CBSE SAMPLE PAPER - 09

Class 11 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. cos 20° cos 40° cos 80° = ? **[1]**

a) $\frac{1}{16}$ b) $\frac{1}{8}$

c) $\frac{\sqrt{3}}{8}$ d) $\frac{\sqrt{3}}{16}$

2. The arithmetic mean of numbers a, b, c, d and e is M. What is the value of (a - M) + (b - M) + (c - M) + (d - M) + (e - M)? **[1]**

a) 5 M b) M

c) a + b + c + d + e d) 0

3. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is: **[1]**

a) $\frac{2}{5}$ b) $\frac{2}{3}$

c) $\frac{8}{17}$ d) $\frac{4}{17}$

4. $\lim\limits_{x \to 3} \frac{\sqrt{x^2+10}-\sqrt{19}}{x-3}$ is equal to **[1]**

a) 1 b) $\frac{6}{\sqrt{19}}$

c) $\frac{3}{\sqrt{19}}$ d) 0

5. The centroid of a triangle is (2, 7) and two of its vertices are (4, 8) and (-2, 6). The third vertex is **[1]**

a) (0, 0) b) (4, 7)

c) (7, 7) d) (7, 4)

6. If n(U) = 700, n(A) = 200, n(B) = 300 and n(A ∩ B) = 100, then n (A' ∩ B') is **[1]**

a) 600
b) 300
c) 200
d) 400

7. If $z = \left(\frac{1+i}{1-i}\right)$, then z^4 equals. **[1]**

a) 0
b) - 1
c) None of these
d) 1

8. If R is a relation on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} given by x R y ⇔ y = 3 x, then R = **[1]**

a) {(3, 1), (6, 2), (8, 2),(9, 3)}
b) {(3, 1), (2, 6), (3, 9)}
c) None of these
d) {(3, 1), (6, 2), (9, 3)}

9. Solution of a linear inequality in variable x is represented on the number line as follow: **[1]**

a) $x \in \left(\frac{7}{2}, \infty\right)$
b) $x \in \left(-\infty, \frac{7}{2}\right)$
c) $x \in \left(-\infty, \frac{7}{2}\right]$
d) $x \in \left(\frac{7}{2}, -\infty\right)$

10. The length of a pendulum is 60 cm. The angle through which it swings when its tip describes an arc of length 16.5 cm is **[1]**

a) 16° 15'
b) 15° 30'
c) 16° 30'
d) 15° 45'

11. If ${}^{20}C_r = {}^{20}C_{r-10}$, then ${}^{18}C_r$ is equal to **[1]**

a) 4896
b) 816
c) 1632
d) None of these

12. If a, b, c are in G.P. and log a - log 2b, log 2b - log 3c and log 3c - log a are in A.P., then a, b, c are the lengths of the sides of a triangle which is: **[1]**

a) acute angled
b) right angled
c) equilateral
d) obtuse angled

13. $\{C_0 + 3C_1 + 5C_2 + ...+ (2n + 1)C_n\} = ?$ **[1]**

a) None of these
b) (n - 1) (n + 2)
c) $(n + 2) . 2^{n-1}$
d) $(n + 1)2^n$

14. Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23. **[1]**

a) (3, 5), (5, 7), (7, 9)
b) (6, 8),(8, 10), (10, 12)
c) none of these
d) (4, 6), (6, 8), (8, 10)

15. The smallest set A such that A ∪ {1, 2} = {1, 2, 3, 5, 9} is **[1]**

a) {1, 2, 5, 9}

b) {4, 5, 6}

c) {3, 5, 9}

d) {2, 3, 5}

16. If tan x = $\frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$, then tan $\frac{x}{2}$ = ? **[1]**

a) 3

b) $\frac{-1}{3}$

c) $\frac{1}{3}$

d) -3

17. $(z+1)(\bar{z}+1)$ can be expressed as **[1]**

a) $|z|^2 + 1$

b) $|z|^2 + 2$

c) none of these

d) $|z + 1|^2$

18. If ${}^{(a^2-a)}C_2 = {}^{(a^2-a)}C_4$, then a = **[1]**

a) None of these

b) 2

c) 3

d) 4

19. If a, b $\in$ R and x $\in$ N, then **[1]**

$(a + b)^n = {}^nc_0a^n + {}^nc_1a^{n-1}b + {}^nc_2a^{n-2}b^2 + {}^nc_nb^n$

Assertion (A): The no of terms in binomial expansion is (n + 1)

Reason (R): Sum of indices of a and b in each term is (n + 1)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If A = {1, 2, 3}, B = {2, 4}, then the number of relation from A to B is equal to 64. **[1]**

Reason (R): The total number of relation from set A to set B is equal to $\{2^{n(A) \cdot n(B)}\}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. If y = f(x) = $\frac{ax-b}{bx-a}$ and $a^2 \neq b^2$ then show that x = f(y). **[2]**

22. Evaluate: $\lim_{x \to a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{x-a}$. **[2]**

23. Find the equation of the parabola whose: focus is (1, 1) and the directrix is x + y + 1 = 0. **[2]**

OR

Find the equation of hyperbola having Foci (0, $\pm$13) and the conjugate axis is of length 24.

24. For any two sets of A and B, prove that: B' $\subset$ A' $\Rightarrow$ A $\subset$ B. **[2]**

25. Find the inclination of the line $x + \sqrt{3}y + 6 = 0$. **[2]**

Section C

26. Let f : R $\rightarrow$ R be the function defined by $f(x) = \frac{1}{2-\cos x}$, $\forall x \in R$. Then, find the range of f. **[3]**

27. Find the length of the medians of the triangle with vertices A(0, 0, 6), B (0, 4, 0) and C (6, 0, 0). **[3]**

OR

Find the equation of the curve formed by the set of all points which are equidistant from the points A (-1, 2, 3) and B(3, 2, 1).

28. Expand the given expression $\left(x+\frac{1}{x}\right)^6$ **[3]**

OR

Using binomial theorem, expand: $(\sqrt[3]{x}-\sqrt[3]{y})^6$

29. $Express\ \frac{2-\sqrt{-25}}{1-\sqrt{-16}}\ in\ standard\ form$ **[3]**

OR

If $z_1 = 3 + i$ and $z_2 = 1 + 4i$, then verify that $|z_1 - z_2| > |z_1| - |z_2|$.

