

NUCLEI

12th Standard CBSE

Date : 17-Nov-22

PhysicsReg.No. :

Exam Time : 02:00:00 Hrs

Total Marks : 100

64 x 5 = 320

- 1) The nuclear mass is $_{26}^{56}\text{Fe}$ 55.85 u. Calculate its nuclear density.

- 2) Calculate number of protons and neutrons in $_{92}^{235}\text{U}$ and the isotope $_{88}^{236}\text{Ra}$

- 3) Express 1 Joule in eV. Taking 1 a.m.u.=931 MeV, calculate the mass of $_{6}^{12}\text{C}$.

- 4) Find the effective mass of a photon if the wavelength of radiation is 3000 \AA

- 5) Calculate the nuclear mass density of $_{92}^{238}\text{U}$ Given $R_0 = 1.5 \text{ fermi}$ and mass of each nucleon is $1.6 \times 10^{-27} \text{ kg}$

- 6) Calculate the binding energy per nucleon of the nucleus $_{26}^{56}\text{Fe}$. Given that mass of $_{26}^{56}\text{Fe} = 55.934939 \text{ u}$, the mass of proton = 1.007825 u and mass of neutron = 1.008665 u and $1 \text{ u} = 931 \text{ MeV}$.

- 7) Calculate the B.E/nucleon of $_{17}^{35}\text{Cl}$ the nucleus. Given that mass of proton = 1.007825 u, mass of neutron = 1.008665 u, mass of $_{17}^{35}\text{Cl} = 34.980000 \text{ u}$; $1 \text{ u} = 931 \text{ MeV}$.

- 8) Calculate the binding energy per nucleon of $_{20}^{40}\text{Ca}$ the nucleus. Given $m(_{20}^{40}\text{Ca}) = 39.962589 \text{ u}$; $m_n = 1.008665 \text{ u}$; $m_p = 1.007825 \text{ u}$
Take $1 \text{ a.m.u.} = 931 \text{ MeV}$

- 9) Calculate binding energy per nucleon of $_{83}^{209}\text{Bi}$ Given that $m_p = 1.00727 \text{ amu}$, $m_n = 1.00866 \text{ amu}$, $m(_{83}^{209}\text{Bi}) = 208.980388 \text{ amu}$
 $m(\text{neutron}) = 1.008665 \text{ amu}$
 $m(\text{proton}) = 1.007825 \text{ amu}$

- 10) The decay constant for a given radioactive sample is 0.3465 day^{-1} What percentage of this sample will get decayed in a period of 4 days?

- 11) The half-life of radium is 1500 years. After how many years will one gram of pure radium
(i) reduce to 1 centigram?
(ii) lose one milligram?
-
- 12) It is observed that only 6.25% of a given radioactive sample is left undecayed after a period of 16 days. What is the decay constant of this sample in $\{\text{day}\}^{-1}$?
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- 13) A radioactive material is reduced to $\frac{1}{16}$ of its original amount in 4 days. How much material should one begin with so that 4×10^{-3} kg of the material is left over after 6 days?
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- 14) One MeV positron encounters one MeV electron traveling in opposite direction. What is the wavelength of photons produced, given rest mass energy of electron or positron = 0.512 MeV? Take $h = 6.62 \times 10^{-34}$ Js
-
- 15) Complete the decay reaction
 ${}_{10}^{23}\text{Ne} \rightarrow ? + {}_{-1}^0\text{e} + ?$
 Also, find the maximum KE of electrons emitted during this decay. Given of ${}_{10}^{23}\text{Ne} = 22.994465$ u, the mass of ${}_{11}^{23}\text{Na} = 22.989768$ u.
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- 16) A neutron is absorbed by a ${}_{3}^6\text{Li}$ nucleus with subsequent emission of α -particle. Write corresponding nuclear reaction. Calculate the energy released in this reaction.
 Given $m({}_{3}^6\text{Li}) = 6.015126$ u;
 $m({}_{2}^4\text{He}) = 4.00026044$ u
 $m({}_{0}^1\text{n}) = 1.0086654$ u
 $m({}_{1}^3\text{H}) = 3.016049$ u
-
- 17) In a star, three alpha particles join in a single reaction to form nucleus ${}_{6}^{12}\text{C}$. Calculate the energy released in the reaction.
 Given $m({}_{2}^4\text{He}) = 4.00026044$ u
 $m({}_{6}^{12}\text{C}) = 12.000000$ u
 $1 \text{ amu} = 931 \text{ MeV}$
-
- 18) If in a nuclear fusion reaction, mass defect is 0.3%, then find energy released in fusion of 1 kg mass.
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- 19) If 200 MeV energy is released in the fission of a single nucleus of ${}_{92}^{235}\text{U}$, how many fissions must occur per second to produce a power of 1 kW?
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- 20) The mean lives of a radioactive substance are 1620 years and 405 years for α -emission and β -emission respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by α -emission and β -emission simultaneously.
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- 21) A nuclear reactor is a powerful device, wherein nuclear energy is utilised for peaceful purposes. It is based upon controlled nuclear chain reaction. The nuclear chain reaction is controlled by the use

of control rods (of boron or cadmium) and moderators like heavy water, graphite, etc. The whole reactor is protected with concrete walls 2 to 2.5 metre thick, so that radiations emitted during nuclear reactions may not produce harmful effects.

Read the above passage and answer the following questions:

- (i) Give any two merits of nuclear reactors.
 - (ii) What is radioactive waste?
 - (iii) Why do people often oppose the location site of a nuclear reactor? What do you suggest?
-

22) Einstein was the first to establish the equivalence between mass energy. According to him, whenever a certain mass $\left(\Delta m \right)$ disappears in some process, the amount of energy released is $E = \left(\Delta m \right) \{ c \}^2$, where c is velocity of light vacuum $\left(= 3 \times 10^8 \text{ m/s} \right)$. The reverse is also true, i.e., whenever energy E disappears, an equivalent mass $\left(\Delta m \right) = \{ E/c \}^2$ appears.

Read the above passage and answer the following questions :

- (i) What is the energy released when 1 a.m.u. of mass disappears in a nuclear reaction?
 - (ii) Do you know any phenomenon in which energy materialises?
 - (iii) What values of life do you learn from this famous relation?
-

23) Poonam's mother is diagnosed cancer. The attending physician told her that she has to undergo radiotherapy. While telling her the side effects of the treatment, the doctor told that her beautiful hair may fall and she may become bald. Poonam's mother refuses to get the treatment.

Read the above passage and answer the following questions:

- (i) What would you do if you were in Poonam's place?
 - (ii) What values are associated with your attitude?
-

24) Marie Curie and her teacher turned husband Pierre Curie worked hard to extract radium chloride (RaCl_2) from uranium ore. They succeeded in 192 after a long struggle. About 0.19 g of RaCl_2 was extracted and its radioactivity was studied. They were awarded by the noble prize, which they shared it which they shared it which they shared it with Henri Becquerel.

Read the above passage and answer following questions:

- (i) What are the values shown by Marie Curie and her husband?
 - (ii) What do you understand by radioactivity? How the half-life period is related to the disintegration constant?
 - (iii) How is average-life of radioactive element related to half-life?
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25) Shyamsaw his younger brother wondering with a question which deals with emission of light from a vapour lamp. He was anxious to know how different colours were being emitted by different lights. He also saw mercury and sodium vapour lamps in the Physics lab and was curious to know what is inside the lamps. On seeing his anxiety to know more about it, Shyam explained about absorption of energy and re-emission of photons in the visible region. He also advised him not to touch or break any items in the lab for the knowledge.

