt{3}} = \frac{1}{2}$$