30. Solve the following system of inequations: **[3]**

$\frac{5x}{4}+\frac{3x}{8}>\frac{39}{8}$

$\frac{2x-1}{12}-\frac{x-1}{3}<\frac{3x+1}{4}$

31. Prove that : ${}^{2n}C_n = \frac{2^n[1\cdot3\cdot5.....(2n-1)]}{n!}$. **[3]**

Section D

32. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that: **[5]**

i. one is red and two are white

ii. two are blue and one is red

iii. one is red.

33. i. Find the derivative of $\frac{\sin x+\cos x}{\sin x-\cos x}$. **[5]**

ii. Let $f(x)=\begin{cases}x^2-1, & 0<x<2\\ 2x+3, & 2\le x<3\end{cases}$, find quadratic equation whose roots are $\lim\limits_{x\to2^-} f(x)$ and $\lim\limits_{x\to2^+} f(x)$.

OR

Evaluate the following limits: $\lim\limits_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-(\sqrt{3}+\sqrt{2})}{x^2-6}$.

34. Prove that: $\sin 20^o \sin 40^o \sin 80^o = \frac{\sqrt{3}}{8}$ **[5]**

OR

If $A + B + C = \pi$, prove that $\sin^2 A - \sin^2 B + \sin^2 C = 2\sin A \cos B \sin C$

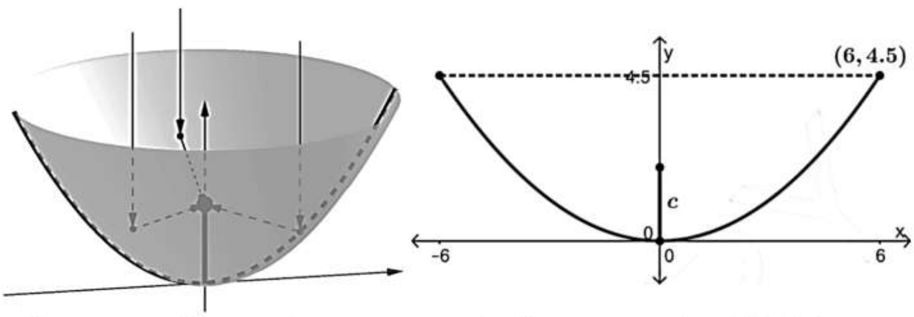
35. Calculate the mean and standard deviation for the following data: **[5]**

Wages upto (₹)	15	30	45	60	75	90	105	120
No. of workers	12	30	65	107	157	202	222	230

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across, and 4.5 ft deep at the vertex.



(i) Name the type of curve given in the above paragraph and find the equation of curve?

(ii) Find the equation of parabola whose vertex is (3, 4) and focus is (5, 4).

(iii) Find the equation of parabola Vertex (0, 0) passing through (2, 3) and axis is along x-axis. and also find the length of latus rectum.

OR

Find focus, length of latus rectum and equation of directrix of the parabola $x^2 = 8y$.

37. **Read the text carefully and answer the questions:** **[4]**

A company produces 500 computers in the third year and 600 computers in the seventh year. Assuming that the production increases uniformly by a constant number every year.

(i) The difference in number of computers produced in 10th year and 8th year is

a) 50 b) 25

c) 75 d) 100

(ii) The number of computers produced in 21 st year is

a) 650 b) 850

c) 700 d) 950

(iii) The total production in 10 years is

a) 5265 b) 5625

c) 2655 d) 6525

OR

The production in first year is

a) 250 b) 300

c) 400 d) 450

38. **Read the text carefully and answer the questions:** **[4]**

The school organised a farwell party for 100 students and school management decided three types of drinks will be distributed in farewell party i.e., Milk (M), Coffee (C) and Tea (T).

Organiser reported that 10 students had all three drinks M, C, T. 20 students had M and C; 30 students and C and T; 25 students had M and T. 12 students had M only; 5 students had C only; 8 students had T only.

(i) Find the number of students who prefer Milk and Coffee but not tea?

(ii) Find the number of students who prefer Tea.

Solution

CBSE SAMPLE PAPER - 09

Class 11 - Mathematics

Section A

1. **(b)** $\frac{1}{8}$

   Explanation: Given exp. $= \frac{1}{2}$ (2 cos 20° cos 80°) cos 40°

   $= \frac{1}{2}$ [cos (80° + 20°)+ cos(80° - 20°)] cos 40°

   $= \frac{1}{2}$ [(cos 100° + cos 60°) cos 40°] $= \frac{1}{2}\left[\left(\cos 100^{\circ}+\frac{1}{2}\right)\cos 40^{\circ}\right]$

   $= \frac{1}{4}$ (2 cos 100° cos 40°) + $\frac{1}{4}$ cos 40°

   $= \frac{1}{4}$ cos (100° + 40°) + cos (100° - 40°)] + $\frac{1}{4}$ cos 40°

   $= \frac{1}{4}$ cos 140° + cos 60°) + $\frac{1}{4}$ cos 40° = $\frac{1}{4}$ (cos 140° + cos 40°) $+ \left(\frac{1}{4}\times\frac{1}{2}\right)$

   $= \frac{1}{4}$ [cos(180° - 40°) + cos 40°] $+\frac{1}{8} = \frac{1}{4}$(-cos 40°+ cos 40°) $+\frac{1}{8} = \frac{1}{8}$.

2. **(d)** 0

   Explanation: Given, $\frac{a+b+c+d+e}{5}$ = M

   $\Rightarrow$ a + b + c + d + e = 5 M

   $\therefore$ (a - M) + (b - M) + (c - M) + (d - M) + (e - M)

   = (a + b + c + d + e) - 5M = 5M - 5M = 0

3. **(c)** $\frac{8}{17}$

   Explanation: Let B_1 and B_2 be the boxes and N be the number of the non-prime number.