Read the above passage and answer the following questions:

- (i) What is the moral you derive from Shyam's behaviour?
 - (ii) Which series in the hydrogen spectrum is in the visible region?
 - (iii) Write the quality displayed by Shyam's brother.
-

26) (a) Define the terms

(i) half life ($T_{1/2}$) and

(ii) average life (τ). Find their relationships with the decay constant (λ)

(b) A radioactive nucleus has a decay constant, $\lambda = 0.3465 \text{ (day)}^{-1}$. How long would it take the nucleus to decay to 75% of its initial amount?

27) A radioactive nucleus has a decay constant, $\lambda = 0.3465 \text{ (day)}^{-1}$. How long would it take the nucleus to decay to 75% of its initial amount?

28) For the past some time, Arti has been observing some erratic body movement, unsteadiness and lack of coordination in the activities of her sister Radha, who also used to complain of severe headache occasionally. Arti suggested to her parents to get a medical check-up of Radha. The doctor thoroughly examined Radha and diagnosed that she has a brain tumour

(a) What, according to you, are the values displayed by Arti?

(b) How can radioisotopes help a doctor to diagnose brain tumour?

29) Draw the plot of binding energy per nucleon (BE/A) as a function of mass number A. Write two important conclusions that can be drawn regarding the nature of nuclear force.

Use this graph to explain the release of energy in both the processes of nuclear fusion and fission.

Write the basic nuclear process of neutron undergoing β -decay. Why is the detection of neutrinos found very difficult?

30) Define the Q-value of a nuclear process. When can a nuclear process not proceed spontaneously? If both the number of protons and the number of neutrons are conserved in a nuclear reaction in what way is mass converted into energy (or vice-versa) in the nuclear reaction?

31) Asha's mother read an article in the newspaper about a disaster that took place at Chernobyl. She could not understand much from the article and asked a few questions from Asha regarding the article.

Asha tried to answer her mother's questions based on what she learnt in Class XII Physics?

32) (i) In Rutherford scattering experiment, draw the trajectory traced by α -particles in the Coulomb field of target nucleus and explain how this led to estimate the size of the nucleus.

(ii) Describe briefly how wave nature of moving electrons was established experimentally.

(iii) Estimate the ratio of de Broglie wavelengths associated with deuterons and α -particles when they are accelerated from rest through the same accelerating potential V.

33) We are given the following atomic masses:

${}_{92}^{238} \text{U} = 238.05079 \text{ u}$ & ${}_{2}^{4} \text{He}$

${}_{90}^{234} \text{Th} = 234.04363 \text{ u}$ & ${}_{1}^{1} \text{H} = 1.00783 \text{ u}$

${}_{91}^{237} \text{Pa} = 237.05121 \text{ u}$

Here the symbol Pa is for the element protactinium ($Z = 91$).

- (a) Calculate the energy released during the alpha decay of ${}_{92}^{238}\text{U}$.
 (b) Show that ${}_{92}^{238}\text{U}$.can not spontaneously emit a proton.

34) Answer the following questions:

- (a) Are the equations of nuclear reactions (such as those given in Section 13.7) 'balanced' in the sense a chemical equation (e.g., $2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$) is? If not, in what sense are they balanced on both sides?
 (b) If both the number of protons and the number of neutrons are conserved in each nuclear reaction, in what way is mass converted into energy (or vice-versa) in a nuclear reaction?
 (c) A general impression exists that mass-energy interconversion takes place only in nuclear reaction and never in chemical reaction. This is strictly speaking, incorrect. Explain

- 35) (a) Two stable isotopes of lithium ${}^6_3\text{Li}$ and ${}^7_3\text{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.
 (b) Boron has two stable isotopes, ${}^{10}_5\text{B}$ and ${}^{11}_5\text{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundance of ${}^{10}_5\text{B}$ and ${}^{11}_5\text{B}$.

- 36) Obtain the binding energy (in MeV) of a nitrogen nucleus ${}^{14}_7\text{N}$ given $m({}^{14}_7\text{N}) = 14.00307\text{ u}$

- 37) Obtain the binding energy of the nuclei ${}^{56}_{26}\text{Fe}$ and ${}^{209}_{83}\text{Bi}$ in units of MeV from the following data:
 $m({}^{56}_{26}\text{Fe}) = 55.934939\text{ u}$
 $m({}^{209}_{83}\text{Bi}) = 208.980388\text{ u}$

- 38) A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u).

- 39) Write nuclear reaction equations for
 (i) α -decay of ${}^{226}_{88}\text{Ra}$
 (ii) α -decay of ${}^{242}_{94}\text{Pu}$
 (iii) β -decay of ${}^{32}_{15}\text{P}$
 (iv) β^- -decay of ${}^{210}_{83}\text{Bi}$
 (v) β^+ -decay of ${}^{11}_6\text{C}$
 (vi) β^+ -decay of ${}^{97}_{43}\text{Tc}$
 (vii) Electron capture of ${}^{120}_{54}\text{Xe}$

- 40) A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

41) Obtain the amount of ${}_{27}^{60}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}_{27}^{60}\text{Co}$ is 5.3 years.

42) Find the Q-value and the kinetic energy of the emitted α -particle in the α -decay of (a) ${}_{88}^{226}\text{Ra}$ and (b) ${}_{86}^{222}\text{Rn}$.

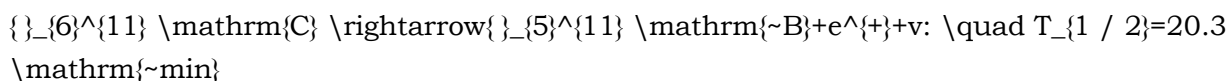
$$m\left({}_{88}^{226}\text{Ra}\right)=226.02540 \text{ u}$$

$$m\left({}_{86}^{222}\text{Rn}\right)=222.01750 \text{ u}$$

$$m\left({}_{86}^{222}\text{Rn}\right)=220.01137 \text{ u}$$

$$m\left({}_{84}^{216}\text{Po}\right)=216.00189 \text{ u}$$

43) The radionuclide ${}^{11}\text{C}$ decays according to



The maximum energy of the emitted positron is 0.960 MeV. Given the mass values:

$$m\left({}_{6}^{11}\text{C}\right)=11.011434 \text{ u} \quad \text{and} \quad m\left({}_{6}^{11}\text{B}\right)=11.009305 \text{ u},$$

$$m\left({}_{5}^{11}\text{B}\right)=11.009305 \text{ u},$$

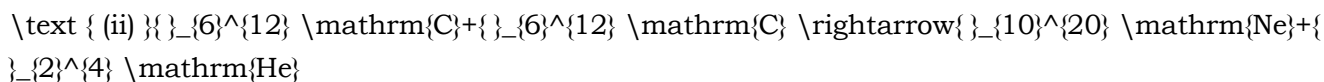
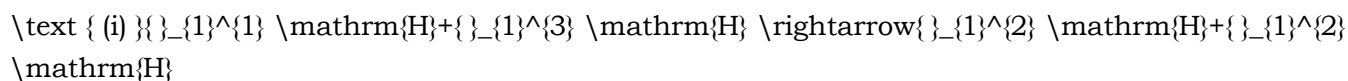
calculate Q and compare it with the maximum energy of the positron emitted.

44) The nucleus ${}_{10}^{23}\text{Ne}$ decays by β - emission. Write down the β -decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$$m\left({}_{10}^{23}\text{Ne}\right)=22.994466 \text{ u}$$

$$m\left({}_{11}^{23}\text{Na}\right)=22.089770 \text{ u} .$$

45) The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by $Q = [m_A + m_b - m_C - m_d] c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.