   $\because$ $P(B_1) = P(B_2) = \frac{1}{2}$

   and P (non-prime number)

   $= P(B_1) \times P\left(\frac{N}{B_1}\right) + P(B_2) \times P\left(\frac{N}{\bar{B}_2}\right) = \frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}$

   So,

   $P\left(\frac{B_1}{N}\right) = \frac{P(B_1)\times P\left(\frac{N}{B_1}\right)}{P(B_1)\times P\left(\frac{N}{B_1}\right)+P(B_2)\times P\left(\frac{N}{B_2}\right)}$

   $= \frac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{15}{40}} = \frac{8}{17}$

4. **(c)** $\frac{3}{\sqrt{19}}$

   Explanation: Using L'Hospital,

   $\lim_{x\to 3} \frac{\frac{2x}{2\sqrt{x^2+10}}}{1}$

   Substituting x = 3 in $\frac{\frac{2x}{2\sqrt{x^2+10}}}{1}$

   We get $\frac{3}{\sqrt{19}}$

5. **(b)** (4, 7)

   Explanation: Let A (4, 78) and B (-2, 6) be the given vertex. Let C(h, k) be the third vertex.

   The centroid of $\triangle$ ABC is $\left(\frac{4-2+h}{3}, \frac{8+6+k}{3}\right)$

   It is given that the centroid of triangle ABC is (2, 7) as obtained from above formula,

   $\therefore \frac{4-2+h}{3} = 2$, $\frac{8+6+k}{3} = 7$

   $\Rightarrow$ h = 4, k = 7

   Thus, the third vertex is (4, 7)

6. **(b)** 300

   Explanation: n(A $\cup$ B) = n(A) + n(B) - n(A $\cap$ B)

   = 200 + 300 - 100 = 400

n(A' ∩ B') = n(U) - n(A ∪ B)

= 700 - 400 = 300

7. **(d)** 1

Explanation: 1

Let z = $\frac{1+i}{1-i}$

$z=\frac{1+i}{1-i}\times\frac{1+i}{1+i}$

$\Rightarrow z=\frac{1+i^2+2i}{1-i^2}$

$\Rightarrow z=\frac{2i}{2}$

⇒ z = i

⇒ $z^4 = i^4$

Since i2 = -1, we have:

$\Rightarrow z^4=i^2\times i^2$

⇒ $z^4 = 1$

8. **(c)** None of these

Explanation: Here, y = 3x;

If x = 1; then y = 3.

If x = 2; then y = 6.

If x = 3; then y = 9.

Therefore the required relation will be R = {(1, 3), (2, 6), (3, 9)}.

9. **(b)** $x\in\left(-\infty,\frac{7}{2}\right)$

Explanation: The given graph represents all the values of x less than $\frac{7}{2}$ on a real number line.

So, $x\in\left(-\infty,\frac{7}{2}\right)$

10. **(d)** 15° 45'

Explanation: Here, r = 60 cm and l = 16.5 cm.

$\therefore\theta=\frac{l}{r}=\left(\frac{16.5}{60}\right)^c=\left(\frac{16.5}{60}\times\frac{180}{\pi}\right)^\circ=\left(\frac{16.5}{60}\times180\times\frac{7}{22}\right)^\circ=\left(\frac{63}{4}\right)^\circ$ = 15° 45'

11. **(b)** 816

Explanation: r + r - 10 = 20 [$\because{}^nC_x={}^nC_y\Rightarrow$ n = x + y or x = y]

⇒ 2r - 10 = 20

⇒ 2r = 30

⇒ r = 15

Now,

${}^{18}C_r={}^{18}C_{15}$

$\therefore{}^{18}C_{15}={}^{18}C_3$

$\therefore{}^{18}C_3$ = $\frac{18}{3}\times\frac{17}{2}\times16$ = 816.

12. **(d)** obtuse angled

Explanation: According to the given conditions,

b^2 = ac and 2(log2b - log3c) = loga - log2b + log3c - loga

⇒ 2 log $\left(\frac{2b}{3c}\right)$ = log $\left(\frac{3c}{2b}\right)$

⇒ log$\left(\frac{2b}{3c}\right)^2$ = log $\left(\frac{3c}{2b}\right)$

$\Rightarrow\left(\frac{b}{c}\right)^3=\left(\frac{3}{2}\right)^3$

$\Rightarrow\frac{b}{c}=\frac{3}{2}$

Now, b^2 = ac and 2b = 3c

⇒ b = $\frac{2a}{3}$ and c = $\frac{4a}{9}$

Since, a + b = $\frac{5a}{3}$ > c, b + c = $\frac{10a}{9}$ > a, c + a = $\frac{13a}{9}$ > b

It implies that a, b, c form a triangle with a as the greatest side.

Let us find the greatest angle A of △ABC by using the cosine formula.

cos A = $\frac{b^2+c^2-a^2}{2bc}$

$= -\frac{29}{48} < 0$

$\Rightarrow$ The angle A is obtuse.

13. **(d)** $(n + 1)2^n$

Explanation: We have, $C_0 + 3C_1 + 5C_2 + ...+ (2n + 1)C_n$

$= (C_0 + C_1 + C_2 + ... + C_n) + 2(C_1 + 2C_2 + ... + nC_n)$

$= 2^n + 2(n \cdot 2^{n-1}) = (n + 1) \cdot 2^n$

14. **(b)** (6, 8),(8, 10), (10, 12)

Explanation: Let the consecutive even positive integers be x and x + 2.

By data, x > 5 and x + (x + 2) < 23

Now x + (x + 2) < 23

$\Rightarrow$ 2x + 2 < 23

$\Rightarrow$ 2x < 21

$\Rightarrow x < \frac{21}{2} = 10\frac{1}{2}$

So we have the least possible value of x is 6 and the maximum value of x is 10.

Therefore the possible pairs of consecutive even positive integers are (6, 8), (8, 10), (10, 12).

15. **(c)** {3, 5, 9}

Explanation: The union of two sets A and B is the set of elements in A, or B, or both.

So smallest set A = {3, 5, 9}

16. **(d)** -3

Explanation: Since x lies in quadrant III, we have : cos x < 0

Now, tan x = $\frac{3}{4} \Rightarrow \sec^2 x = (1 + \tan^2 x) = \left(1 + \frac{9}{16}\right) = \frac{25}{16}$

$\Rightarrow \cos^2 x = \frac{16}{25} \Rightarrow \cos x = -\sqrt{\frac{16}{25}} = \frac{-4}{5}$

Also, $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \frac{x}{2}$ lies in quadrant II

$\Rightarrow \sin\frac{x}{2} > 0$ and $\cos\frac{x}{2} < 0$

$2\sin^2\frac{x}{2}$ = (1 - cos x) $= \left(1 + \frac{4}{5}\right) = \frac{9}{5} \Rightarrow \sin^2\frac{x}{2} = \frac{9}{10}$

$\therefore \sin\frac{x}{2} = +\sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \quad \left[\because \sin\frac{x}{2} > 0\right]$

$2\sin^2\frac{x}{2}$ = (1 - cos x) $= \left(1 - \frac{4}{5}\right) = \frac{1}{5} \Rightarrow \cos^2\frac{x}{2} = \frac{1}{10}$

$\therefore \cos\frac{x}{2} = -\sqrt{\frac{1}{10}} = \frac{-1}{\sqrt{10}}$

$\therefore \tan\frac{x}{2} = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{-1}$ = -3.

17. **(d)** $|z + 1|^2$

Explanation: We have $z\bar{z} = |z|^2$

Now $(z+1)(\bar{z}+1) = (z+1)\left(\overline{z+1}\right) = |z + 1|^2$

18. **(c)** 3

Explanation: $a^2 - a = 2 + 4$ [$\because {}^nC_x = {}^nC_y \Rightarrow$ n = x + y or x = y]

$\Rightarrow a^2 - a - 6 = 0$

$\Rightarrow a^2 - 3a + 2a - 6 = 0$

$\Rightarrow$ a(a - 3) + 2(a - 3) = 0

$\Rightarrow$ (a + 2)(a - 3) = 0

$\Rightarrow$ a = -2 or a = 3

But,

a = -2 is not possible.

$\therefore$ a = 3.

19. **(c)** A is true but R is false.

Explanation: Assertion is true

∵ It is one of the observation of binomial expansion.

Reason:

Not true. AS sum of indices of a and b in each term is n.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: We know by the property of relation, the total number of relation from set A to set B is $2^{n(A) \cdot n(B)}$.

$2^{3 \times 2} = 64$

Section B

21. Here we are given that, y = f(x) = $\frac{ax-b}{bx-a}$

$\therefore$ f(y) = f{f(x)} = $f\left(\frac{ax-b}{ax-a}\right) = \frac{\left\{a\left(\frac{ax-b}{bx-a}\right)-b\right\}}{\left\{b\left(\frac{ax-b}{bx-a}\right)-a\right\}}$

$= \frac{\left\{\left(a^2x-ab\right)-\left(b^2x-ab\right)\right\}}{(bx-a)} \times \frac{(bx-a)}{\left(\left(abx-b^2\right)-\left(abx-a^2\right)\right\}}$

$= \frac{\left(a^2x-b^2x\right)}{\left(a^2-b^2\right)} = \frac{\left(a^2-b^2\right)x}{\left(a^2-b^2\right)} = x$

Hence, x = f(y)

22. $\lim\limits_{x \to a} \frac{(x+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{x-a}$

$= \lim\limits_{(x+2)\to(a+2)} \frac{(x+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{(x+2)-(a+2)}$ [I as x → a ∴ x + 2 → a + 2]

$= \frac{3}{2}(a+2)^{\frac{3}{2}-1} \left[\because \lim\limits_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}\right]$

$= \frac{3}{2}(a+2)^{\frac{1}{2}}$

23. Let P(x, y) be any point on the parabola whose focus is S (1, 1) and the directrix is x + y + 1 = 0

Draw PM perpendicular to x + y + 1 = 0

Thus, we have:

SP = PM

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x - 1)^2 + (y - 1)^2 = \left|\frac{x+y+1}{\sqrt{1+1}}\right|^2$

$\Rightarrow (x - 1)^2 + (y - 1)^2 = \left(\frac{x+y+1}{\sqrt{2}}\right)^2$

$\Rightarrow 2(x^2 + 1 - 2x + y^2 + 1 - 2y) = x^2 + y^2 + 1 + 2xy + 2y + 2x$

$\Rightarrow (2x^2 + 2 - 4x + 2y^2 + 2 - 4y) = x^2 + y^2 + 1 + 2xy + 2y + 2x$

$\Rightarrow x^2 + y^2 - 2xy - 6x - 6y + 3 = 0$,

which is the required equation of parabola.

OR

Here foci are $(0, \pm 13)$ which lie on y-axis.

So the equation of hyperbola in standard form is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\therefore (13)^2 = a^2 + (12)^2$

$\Rightarrow a^2 = 169 – 144 = 25$

Thus required equation of hyperbola is

$\frac{y^2}{25} - \frac{x^2}{(12)^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{144} = 1$

24. We have B’⊂ A’

We have to prove: A ⊂ B

Let, x ∈ A

⇒ x ∉ A’ [∵ A ∩ A’ = ϕ]

⇒ x ∉ B’ [∵ B’ ⊂ A’]

⇒ x ∈ B [∵ B ∩ B’ = ϕ]

Therefore, x ∈ A ⇒ x ∈ B

This is true for all x ∈ A

∴ A ⊂ B.

25. Here,it is given equation is $x+\sqrt{3}y+6=0$

We can rewrite it as $\sqrt{3}y=-x-6$

$\Rightarrow \mathrm{y}=\frac{-1}{\sqrt{3}}\mathrm{x}+\frac{-6}{\sqrt{3}}$

It is in the form of $y=x\times\tan\alpha+c$

Where $\tan\alpha=-\frac{1}{\sqrt{3}}$ and $c=-\frac{6}{\sqrt{3}}$

The inclination of the line is α

Therefore $\alpha=\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$=\frac{5\pi}{6}$

Section C

26. Given function, $f(x)=\frac{1}{2-\cos x},\ \forall x\in R$

Let $y=\frac{1}{2-\cos x}$

⇒ 2y - y cosx = 1

⇒ y cosx = 2y - 1

$\Rightarrow \cos x=\frac{2y-1}{y}=2-\frac{1}{y}\Rightarrow \cos x=2-\frac{1}{y}$

$\Rightarrow -1\leqslant\cos x\leqslant 1\Rightarrow -1\leqslant 2-\frac{1}{y}\leqslant 1$

$\Rightarrow -3\leqslant-\frac{1}{y}\leqslant-1$

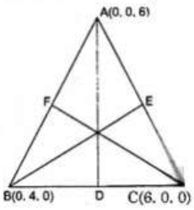
$\Rightarrow 3\geqslant\frac{1}{y}\geqslant 1$