Atomic masses are given to be

$$m\left({}_{1}^{2}\text{H}\right)=2.014102 \text{ u}$$

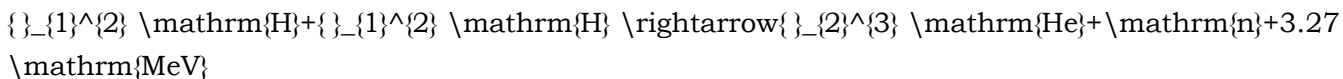
$$m\left({}_{1}^{3}\text{H}\right)=3.016049 \text{ u}$$

$$m\left({}_{6}^{12}\text{C}\right)=12.000000 \text{ u}$$

$$m\left({}_{10}^{20}\text{Ne}\right)=19.992439 \text{ u}$$

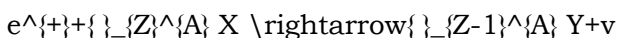
46) A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}_{92}^{235}\text{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}_{92}^{235}\text{U}$ and that this nuclide is consumed only by the fission process.

47) How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as



48) Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

49) For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted).



Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

50) In a periodic table given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $\{_{12}^{24} \mathrm{Mg}$ (23.98504u), $\{_{12}^{25} \mathrm{Mg}$ (24.98584u) and $\{_{12}^{26} \mathrm{Mg}$ (25.98259u). The natural abundance of $\{_{12}^{24} \mathrm{Mg}$ is 78.99% by mass. Calculate the abundances of other two isotopes.

51) The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $\{_{20}^{41} \mathrm{Ca}$ and $\{_{13}^{27} \mathrm{Al}$ from the following data:

$$m(\begin{array}{c} 40 \\ 20 \end{array} \mathrm{Ca}) = 39.962591 \mathrm{u}$$

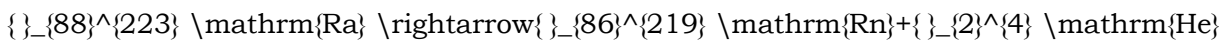
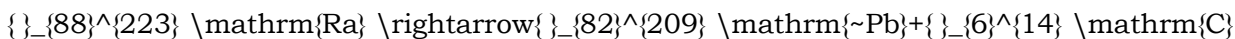
$$m(\begin{array}{c} 41 \\ 20 \end{array} \mathrm{Ca}) = 40.962278 \mathrm{u}$$

$$m(\begin{array}{c} 26 \\ 13 \end{array} \mathrm{Al}) = 25.986895 \mathrm{u}$$

$$m(\begin{array}{c} 27 \\ 13 \end{array} \mathrm{Al}) = 26.981541 \mathrm{u}$$

52) A source contains two phosphorous radio nuclides $\{_{15}^{32} \mathrm{P}$ ($T_{1/2} = 14.3 \mathrm{d}$) and $\{_{15}^{33} \mathrm{P}$ ($T_{1/2} = 25.3 \mathrm{d}$). Initially, 10% of the decays come from $\frac{33}{15} \mathrm{P}$. How long one must wait until 90% do so?

53) Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α -particle. Consider the following decay processes:



Calculate the Q-values for these decays and determine that both are energetically allowed.

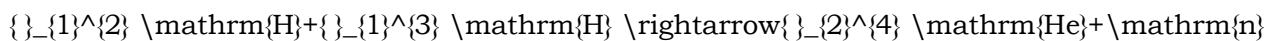
54) Consider the fission of $\begin{array}{c} 238 \\ 92 \end{array} \mathrm{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are $\begin{array}{c} 140 \\ 58 \end{array} \mathrm{Ce}$ and $\begin{array}{c} 99 \\ 44 \end{array} \mathrm{Ru}$. Calculate Q for this fission process. The relevant atomic and particle masses are

$$m(\begin{array}{c} 238 \\ 92 \end{array} \mathrm{U}) = 238.05079 \mathrm{u}$$

$$m\left({}_{58}^{140}\text{Ce}\right)=139.90543\text{ u}$$

$$m\left({}_{99}^{244}\text{Rf}\right)=98.90594\text{ u}$$

55) Consider the D-T reaction (deuterium-tritium fusion)



(a) Calculate the energy released in MeV in this reaction from the data:

$$m\left({}_{1}^{2}\text{H}\right)=2.014102\text{ u}$$

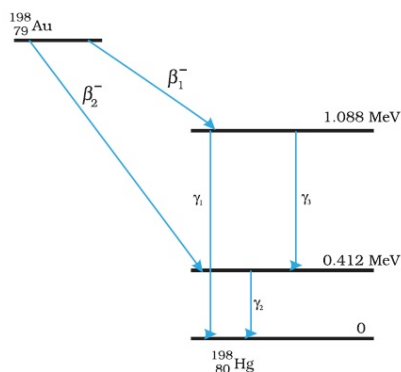
$$m\left({}_{1}^{3}\text{H}\right)=3.016049\text{ u}$$

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction?

56) Obtain the maximum kinetic energy of β -particles, and the radiation frequencies of γ decays in the decay scheme. You are given that

$$m\left({}_{79}^{198}\text{Au}\right)=197.968233\text{ u}$$

$$m\left({}_{80}^{198}\text{Hg}\right)=197.966760\text{ u}$$



57) Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ${}^{235}\text{U}$ in a fission reactor.

58) Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ${}^{235}\text{U}$ to be about 200MeV.

59) If both the number of protons and the number of neutrons are conserved in a nuclear reaction, in what way is mass converted into energy (or vice-versa) in nuclear reaction

60) Supposing that protons and neutrons have equal masses. Calculate how many times nuclear matter is denser than water? Take, mass of a nucleon = 1.67×10^{-27} kg and $R_0 = 1.2 \times 10^{-15}$ m.

61) Find the binding energy per nucleon of ${}_{20}\text{Ca}^{40}$ nucleus. Given, $m({}_{20}\text{Ca}^{40}) = 39.962589$ u, $m_n = 1.008665$ u and $m_p = 1.007825$ u. Take, $1\text{ amu} = 931\text{ MeV}/c^2$

- 62) The half-life of radon is 3.8 days. Calculate how much of 15 mg of radon will remain after 38 days?
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- 63) Calculate the decay constant of a radioactive sample, if number of atoms present in it drops to 1/16 th of its initial value in 40 years.
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- 64) ${}_{86}\text{Rn}^{222}$ is converted into ${}_{84}\text{Po}^{218}$ Name the particle emitted in this case and write down the corresponding equation.
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64 x 5 = 320

- 1) Here, $M = 55.85 \text{ u}$
 $= 55.85 \times 1.66 \times 10^{-27} \text{ kg} = 9.27 \times 10^{-26} \text{ kg}$
 $R = \left(\frac{R_0}{A} \right)^{1/3} = 1.2 \times 10^{-15} \left(\frac{56}{\text{right}} \right)^{1/3} \text{ metre}$
 Nuclear density,
 $\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3} \pi R^3} = \frac{3M}{4\pi R^3}$
 $= \frac{3 \times 9.27 \times 10^{-26}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}$
 $= 2.29 \times 10^{17} \text{ kg/m}^3$
-
- 2) For ${}_{92}\text{U}^{235}$, $Z=92$, $A=235$
 No. of protons, $Z=92$
 No. of neutrons $= A - Z = 235 - 92 = 143$
 For ${}_{88}\text{Ra}^{236}$, $Z=88$, $A=236$
 No. of protons $= Z = 88$
 No. of neutrons $= A - Z = 236 - 88 = 148$
-
- 3)
 $1 \text{ eV} = 1.602 \times 10^{-19} \text{ joule}$
 $1.602 \times 10^{-19} \text{ joule} = 1 \text{ eV}$
 $1 \text{ joule} = \frac{1}{1.602 \times 10^{-19}} \text{ eV}$
 $1 \text{ joule} = 6.242 \times 10^{18} \text{ eV}$
 (ii) From $E = mc^2$
 $m = \frac{E}{c^2} \therefore 1 \text{ a.m.u.} = \frac{931 \times 1.602 \times 10^{-13}}{(3 \times 10^8)^2}$
 $= 1.66 \times 10^{-27} \text{ kg}$
 Now, definition, Mass of ${}_{6}\text{C}^{12} = 12 \text{ a.m.u.}$
 $= 12 \times 1.657 \times 10^{-27} \text{ kg} = 1.99 \times 10^{-26} \text{ kg}$
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- 4) Here, $m = ?$, $\lambda = 3000 \text{ m}$
 From $E = mc^2$, $m = \frac{E}{c^2} = \frac{hc}{\lambda c^2}$
 $= \frac{h}{c\lambda} = \frac{6.63 \times 10^{-34}}{3 \times 10^8 \times 3000}$
 $= 7.367 \times 10^{-36} \text{ kg}$
-
- 5)

$$\rho = \frac{\text{nuclear mass}}{\text{nuclear volume}}$$

$$= \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3m}{4\pi R^3} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.5 \times 10^{-15})^3}$$

$$= 1.18 \times 10^{17} \text{ kg/m}^3$$

6) In ${}_{26}^{56}\text{Fe}$, no. of protons = 26;
 no. of neutrons = $56 - 26 = 30$
 therefore Mass defect = $26m_p + 30m_n - M_{\text{Fe}}$
 $= 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$
 $= 26.20345 + 30.25995 - 55.934939$
 $= 0.528461 \text{ u}$
 B.E/nucleon = $\frac{0.528461 \times 931}{56} = 8.79 \text{ MeV/N}$

7) In ${}_{17}^{35}\text{Cl}$, no. of protons = 17;
 no. of neutrons = $35 - 17 = 18$
 therefore Mass defect = $17m_p + 18m_n - M_{\text{Cl}}$
 $= 17 \times 1.007825 + 18 \times 1.008665 - 34.980000$
 $= 17.133025 + 18.155970 - 34.980000$
 $= 0.308995 \text{ u}$
 B.E/nucleon = $\frac{0.308995 \times 931}{35} = 8.22 \text{ MeV/N}$

8) In a nucleus ${}_{20}^{40}\text{Ca}$, number of protons = 20, number of neutrons = $40 - 20 = 20$
 Total mass of 20 protons and 20 neutrons = $20m_p + 20m_n = (m_p + m_n)$
 $= 20(1.007825 + 1.008665) = 40.3298 \text{ u}$
 Mass defect, $\Delta m = 40.3298 - 39.962589$
 $= 0.367211 \text{ u}$
 Total B.E. = $0.367211 \times 931 \text{ MeV}$
 $= 341.873441 \text{ MeV}$
 B.E./nucleon = $\frac{341.873441}{40} = 8.547 \text{ MeV/N}$

9) In ${}_{83}^{209}\text{Bi}$, number of protons = 83
 number of neutrons = $209 - 83 = 126$
 Mass defect,
 $\Delta m = 83m_p + 126m_n - M(\text{Bi})$
 $= 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$
 $= 83.649475 + 127.091790 - 208.980388$
 $= 1.760877 \text{ amu}$
 B.E/nucleon = $\frac{1.760877 \times 931}{209}$
 $= 7.85 \text{ MeV/N}$

10) Here, $\lambda = 0.3465 \text{ day}^{-1}$, $t = 4 \text{ days}$
 Half life, $T = \frac{0.693}{\lambda} = \frac{0.693}{0.3465} = 2 \text{ days}$
 No. of half lives, $n = \frac{t}{T} = \frac{4}{2} = 2$
 $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 25\%$
 Percentage of sample that will get decayed
 $= 100 - 25 = 75\%$
 75%

11) Here, $T = 1500 \text{ yrs}$, $t = ?$
 $\frac{N}{N_0} = \frac{10^{-2} \text{ g}}{1 \text{ g}} = 10^{-2} = \left(\frac{1}{2}\right)^n = \frac{1}{100}$
 $\therefore n(\log 1 - \log 2) = \log 1 - \log 100$
 $n(0 - 0.3010) = 0 - 2$, $n = \frac{2}{0.3010} = 6.6445$
 As $n = \frac{t}{T}$
 $\therefore t = nT = 6.6445 \times 1500 = 9967 \text{ years}$
 (ii) $\frac{N}{N_0} = \frac{1 - 10^{-3}}{1} = 0.999 = \left(\frac{1}{2}\right)^n$
 $\therefore n(\log e - \log 2) = \log 0.999$
 or $n(0 - 0.6931) = -1.0005$,
 $n = \frac{1.0005}{0.6931} = 1.4435$
 As $n = \frac{t}{T}$ $\therefore t = nT = 1.4435 \times 1500 \text{ yrs}$
 $= 2165.2 \text{ years}$

12) Here, $\frac{N}{N_0} = \frac{6.25}{100} = \frac{0.25}{4}$,
 $t = 16 \text{ days}$, $\lambda = ?$
 As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{0.25}{4} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$
 $\therefore n = 4$ As $n = \frac{t}{T}$, $T = \frac{t}{n} = \frac{16}{4} = 4 \text{ days}$
 Decay constant $\lambda = \frac{0.693}{T} = \frac{0.693}{4}$
 $= 0.173 \text{ day}^{-1}$

13) Here, $\frac{N}{N_0} = \frac{1}{16}$, $t = 4 \text{ days}$
 As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{16} = \left(\frac{1}{2}\right)^4$ $\therefore n = 4$
 Half-life $T = \frac{t}{n} = \frac{4}{4} = 1 \text{ day}$
 Now, $N = 4 \times 10^{-3} \text{ kg}$, $t = 6 \text{ days}$, $N_0 = ?$
 As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{6/1} = \frac{1}{64}$
 $\therefore N_0 = 64N = 64 \times 4 \times 10^{-3} = 0.256 \text{ kg}$

14)

Two photons are produced when a positron annihilates an electron.

$$\text{i.e., } \gamma_{-1} + \gamma_{+1} = 2\gamma$$

Total energy involved

$$= \text{Rest mass energy} + \text{KE of both}$$

$$= 2(0.512 + 1) = 2 \times 1.512 \text{ MeV}$$

Therefore Energy of each photon

$$= \frac{2 \times 1.512}{2} = 1.512 \text{ MeV}$$

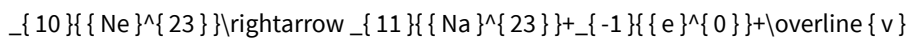
$$E = 1.512 \times 1.6 \times 10^{-13} \text{ J}$$

$$\text{As } E = \frac{hc}{\lambda}$$

$$\therefore \lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.512 \times 1.6 \times 10^{-13}} \text{ m}$$

$$= 8.21 \times 10^{-13} \text{ m}$$

- 15) Applying conservation of charge number and mass number, we can write the decay reaction as



Neglecting mass of ${}_{-1}^0\text{e}$, mass defect,

$$\Delta m = m({}_{10}^{23}\text{Ne}) - m({}_{11}^{23}\text{Na})$$

$$= 22.994465 - 22.989768$$

$$\Delta m = 0.004697 \text{ u}$$

Max K.E. of electron = Q value of the reaction

$$0.004697 \times 931 \text{ MeV}$$

$$= 4.372 \text{ MeV}$$

- 16) The nuclear reaction is given by



Mass defect

$$\Delta m = m({}_{3}^6\text{Li}) + m({}_0^1\text{n}) - m({}_{2}^4\text{He}) - m({}_{1}^3\text{H})$$

$$= 6.015126 + 1.0086654 - 4.0026044 - 3.016049$$

$$= 7.0237914 - 7.0186534$$

$$\Delta m = 0.005138 \text{ u}$$

$$\text{Energy released} = \Delta m \times 931 \text{ MeV}$$

$$= 0.005138 \times 931 = 4.783 \text{ MeV}$$

- 17) The nuclear reaction is



Mass defect $\Delta m = 3 \times 4.002604 - 12.000000$

$$= 0.007812 \text{ u}$$

$$\text{Energy released} = 0.007812 \times 931 \text{ MeV}$$

$$= 7.27 \text{ MeV}$$

- 18)

$$\begin{aligned} \text{Here, } \Delta m &= 0.3\% (1\text{kg}) = \frac{0.3}{100} \text{kg} = 3 \times 10^{-3} \text{kg} \\ E &= (\Delta m) c^2 = (3 \times 10^{-3}) \left(3 \times 10^8 \right)^2 \\ &= \frac{0.3}{100} \text{kg} \\ &= 3 \times 10^{-3} \text{kg} \\ E &= (\Delta m) c^2 = (3 \times 10^{-3}) \left(3 \times 10^8 \right)^2 \\ &= 27 \times 10^{13} \text{J} \end{aligned}$$