$\Rightarrow \frac{1}{3}\leqslant y\leqslant 1$

So, Range of y, that is f(x) is $\left[\frac{1}{3},1\right]$

27. Here A(0, 0, 6), B (0, 4, 0) and C (6, 0, 0) are vertices of $\Delta\mathrm{AB}C$

Now D is mid point of BC



∴ Coordinates of D is $\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0+0}{2}\right)$

= (3 ,2 , 0)

$\therefore AD=\sqrt{(0-3)^2+(0-2)^2+(6-0)^2}$

$=\sqrt{9+4+36}=\ 7$ units

Also E is mid point of AC

∴ Coordinates of E is $\left(\frac{0+6}{2},\frac{0+0}{2},\frac{0+6}{2}\right)$

= (3, 0, 3)

$\therefore BE=\sqrt{(0-3)^2+(4-0)^2+(0-3)^3}$

$=\sqrt{9+16+9}=\sqrt{34}$ units

Also F is mid point of AB

∴ Coordinates of F is $\left(\frac{0+0}{2},\frac{0+4}{2},\frac{6+0}{2}\right)$

= (0, 2, 3)

$\therefore CF=\sqrt{(6-0)^2+(0-2)^2+(0-3)^2}\ =\sqrt{36+4+9}$ = 7 units

OR

Consider, C(x, y, z) point equidistant from points A(-1, 2, 3) and B(3, 2, 1).

∴ AC = BC

$\sqrt{(x+1)^2+(y-2)^2+(z-3)^2}=\sqrt{(x-3)^2+(y-2)^2+(z-1)^2}$

Squaring both sides,

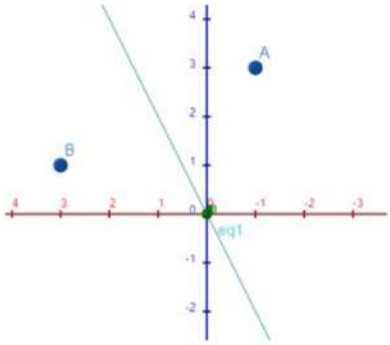
$\Rightarrow (x+1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z-1)^2$

$\Rightarrow x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 - 2z + 1$

$\Rightarrow 8x - 4z = 0$

$\Rightarrow 2x - z = 0$

$\Rightarrow z = 2x$



Equation of curve is z = 2x

28. Using binomial theorem for the expansion of $\left(x + \frac{1}{x}\right)^6$ we have

$\left(x + \frac{1}{x}\right)^6 = = {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3$

$+ {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{6}\right)^6$

$= x^6 + 6 \cdot x^5 \cdot \frac{1}{x} + 15 \cdot 4x^4 \cdot \frac{1}{x^2} + 20 \cdot x^3 \cdot \frac{1}{x^3} + 15 \cdot x^2 \cdot \frac{1}{x^4} + 6 \cdot x \cdot \frac{1}{x^5} + \frac{1}{x^6}$

$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

OR

To find: Expension of $(\sqrt[3]{x} - \sqrt[3]{y})^6$ by means of binomial theorem..

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + nC_nb^n$

We have, $(\sqrt[3]{x} - \sqrt[3]{y})^6$

We can write $\sqrt[3]{x}$, as $x^{\frac{1}{3}}$, and $\sqrt[3]{y}$, as $y^{\frac{1}{3}}$,

Now, we have to solve for $\left(x^{\frac{1}{3}} - y^{\frac{1}{3}}\right)^6$

$\Rightarrow \left[6C_0\left(x^{\frac{1}{3}}\right)^{6-0}\right] + \left[6C_1\left(x^{\frac{1}{3}}\right)^{6-1}\left(-y^{\frac{1}{3}}\right)^1\right] + \left[6C_2\left(x^{\frac{1}{3}}\right)^{6-2}\left(-y^{\frac{1}{3}}\right)^2\right] + \left[6C_3\left(x^{\frac{1}{3}}\right)^{6-3}\left(-y^{\frac{1}{3}}\right)^3\right]$

$+ \left[6C_4\left(x^{\frac{1}{3}}\right)^{6-4}\left(-y^{\frac{1}{3}}\right)^4\right] + \left[6C_5\left(x^{\frac{1}{3}}\right)^{6-5}\left(-y^{\frac{1}{3}}\right)^5\right] + \left[6C_6\left(-y^{\frac{1}{3}}\right)^6\right]$

$\Rightarrow \left[{}^6C_0\left(\frac{6}{x^3}\right)\right] - \left[{}^6C_1\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[6C_2\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - \left[6C_3\left(x^{\frac{3}{3}}\right)\left(y^{\frac{3}{3}}\right)\right]$

$+ \left[{}^6C_4\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right] - \left[6C_5\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[6C_6\left(\frac{6}{y^3}\right)\right]$

$\Rightarrow \left[\frac{6!}{0!(6-0)!}(x^2)\right] - \left[\frac{6!}{1!(6-1)!}\left(x^{\frac{5}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] + \left[\frac{6!}{2!(6-2)!}\left(x^{\frac{4}{2}}\right)\left(x^{\frac{2}{3}}\right)\right]$

$- \left[\frac{6!}{3!(6-3)!}(x)(y)\right] + \left[\frac{6!}{4!(6-4)!}\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{5}}\right)\right] - \left[\frac{6!}{5!(6-5)!}\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[\frac{6!}{6!(6-6)!}(y^2)\right]$

$\Rightarrow \left[1\left(x^2\right)\right] - \left[6\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[15\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - [20(x)(y)] + \left[15\left(x^{\frac{2}{3}}\right)\left(\frac{4}{y^3}\right)\right]$

$- \left[6\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[1\left(y^2\right)\right]$

$\Rightarrow x^2 - 6x^{\frac{5}{2}}y^{\frac{1}{3}} + 15x^{\frac{4}{3}}y^{\frac{2}{3}} - 20xy + 15x^{\frac{2}{3}}y^{\frac{4}{3}} - 6x^{\frac{1}{3}}y^{\frac{5}{3}} + y^2$

Hence the result.

sol) $\frac{2-\sqrt{-25}}{1-\sqrt{-16}} = \frac{2-5i}{1-4i} = \frac{2-5i}{1-4i} \times \frac{1+4i}{1+4i}$