19)

Here, energy released/fission
 $= 200 \text{MeV}$
 $= 200 \times 1.6 \times 10^{-13} \text{J} = 3.2 \times 10^{-11} \text{J}$
 Total energy required/sec $= 1 \text{ kW} = 1000 \text{w}$
 $= 1000 \text{J/s.}$
 Number of fissions/sec
 $= \frac{\text{Energy reqd. per sec.}}{\text{energy released/fission}}$
 $= \frac{1000}{3.2 \times 10^{-11}} = 3.125 \times 10^{13} \text{s}$

20)

Here, $\tau_{\alpha} = 1620 \text{years}$, $\tau_{\beta} = 405 \text{years}$
 $t = N_0 - \frac{N}{4} \Rightarrow \frac{N}{4} = N_0 - t$
 If λ_{α} and λ_{β} are decay constants for α and β emission respectively,
 then $\lambda_{\alpha} = \frac{1}{\tau_{\alpha}} = \frac{1}{1620} \text{yr}^{-1}$
 and $\lambda_{\beta} = \frac{1}{\tau_{\beta}} = \frac{1}{405} \text{yr}^{-1}$
 Total decay constant $\lambda = \lambda_{\alpha} + \lambda_{\beta}$
 $= \frac{1}{1620} + \frac{1}{405} = \frac{1+4}{1620} = \frac{1}{324} \text{yr}^{-1}$
 from $N = N_0 e^{-\lambda t}$; $\frac{1}{4} N_0 = N_0 e^{-\lambda t}$
 $t = \frac{\log_e 4}{\lambda} = \frac{1.386}{\frac{1}{324}} \text{yr} = 449.1 \text{yr}$

21)

(i) Nuclear reactors are used in electric power generation. They are also used to produce radioactive isotopes which have applications in medicine, industry and agriculture.

(ii) Radioactive waste consists of fission products and transuranic elements such as plutonium and americium. This waste is extremely hazardous to all forms of life on earth.

(iii) People often oppose the location site of a nuclear reactor because any leakage of nuclear radiations can affect adversely miles of area surrounding it. Elaborate safety measures are needed not only for reactor operations, but also for handling and disposal of radioactive waste.

I would suggest that Government must take stringent safety measures and assure people of safeguards in the event of nuclear accidents. At the same time, people must also be educated accordingly.

22)

(i) Here, $\Delta m = 1 \text{ a.m.u.} = 1.66 \times 10^{-27} \text{ kg}$

$$E = (\Delta m) c^2 = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 1.49 \times 10^{-10} \text{ J}$$

(ii) Yes, in the phenomenon of pair production. Under suitable conditions, a photon materialises into an electron and a positron: $\gamma = e^{-} + e^{+}$

(iii) Einstein's relation, $E = (\Delta m) c^2$ emphasises that when certain mass disappears, an equivalent amount of energy appears. The reverse is also true. It implies that to gain something, you have to lose another in an equivalent amount. No one can have all gains together or all losses together. It also implies that nothing comes for free. You have to pay the price in one form and acquire something in the desired form.

23)

(i) If I were in Poonam's place, I would tell my mother that treatment of cancer is a must. Hair fall is a temporary effect and the hair would grow slowly after the therapy. Convincing my mother to get the treatment, using all sorts of reasoning will be my top priority.

(ii) Concern for my mother's health is of utmost importance. I will not mind taking help from specialists to convince my mother for getting the treatment.

24)

(i) Values shown here are dedication, hard working nature, brilliance, true justification of the help when helped by another, honesty and gratitude.

(ii) The phenomenon of spontaneous emission of α , β and γ -particles by the nuclide is known as radioactivity. Half-life period is related with the disintegration constant as,

$$T_{1/2} = \frac{0.693}{\lambda}$$

(iii) Average life of radioactive element,

$$\tau = 1.44 T_{1/2}$$

25)

(i) Concern for his brother, care about the school thorough subject knowledge.

(ii) Balmer series.

(iii) His brother has zeal to learn.

26)

(a) Definition

(i) Half life: Time taken by a radioactive nuclei to reduce to half of the initial number of radio nuclei.

(ii) Average life: Ratio of total life time of all radioactive nuclei to the total number of nuclei in the sample.

Relation between half life and decay constant:

$$T_{1/2} = \frac{0.693}{\lambda}$$

Relation between average life and decay constant

$$\tau = \frac{1}{\lambda}$$

$$(b) N = N_0 e^{-\lambda t}$$

$$\frac{3}{4} N_0 = N_0 e^{-(0.3465)t} \quad (\text{because } N_0 \text{ is } 75\% \text{ of } N_0)$$

$$N = \frac{3}{4} N_0$$

$$e^{(0.3465)t} = \frac{4}{3}$$

$$0.3465 t = \log_e \left(\frac{4}{3} \right)$$

$$= 2.303 [\log_4 - \log_3]$$

$$= 2.303 [0.6020 - 0.4771]$$

$$= 2.303 \times 0.1249$$

$$t = \frac{2.303 \times 0.1249}{0.3465}$$

$$\therefore t = 0.83 \text{ days or } 19.92 \text{ hours}$$

27)

$$N = N_0 e^{-\lambda t}$$

$$\frac{3}{4} N_0 = N_0 e^{-(0.3465)t} \quad (\text{because } N = \frac{3}{4} N_0)$$

$$e^{(0.3465)t} = \frac{4}{3}$$

$$0.3465 \times t = \log_e \left(\frac{4}{3} \right)$$

$$= 2.303 [\log 4 - \log 3]$$

$$= 2.303 [0.6020 - 0.4771]$$

$$= 2.303 \times 0.1249$$

$$t = \frac{2.303 \times 0.1249}{0.3465}$$

\therefore t = 0.83 days \ or \ 19.92 \ hours

- 28)
- (a) Keen observer/helpful/concerned/responsible/respectful towards elders(Any two)
- (b) The doctor can trace and observe, the difference between the moment of an appropriate through a normal brain and a brain having tumour in it.
-

- 29)
- While drawing the plot. we have to keep in mind that first binding energy will increase sharply and then it will be constant almost.
- For plot of binding energy per nucleon as the function of mass number A
- Following are the two conclusions that can be drawn regarding the nature of the nuclear force.
- The force is attractive and strong enough to produce a binding energy of few MeV per nucleon.
- The two important conclusions regarding the nature of nuclear force are given below.
- (i) The nuclear force is attractive and sufficiently strong to produce a binding energy of a few MeV per nucleon.
- (ii) The constancy of the binding energy in the wide range of mass number $30 < A < 170$ indicate that nuclear force is a short-range force.
- (b) (i) According to the binding energy curve, a very heavy nucleus ($A > 170$), has lower binding energy per nucleon compared to nuclei of middle mass number ($30 < A < 170$).
- Thus, if a heavy nucleus breaks into two nuclei of mass number between 30 and 170, nucleons get more tightly bound. This implies energy would be released in the process. (nuclear fission)
- (ii) When two light nuclei ($A \leq 10$) join to form a heavier nucleus, the binding energy per nucleon of fused heavier nucleus increases.
- Again it indicates that energy would be released in the process (nuclear fusion).
- (c) The basic nuclear process of neutron undergoing β -decay is given as
- $$n \rightarrow p + e^{-} + \bar{\nu}$$
- Here $\bar{\nu}$ is antineutrino.
- Neutrino and antineutrino both are neutral particles with very small (possibly, even zero) mass compared to the electrons. They have only weak interaction with other particles. Therefore, the detection of neutrinos is found very difficult.
-

30)

The Q-value of a nuclear process refers the energy release in the nuclear process which can be determined using Einstein's mass-energy relation, $E = mc^2$. The Q-value is equal to the difference of mass of products and reactant nuclei multiplied by square of velocity of light.

$$Q = [m_{\alpha} - m_{\text{He}}] C^2 \text{ in } \alpha\text{-decay}$$

The nuclear process does not proceed spontaneously when Q- value of a process is negative or sum of masses of product is greater than sum of masses of reactant.