29. $= \frac{(2+20) + i\,(8-5)}{1 - 16i^2}$

$= \frac{22 + 3i}{17} = \frac{22}{17} + \frac{3}{17}i$

OR

Given, $z_1 = 3 + i$ and $z_2 = 1 + 4i$

Now, $z_1 - z_2 = (3 + i) - (1 + 4i) = 2 - 3i$

$\therefore |z_1 - z_2| = |2 - 3i| = \sqrt{(2)^2 + (-3)^2}$

$= \sqrt{4+9} = \sqrt{13} = 3.60$ ...(i)

As $z_1 = 3 + i$

$\Rightarrow |z_1| = \sqrt{3^2 + 1^2} = \sqrt{10}$ and

$z_2 = 1 + 4i$

$\Rightarrow |z_2| = \sqrt{1^2 + 4^2} = \sqrt{17}$

$\therefore |z_1| - |z_2| = \sqrt{10} - \sqrt{17} = 3.16 - 4.12 = -0.96$ ...(ii)

From Eqs. (i) and (ii), we get

$|z_1 - z_2| > |z_1| - |z_2|$

30. The given system of inequation is

$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ ...(i)

$\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$ ...(ii)

Now, $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$

$\Rightarrow \frac{10x+3x}{8} > \frac{39}{8}$

$\Rightarrow 13x > 39$

$\Rightarrow x > 3$

$\Rightarrow x \in (3, \infty)$

So, the solution set of inequation (i) is the interval $(3, \infty)$.

and, $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

$\Rightarrow \frac{(2x-1)-4(x-1)}{12} < \frac{3x+1}{4}$

$\Rightarrow \frac{-2x+3}{12} < \frac{3x+1}{4}$

$\Rightarrow -2x + 3 < 3(3x + 1)$ [Multiplying both sides by 12]

$\Rightarrow -2x + 3 < 9x + 3$

$\Rightarrow -2x - 9x < 3 - 3$

$\Rightarrow -11x < 0$

$\Rightarrow x > 0$

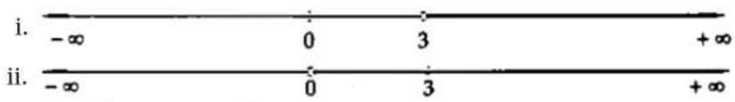
$\Rightarrow x \in (0, \infty)$

So, the solution set of inequation (ii) is the interval $(0, \infty)$.

These solution sets are graphed on the real line in Figure (i) and (ii) respectively.

From Fig. (i) and (ii), we observe that the intersection of the solution sets of inequations (i) and (ii) is the interval $(3, \infty)$ represented by the common thick line.

Hence, the solution set of the given system of inequations is the interval $(3, \infty)$.



31. $^{2n}C_n = \frac{2^n[1\cdot35.....(2n-1)]}{n!}$

$= \frac{(2n)!}{n!n!}$

$= \frac{\frac{(2n)(2n-1)(2n-2)(2n-3).....}{4\cdot3\cdot2\cdot1}}{n!n!}$

$= \frac{[2^n....4\cdot2][(2n-1)....3\cdot1]}{n!n!}$

$= \frac{2^n[1\cdot2\cdot....n][1\cdot3\cdot5.....\cdot(2n-1)]}{n!n!}$

$= \frac{2^n\times n![1\cdot3\cdot5....\cdot(2n-1)]}{n!n!}$

$= \frac{2^n[1\cdot3\cdot5....\cdot(2n-1)]}{n!}$

Section D

32. Bag contains:

6 -Red balls

4 -White balls

8 -Blue balls

Since three ball are drawn,

$\therefore n(S) = {}^{18}C_3$

i. Let E be the event that one red and two white balls are drawn.

$\therefore n(E) = {}^6C_1 \times {}^4C_2$

$\therefore P(E) = \frac{{}^6C_1 \times {}^4C_2}{{}^{18}C_3} = \frac{6\times4\times3}{2} \times \frac{3\times2}{18\times17\times16}$

$P(E) = \frac{3}{68}$

ii. Let E be the event that two blue balls and one red ball was drawn.

$\therefore n(E) = {}^8C_2 \times {}^6C_1$

$\therefore P(E) = \frac{{}^8C_2 \times {}^6C_1}{{}^{18}C_3} = \frac{8\times7}{2} \times 6 \times \frac{3\times2\times1}{18\times17\times16} = \frac{7}{34}$

$P(E) = \frac{7}{34}$

iii. Let E be the event that one of the ball must be red.

$\therefore$ E = {(R,W,B) or (R,W,W) or (R,B,B)}

$\therefore n(E) = {}^6C_1 \times {}^4C_1 \times {}^8C_1 + {}^6C_1 \times {}^4C_2 + {}^6C_1 \times {}^8C_2$

$\therefore P(E) = \frac{{}^6C_1\times{}^4C_1\times{}^8C_1+{}^6C_1\times{}^4C_2+{}^6C_1\times{}^8C_2}{{}^{18}C_3} = \frac{6\times4\times8+\frac{6\times4\times3}{2\times1}+\frac{6\times8\times7}{2\times1}}{\frac{18\times17\times16}{3\times2\times1}}$

$= \frac{396}{816} = \frac{33}{68}$

33. i. Let $y = \frac{\sin x+\cos x}{\sin x-\cos x}$

On differentiating both sides of y w.r.t. x, we get

$\frac{dy}{dx} = \frac{\left[(\sin x+\cos x)\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\frac{d}{dx}(\sin x-\cos x)\right]}{(\sin x-\cos x)^2}$

[by quotient rule of derivative]

$= \frac{[\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(cosx+sin]}{(\sin x-\cos x)^2}$

$= \frac{-(\cos x-\sin x)(\cos x-\sin x)-(\cos x+\sin x)^2}{(\sin x-\cos x)^2}$

$= \frac{-(\cos x-\sin x)^2-(\cos x+\sin x)^2}{(\sin x-\cos x)^2}$

$= \frac{[-(\cos^2 x+\sin^2 x-2\cos x\sin x)+(\cos^2 x+\sin^2 x+2\cos x\sin x)]}{(\sin x-\cos x)^2}$

$= \frac{-[1+1]}{(\sin x-\cos x)^2} = \frac{-2}{(\sin x-\cos x)^2}$

ii. Given, $f(x) = \begin{cases} x^2-1, & 0<x<2 \\ 2x+3, & 2\le x<3 \end{cases}$

At x = 2,

RHL = $\lim_{x\to2^+} f(x)$

$= \lim_{h\to0} f(2+h)$

$= \lim_{h\to0}$ 2(2 + h) + 3

= 2(2 + 0) + 3

= 4 + 3 = 7 = α [say]

[$\because$ f(x) = 2x + 3]

LHL = $\lim_{x\to2^-} f(x) = \lim_{h\to0}$ f(2 - h)

$= \lim_{h\to0}$ $(2 - h)^2 - 1 = (2 - 0)^2 - 1$

= 4 - 1 = 3 = β [say] [$\therefore$ f(x) = x^2 - 1]

If a quadratic euation has root α and β, then the equation is

x^2 - (Sum of roots) x + Product of roots = 0

i.e., $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

i.e., x^2 - (7 + 3)x + 7 $\times$ 3 = 0

$\Rightarrow x^2 - 10x + 21 = 0$

OR

We have to find the value $\lim\limits_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-(\sqrt{3}+\sqrt{2})}{x^2-6}$

Re-writing the equation as

$=\lim\limits_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\sqrt{(\sqrt{3}+\sqrt{2})^2}}{x^2-6}$

$=\lim\limits_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\sqrt{3+2+2\sqrt{6}}}{x^2-6}$

$=\lim\limits_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\sqrt{5+2\sqrt{6}}}{x^2-6}$

Now rationalizing the above equation

$=\lim\limits_{x\to\sqrt{6}}\frac{(\sqrt{5+2x}-\sqrt{5+2\sqrt{6}})}{x^2-6}\frac{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$

Formula: (a + b) (a - b) = $a^2 - b^2$

$=\lim\limits_{x\to\sqrt{6}}\frac{(5+2x-(5+2\sqrt{6}))}{x^2-6}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$

$=\lim\limits_{x\to\sqrt{6}}\frac{(2x-2\sqrt{6})}{x^2-6}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$

$=\lim\limits_{x\to\sqrt{6}}\frac{2(x-\sqrt{6})}{(x+\sqrt{6})(x-\sqrt{6})}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$

$=\lim\limits_{x\to\sqrt{6}}\frac{2(1)}{(x+\sqrt{6})(1)}\frac{(1)}{(\sqrt{5+2x}+\sqrt{5+2\sqrt{6}})}$

$=\frac{2}{2\sqrt{6}}\frac{1}{(2\sqrt{5+2\sqrt{6}})}$

$=\frac{1}{2\sqrt{6}}\frac{1}{(\sqrt{5+2\sqrt{6}})}$

34. Given, LHS $= sin20^o sin40^o sin80^o$

$=\frac{1}{2}$ [2 sin 20° · sin 40°] sin 80^0 [multiplying and dividing by 2]

$=\frac{1}{2}[cos(20^o-40^o)-cos(20^o+40^o)]\cdot\sin 80^\circ$ [$\because 2\sin x\cdot\sin y$ = cos (x - y) - cos (x + y)]

$=\frac{1}{2}[cos(-20^o)-cos60^o]sin80^o$

$=\frac{1}{2}$ [cos 20^0 - $\frac{1}{2}$]· sin 80° [$\because$ cos (- θ) = cos θ and cos 60^0 = $\frac{1}{2}$]

$=\frac{1}{2}\times\frac{1}{2}\left[2\left(\cos 20^\circ-\frac{1}{2}\right)\cdot\sin 80^\circ\right]$ [again multiplying and dividing by 2]

$=\frac{1}{4}[2\cos 20^\circ\cdot\sin 80^\circ-\sin 80^\circ]$

$=\frac{1}{4}[sin(20^o+80^o)-sin(20^o-80^o)-sin80^o]$ [$\because 2\cos x\cdot\sin y=sin(x+y)-sin(x-y)$]

$=\frac{1}{4}[sin100^o-sin(-60^o)-sin80^o]$

$=\frac{1}{4}$ [sin 100^0 + sin 60^0 - sin 80^0] [$\because$ sin (- θ) = - sin θ]

$=\frac{1}{4}$ [sin (180^0 - 80^0) + sin 60^0 - sin 80^0] [$\because$ sin 100^0 = sin (180^0 - 80^0)]

$=\frac{1}{4}$ [sin 80^0 + sin 60^0 - sin 80^0] [$\because$ sin ($\pi-\theta$) = sin θ]

$=\frac{1}{4}\times$ sin 60^0 = $\frac{1}{4}\times\frac{\sqrt{3}}{2}$ [$\because$ sin 60^0 = $\frac{\sqrt{3}}{2}$]

$=\frac{\sqrt{3}}{8}$ = RHS

Hence proved.

OR

Here it is given that, A + B + C = π

and we need to prove that $\sin^2$A - $\sin^2$B + $\sin^2$C = 2sin A cos B sin C

Taking LHS, we have

L.H.S = $\sin^2$ A – $\sin^2$ B + $\sin^2$ C

Using formula ,

$\frac{1-\cos 2A}{2}$ = $\sin^2$A, we have

L.H.S = $\frac{1-\cos 2A}{2}$ - $\frac{1-\cos 2B}{2}$ + $\frac{1-\cos 2C}{2}$

$=\frac{1-\cos 2A-1+\cos 2B+1-\cos 2C}{2}$

$=\frac{1-\cos 2A+\cos 2B-\cos 2C}{2}$

Using, $\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

L.H.S = $\frac{1-\cos 2A+\left\{2\sin\left(\frac{2B+2C}{2}\right)\sin\left(\frac{2C-2B}{2}\right)\right\}}{2}$

$= \frac{1-\cos 2A+2\sin(B+C)\sin(C-B)}{2}$

Since A + B + C = π

Or B + C = 180 - A

And $\sin(\pi - A) = \sin A$

L.H.S = $\frac{1-\cos 2A+2\sin(\pi-A)\sin(C-B)}{2}$

$= \frac{1-\cos 2A+2\sin A\sin(C-B)}{2}$

Using, $\cos 2A = 1 - 2\sin^2 A$

L.H.S = $\frac{1-1+2\sin^2 A+2\sin A\sin(C-B)}{2}$

$= \frac{2\sin A\{\sin A+\sin(C-B)\}}{2}$

$= \frac{2\sin A\{\sin A+\sin(C-B)\}}{2}$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

L.H.S = $\frac{2\sin A\left\{2\sin\left(\frac{A+C-B}{2}\right)\cos\left(\frac{A-C+B}{2}\right)\right\}}{2}$

$= \frac{1-2\sin A\left\{2\sin\left(\frac{\pi-B-B}{2}\right)\cos\left(\frac{\pi-C-C}{2}\right)\right\}}{2}$

$= \frac{2\sin A\left\{2\sin\left(\frac{\pi}{2}-\frac{2B}{2}\right)\cos\left(\frac{\pi}{2}-\frac{2C}{2}\right)\right\}}{2}$

As, $\sin\left(\frac{\pi}{2} - A\right) = \cos A$

L.H.S = $\frac{2\sin A\{2\cos B\sin C\}}{2}$

= 2sin A cos B sin C

= R.H.S.