31)

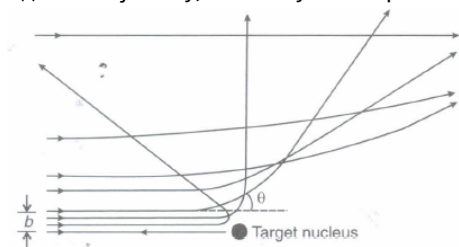
Installation at chernobyl is a pressurised fusion reactor. Possible cause of disaster is melt down of core due to excessive heat development, which occurs when k (multiplication factor) become more than 1.

In a fusion reaction, energy released = (mass defect) $\times c^2$.

According to Asha-subject knowledge and knowledge sharing.

32)

(i) The trajectory, traced by the α -particles in the Coulomb field of target nucleus, has the form shown below.



The size of the nucleus was estimated by observing the distance (d) of closest approach, of the α -particles. This distance is given by :

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze)(2e)}{d} = K$$

where K = kinetic energy of the α -particles when they are far away from the target nuclei.

The wave nature of moving electrons was established through the Davisson-Germer experiment.

In this experiment, it was observed that a beam of electrons, when scattered by a nickel target, showed 'maxima' in certain directions; (like the 'maxima' observed in interference/diffraction experiments with light.)

(iii) We have: $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_d}{\lambda_{\alpha}} = \sqrt{\frac{m_{\alpha} q_{\alpha}}{m_d q_d}} \sqrt{2} = 2$$

33)

(a) The alpha decay of ${}_{92}^{238}\text{U}$ is given by

The energy released in this process is given by

$$Q = \left(M_{\text{U}} - M_{\text{Th}} - M_{\text{He}} \right) c^2$$

Substituting the atomic masses as given in the data, we find

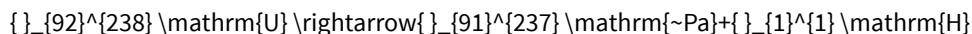
$$Q = (238.05079 - 234.04363 - 4.00260) \text{ u} \times c^2$$

$$= (0.00456 \text{ u}) c^2$$

$$= (0.00456 \text{ u})(931.5 \text{ MeV} / \text{u})$$

$$= 4.25 \text{ MeV}$$

(b) If ${}_{92}^{238}\text{U}$ spontaneously emits a proton, the decay process would be



The Q for this process to happen is

$$= (M_{\text{U}} - M_{\text{Pa}} - M_{\text{H}}) c^2$$

$$= (238.05079 - 237.05121 - 1.00783) \text{ u} \times c^2$$

$$= (-0.00825 \text{ u}) c^2$$

$$= (-0.00825 \text{ u})(931.5 \text{ MeV} / \text{u})$$

$$= -7.68 \text{ MeV}$$

Thus, the Q of the process is negative and therefore it cannot proceed spontaneously. We will have to supply an energy of 7.68 MeV to a ${}_{92}^{238}\text{U}$ nucleus to make it emit a proton.

34)

(a) A chemical equation is balanced in the sense that the number of atoms of each element is the same on both sides of the equation. A chemical reaction merely alters the original combinations of atoms. In a nuclear reaction, elements may be transmuted. Thus, the number of atoms of each element is not necessarily conserved in a nuclear reaction. However, the number of protons and the number of neutrons are both separately conserved in a nuclear reaction. [Actually, even this is not strictly true in the realm of very high energies – what is strictly conserved is the total charge and total ‘baryon number’. We need not pursue this matter here.] In nuclear reactions, the number of protons and the number of neutrons are the same on the two sides of the equation.

(b) We know that the binding energy of a nucleus gives a negative contribution to the mass of the nucleus (mass defect). Now, since proton number and neutron number are conserved in a nuclear reaction, the total rest mass of neutrons and protons is the same on either side of a reaction. But the total binding energy of nuclei on the left side need not be the same as that on the right hand side. The difference in these binding energies appears as energy released or absorbed in a nuclear reaction. Since binding energy contributes to mass, we say that the difference in the total mass of nuclei on the two sides get converted into energy or vice-versa. It is in these sense that a nuclear reaction is an example of mass-energy interconversion.

(c) From the point of view of mass-energy interconversion, a chemical reaction is similar to a nuclear reaction in principle. The energy released or absorbed in a chemical reaction can be traced to the difference in chemical (not nuclear) binding energies of atoms and molecules on the two sides of a reaction. Since, strictly speaking, chemical binding energy also gives a negative contribution (mass defect) to the total mass of an atom or molecule, we can equally well say that the difference in the total mass of atoms or molecules, on the two sides of the chemical reaction gets converted into energy or vice-versa. However, the mass defects involved in a chemical reaction are almost a million times smaller than those in a nuclear reaction. This is the reason for the general impression, (which is incorrect) that mass-energy interconversion does not take place in a chemical reaction.

- 35) Mass of lithium isotope ${}^6_3\text{Li}$, $m_1 = 6.01512 \text{ u}$
 Mass of lithium isotope ${}^7_3\text{Li}$, $m_2 = 7.01600 \text{ u}$
 Abundance of ${}^6_3\text{Li}$, $\eta_1 = 7.5\%$
 Abundance of ${}^7_3\text{Li}$, $\eta_2 = 92.5\%$
 The atomic mass of lithium atom is given as:

$$m = \frac{m_1 \eta_1 + m_2 \eta_2}{\eta_1 + \eta_2}$$

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{92.5 + 7.5}$$

$$= 6.940934 \text{ u}$$
 Mass of boron isotope ${}^{10}_5\text{B}$, $m_1 = 10.01294 \text{ u}$
 Mass of boron isotope ${}^{11}_5\text{B}$, $m_2 = 11.00931 \text{ u}$
 Abundance of ${}^{10}_5\text{B}$, $\eta_1 = x\%$
 Abundance of ${}^{11}_5\text{B}$, $\eta_2 = (100 - x)\%$
 Atomic mass of boron, $m = 10.811 \text{ u}$
 The atomic mass of boron atom is given as:

$$m = \frac{m_1 \eta_1 + m_2 \eta_2}{\eta_1 + \eta_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$1081.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\therefore x = \frac{19.821}{0.99637} = 19.89\%$$
 And $100 - x = 80.11\%$
 Hence, the abundance of ${}^{10}_5\text{B}$ is 19.89% and that of ${}^{11}_5\text{B}$ is 80.11%.