35.

Class-interval	Cumulative frequency	Mid-values, x_i	Frequency, f_i	$u_i = \frac{x_i-67.5}{15}$	$f_i u_i$	$f_i u_i^2$
0-15	12	7.5	12	-4	-48	192
15-30	30	22.5	18	-3	-54	162
30-45	65	37.5	35	-2	-70	140
45-60	107	52.5	42	-1	-42	42
60-75	157	67.5	50	0	0	0
75-90	202	82.5	45	1	45	45
90-105	222	97.5	20	2	40	80
105-120	230	112.5	8	3	24	72
			$\Sigma f_i = 230$		**$\Sigma f_i u_i = -105$**	**$\Sigma f_i u_i^2 = 733$**

Here, A = 67.5, h = 15, N = 230, $\Sigma f_i u_i = -105$ and $\Sigma f_i u_i^2 = 733$

$\therefore$ Mean = A + h $\left(\frac{1}{N} \Sigma f_i u_i\right)$ = 67.5 + 15 $\left(\frac{-105}{230}\right)$ = 67.5 - 6.85 = 60.65

and, Var (x) = $h^2 \left\{\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right\}$

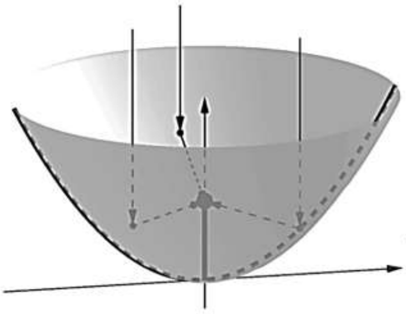
$\Rightarrow$ Var (x) = 225 $\left\{\frac{733}{230} - \left(\frac{-105}{230}\right)^2\right\}$ = 225 (3.18 - 0.2025) = 669.9375

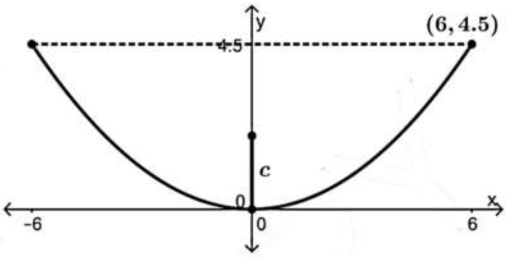
$\therefore$ S.D. = $\sqrt{\text{Var}(X)} = \sqrt{669.9375}$ = 25.883

Section E

36. **Read the text carefully and answer the questions:**

A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across, and 4.5 ft deep at the vertex.





(i) Given curve is a parabola

Equation of parabola is $x^2 = 4ay$

It passes through the point (6, 4.5)

$\Rightarrow 36 = 4 \times a \times 4.5$

$\Rightarrow 36 = 18a$

$\Rightarrow a = 2$

Equation of parabola is $x^2 = 8y$

(ii) Distance between focus and vertex is = a = $\sqrt{(4-4)^2 + (5-3)^2}$ = 2

Equation of parabola is $(y - k)^2 = 4a(x - h)$

where (h, k) is vertex

$\Rightarrow$ Equation of parabola with vertex (3, 4) & a = 2

$\Rightarrow (y - 4)^2 = 8(x - 3)$

(iii) Equation of parabola with axis along x - axis

$y^2 = 4ax$

which passes through (2, 3)

$\Rightarrow 9 = 4a \times 2$

$\Rightarrow 4a = \frac{9}{2}$

hence required equation of parabola is

$y^2 = \frac{9}{2}x$

$\Rightarrow 2y^2 = 9x$

Hence length of latus rectum = 4a = 4.5

OR

$x^2 = 8y$

a = 2

Focus of parabola is (0, 2)

length of latus rectum is $4a = 4 \times 2 = 8$

Equation of directrix y + 2 = 0

37. **Read the text carefully and answer the questions:**

A company produces 500 computers in the third year and 600 computers in the seventh year. Assuming that the production increases uniformly by a constant number every year.

(i) **(a)** 50

Explanation: 50

(ii) **(d)** 950

Explanation: 950

(iii) **(b)** 5625

Explanation: 5625

OR

(d) 450

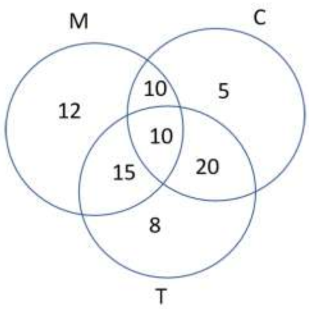
Explanation: 450

38. **Read the text carefully and answer the questions:**

The school organised a farwell party for 100 students and school management decided three types of drinks will be distributed in farewell party i.e., Milk (M), Coffee (C) and Tea (T).

Organiser reported that 10 students had all three drinks M, C, T. 20 students had M and C; 30 students and C and T; 25 students had M and T. 12 students had M only; 5 students had C only; 8 students had T only.

(i)

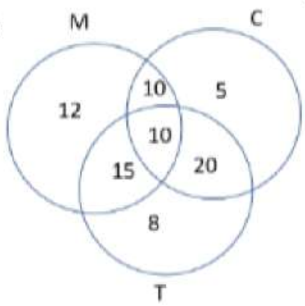


only n(M ∩ C) not tea = n(M ∩ C) - n(M ∩ C ∩ T)

⇒ only n(M ∩ C) not tea = 20 - 10 = 10

The number of students who prefer Milk and Coffee but not tea = 10

(ii)



n(T) = only n(T) + n(M ∩ T) + n(T ∩ C) - n(M ∩ C ∩ T)

⇒ n(T) = 8 + 25 + 30 - 10 = 53

The number of students who prefer Tea = 53