- 36) Atomic mass of nitrogen ${}^{14}_7\text{N}$, $m = 14.00307 \text{ u}$
 A nucleus of nitrogen ${}^{14}_7\text{N}$ contains 7 protons and 7 neutrons.
 Hence, the mass defect of this nucleus, $\Delta m = 7m_{\text{H}} + 7m_{\text{n}} - m$
 Where,
 Mass of a proton, $m_{\text{H}} = 1.007825 \text{ u}$
 Mass of a neutron, $m_{\text{n}} = 1.008665 \text{ u}$

$$\therefore \Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$$

$$= 7.054775 + 7.060655 - 14.00307$$

$$= 0.11236 \text{ u}$$
 But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$$\therefore \Delta m = 0.11236 \times 931.5 \text{ MeV}/c^2$$
 Hence, the binding energy of the nucleus is given as:

$$E_{\text{b}} = \Delta mc^2$$
 Where,
 $c = \text{Speed of light}$

$$\therefore E_{\text{b}} = 0.11236 \times 931.5 \left(\frac{\text{MeV}}{c^2} \right) \times c^2$$

$$= 104.66334 \text{ MeV}$$
 Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

- 37) Atomic mass of ${}_{26}^{56}\text{Fe}$, $m_1 = 55.934939 \text{ u}$
 ${}_{26}^{56}\text{Fe}$ nucleus has 26 protons and $(56 - 26) = 30$ neutrons
 Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of a proton, $m_H = 1.007825 \text{ u}$

Mass of a neutron, $m_n = 1.008665 \text{ u}$

$$\therefore \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$$

$$= 26.20345 + 30.25995 - 55.934939$$

$$= 0.528461 \text{ u}$$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$$\therefore \Delta m = 0.528461 \times 931.5 \text{ MeV}/c^2$$

The binding energy of this nucleus is given as:

$$E_{b1} = \Delta m c^2$$

Where,

c = Speed of light

$$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$$

$$= 492.26 \text{ MeV}$$

$$\text{Average binding energy per nucleon} = \frac{492.26}{56} = 8.79 \text{ MeV}$$

Atomic mass of ${}_{83}^{209}\text{Bi}$, $m_2 = 208.980388 \text{ u}$

${}_{83}^{209}\text{Bi}$ nucleus has 83 protons and $(209 - 83) = 126$ neutrons.

Hence, the mass defect of this nucleus is given as:

$$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$$

Where,

Mass of a proton, $m_H = 1.007825 \text{ u}$

Mass of a neutron, $m_n = 1.008665 \text{ u}$

$$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$$

$$= 83.649475 + 127.091790 - 208.980388$$

$$= 1.760877 \text{ u}$$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$$\therefore \Delta m' = 1.760877 \times 931.5 \text{ MeV}/c^2$$

Hence, the binding energy of this nucleus is given as:

$$E_{b2} = \Delta m' c^2$$

$$= 1.760877 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$$

$$= 1640.26 \text{ MeV}$$

$$\text{Average binding energy per nucleon} = \frac{1640.26}{209} = 7.848 \text{ MeV}$$

38)

Mass of a copper coin, $m' = 3 \text{ g}$

Atomic mass of ${}_{29}^{63}\text{Cu}$ atom, $m = 62.92960 \text{ u}$

The total number ${}_{29}^{63}\text{Cu}$ of atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where,

$N_A = \text{Avogadro's number} = 6.023 \times 10^{23} \text{ atoms/g}$

Mass number = 63 g

$\therefore N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22} \text{ atoms/g}$

${}_{29}^{63}\text{Cu}$ nucleus has 29 protons and $(63 - 29) = 34$ neutrons

\therefore Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, $m_H = 1.007825 \text{ u}$

Mass of a neutron, $m_n = 1.008665 \text{ u}$

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

$= 0.591935 \text{ u}$

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22} \text{ u}$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5 \text{ MeV}/c^2$

Hence, the binding energy of the nuclei of the coin is given as:

$E_b = \Delta mc^2$

$= 1.69766958 \times 10^{22} \times 931.5 \left(\frac{\text{MeV}}{c^2} \right) \times c^2$

$= 1.581 \times 10^{25} \text{ MeV}$

But $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$= 2.5296 \times 10^{12} \text{ J}$

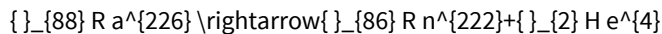
This much energy is required to separate all the neutrons and protons from the given coin.

39)

$\text{{ (i) } \alpha \text{{ -decay of } }_{88}^{226} \text{{Ra}}$

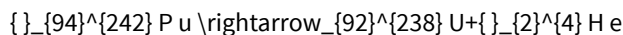
α is a nucleus of helium $\left({}_{2}^{4}\text{He}\right)$ and β is an electron (e^{-} for β^{-} and e^{+} for β^{+}). In every α -decay, there is a loss of 2 protons and 4 neutrons. In every β^{+} -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^{-} -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:



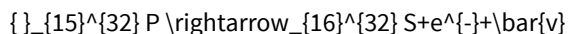
$\text{{ (ii) } \alpha \text{{ -decay of } }_{94}^{242} \text{{Pu}}$

α is a nucleus of helium $\left({}_{2}^{4}\text{He}\right)$ and β is an electron (e^{-} for β^{-} and e^{+} for β^{+}). In every α -decay, there is a loss of 2 protons and 4 neutrons. In every β^{+} -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^{-} -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.



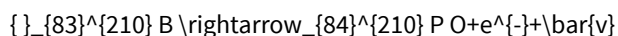
$\text{{ (iii) } \beta \text{{ -decay of } }_{15}^{32} \text{{P}}$

α is a nucleus of helium $\left({}_{2}^{4}\text{He}\right)$ and β is an electron (e^{-} for β^{-} and e^{+} for β^{+}). In every α -decay, there is a loss of 2 protons and 4 neutrons. In every β^{+} -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^{-} -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.



$\text{{ (iv) } \beta \text{{ -decay of } }_{83}^{210} \text{{Bi}}$

α is a nucleus of helium $\left({}_{2}^{4}\text{He}\right)$ and β is an electron (e^{-} for β^{-} and e^{+} for β^{+}). In every α -decay, there is a loss of 2 protons and 4 neutrons. In every β^{+} -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^{-} -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.



$\text{{ (v) } \beta \text{{ +decay of } }_{6}^{11} \text{{C}}$

α is a nucleus of helium $\left({}_{2}^{4}\text{He}\right)$ and β is an electron (e^{-} for β^{-} and e^{+} for β^{+}). In every α -decay, there is a loss of 2 protons and 4 neutrons. In every β^{+} -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^{-} -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, nuclear reactions can be written as:



$\text{{ (vi) } \beta \text{{ +decay of } }_{43}^{97} \text{{Tc}}$

α is a nucleus of helium $\left({}_{2}^{4}\text{He}\right)$ and β is an electron (e^{-} for β^{-} and e^{+} for β^{+}). In every α -decay, there is a loss of 2 protons and 4 neutrons. In every β^{+} -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^{-} -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, nuclear reactions can be written as:



$\text{{ (vii) Electron capture of } }_{54}^{120} \text{{Xe}}$

α is a nucleus of helium $\left({}_{2}^{4}\text{He}\right)$ and β is an electron (e^{-} for β^{-} and e^{+} for β^{+}). In every α -decay, there is a loss of 2 protons and 4 neutrons. In every β^{+} -decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^{-} -decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, nuclear reactions can be written as:



40)

Half-life of the radioactive isotope = T years

Original amount of the radioactive isotope = N_0

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125% of N_0 remains after decay. Hence, we can write

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

$$\text{But, } \frac{N}{N_0} = e^{-\lambda t}$$

Where,

λ = Decay constant

t = Time

$$\therefore -\lambda t = \frac{1}{32}$$

$$-\lambda t = \ln 1 - \ln 32$$

$$-\lambda t = 0 - 3.4657$$

$$t = \frac{3.4657}{\lambda}$$

$$\text{Since, } \lambda = \frac{0.693}{T}$$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5 \text{ T year}$$

Hence, the isotope will take about 5T years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

$$\text{But } \frac{N}{N_0} = e^{-\lambda t}$$

$$-\lambda t = \ln 1 - \ln 100$$

$$t = \frac{4.6052}{\lambda}$$

$$\text{Since } \lambda = \frac{0.693}{T}$$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645 T \text{ year}$$

Hence, the isotope will take about 6.645T years to reduce to 1% of its original value.

41)

The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$

$$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$$

$$= 29.6 \times 10^7 \text{ decay / s}$$

N = Required number of atoms

$$\text{Half-life of } {}_{27}^{60}\text{Co}, T_{\frac{1}{2}} = 5.3 \text{ years}$$

$$= 5.3 \times 365 \times 24 \times 60 \times 60$$

$$= 1.67 \times 10^8 \text{ s}$$

For decay constant λ , we have the rate of decay as:

$$\frac{dN}{dt} = \lambda N$$

$$\text{Where, } \lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{1.67 \times 10^8} \text{ s}^{-1}$$

$$\therefore N = \frac{1}{\lambda} \frac{dN}{dt}$$

$$= \frac{29.6 \times 10^7}{\frac{0.693}{1.67 \times 10^8}} = 7.133 \times 10^{16} \text{ atom}$$

For ${}_{27}^{60}\text{Co}$

$$\text{Mass of } 6.023 \times 10^{23} \text{ (Avogadro's number) atoms} = 60 \text{ g}$$

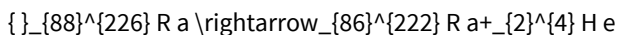
$$\therefore \text{Mass of } 7.133 \times 10^{16} \text{ atoms}$$

$$= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6} \text{ g}$$

Hence, the amount of ${}_{27}^{60}\text{Co}$ necessary for the purpose is $7.106 \times 10^{-6} \text{ g}$

42)

(a) Alpha particle decay of ${}_{88}^{226}\text{Ra}$ emits a helium nucleus. As a result, its mass number reduces to $(226 - 4) = 222$ and its atomic number reduces to $(88 - 2) = 86$. This is shown in the following nuclear reaction.



Q-value of

$$\text{emitted } \alpha\text{-particle} = (\text{Sum of initial mass} - \text{Sum of final mass}) c^2$$

Where,

c = Speed of light

It is given that:

$$m\left({}_{88}^{226}\text{Ra}\right) = 226.02540 \text{ u}$$

$$m\left({}_{86}^{222}\text{Rn}\right) = 222.01750 \text{ u}$$

$$m\left({}_2^4\text{He}\right) = 4.002603 \text{ u}$$

$$Q\text{-value} = [226.02540 - (222.01750 + 4.002603)] u c^2$$

$$= 0.005297 u c^2$$

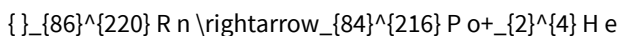
$$\text{But } 1 u = 931.5 \text{ MeV}/c^2$$

$$\therefore Q = 0.005297 \times 931.5 \approx 4.94 \text{ MeV}$$

$$\text{Kinetic energy of the } \alpha\text{-particle} = \frac{\text{Mass number after decay}}{\text{Mass number before decay}} \times Q$$

$$= \frac{222}{226} \times 4.94 = 4.85 \text{ MeV}$$

(b) Alpha particle decay of ${}_{86}^{220}\text{Rn}$ is shown by the following nuclear reaction.



It is given that:

$$\text{Mass of } ({}_{86}^{220}\text{Rn}) = 220.01137 \text{ u}$$

$$\text{Mass of } ({}_{84}^{216}\text{Po}) = 216.00189 \text{ u}$$

$$\therefore Q\text{-value} = [220.01137 - (216.00189 + 0.00260)] \times 931.5$$

$$\approx 641 \text{ MeV}$$

$$\text{Kinetic energy of the } \alpha\text{-particle} = \left(\frac{220-4}{220}\right) \times 6.41$$

$$= 6.29 \text{ MeV}$$

43)

The given nuclear reaction is:



Half life of ${}_{6}^{11}\text{C}$ (nuclei) $T_{\frac{1}{2}} = 20.3 \text{ min}$

Atomic mass of ${}_{6}^{11}\text{C}$ (right) = 11.011434 u

Atomic mass of ${}_{5}^{11}\text{B}$ (right) = 11.009305 u

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the ${}_{6}^{11}\text{C}$ nucleus is given as:

$$\Delta Q = [m({}_{6}^{11}\text{C}) - m({}_{5}^{11}\text{B}) - m_e] c^2 \quad (1)$$

Where,

m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light

m' = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add $6 m_e$ in the case of ${}^{11}\text{C}$ and $5 m_e$ in the case of ${}^{11}\text{B}$

Hence, equation (1) reduces to:

$$\Delta Q = [m({}_{6}^{11}\text{C}) - m({}_{5}^{11}\text{B}) - 2 m_e] c^2$$

(where m are the atomic masses)

(Here $m({}_{6}^{11}\text{C})$ and $m({}_{5}^{11}\text{B})$ are the atomic masses)

$$\therefore \Delta Q = [11.011434 - 11.009305 - 2 \times 0.000548] c^2$$

$$= (0.001033 c^2) \text{ u}$$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

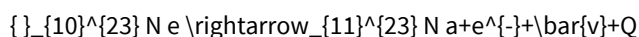
$$\therefore \Delta Q = 0.001033 \times 931.5 \approx 0.962 \text{ MeV}$$

The value of Q is almost comparable to the maximum energy of the emitted positron.

44)

In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus ${}_{10}^{23}\text{Ne}$ is given as:



It is given that:

Atomic mass of ${}_{10}^{23}\text{Ne}$ = 22.994466 u

Atomic mass of ${}_{11}^{23}\text{Na}$ = 22.989770 u

Mass of an electron, $m_e = 0.000548 \text{ u}$

Q-value of the given reaction is given as:

$Q = [m({}_{10}^{23}\text{Ne}) - m({}_{11}^{23}\text{Na}) - m_e] c^2$ There are 10 electrons in ${}_{10}^{23}\text{Ne}$ and 11 electrons in ${}_{11}^{23}\text{Na}$. Hence, the mass of the electron is cancelled in the Q-value equation.

$$\therefore Q = [22.994466 - 22.989770] c^2$$

$$= (0.004696 c^2) \text{ u}$$

But, $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$$\therefore Q = 0.004696 \times 931.5 = 4.374 \text{ MeV}$$

The daughter nucleus is too heavy as compared to e^{-} and $\bar{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e., 4.374 MeV.

45)

The given nuclear reaction is:



It is given that:

$$\text{Atomic mass } m({}_1^1\text{H}) = 1.007825 \text{ u}$$

$$\text{Atomic mass } m({}_1^3\text{H}) = 3.016049 \text{ u}$$

$$\text{Atomic mass } m({}_1^2\text{H}) = 2.014102 \text{ u}$$

According to the question, the Q -value of the reaction can be written as:

$$Q = [m({}_1^1\text{H}) + m({}_1^3\text{H}) - 2m({}_1^2\text{H})] c^2$$

$$= [1.007825 + 3.016049 - 2 \times 2.014102] c^2$$

$$Q = -0.00433 \times 931.5 = -4.0334 \text{ MeV}$$

The negative Q -value of the reaction shows that the reaction is endothermic.

The given nuclear reaction is:



It is given that:

$$\text{Atomic mass of } m({}_{6}^{12}\text{C}) = 12.0 \text{ u}$$

$$\text{Atomic mass of } m({}_{10}^{20}\text{Ne}) = 19.992439 \text{ u}$$

$$\text{Atomic mass of } m({}_{2}^{4}\text{He}) = 4.002603 \text{ u}$$

The Q -value of this reaction is given as:

$$Q = [2m({}_{6}^{12}\text{C}) - m({}_{10}^{20}\text{Ne}) - m({}_{2}^{4}\text{He})] c^2$$

$$= [2 \times 12.0 - 19.992439 - 4.002603] c^2$$

$$= -0.004958 c^2 \text{ u}$$

$$= -0.004958 \times 931.5 = -4.618377 \text{ MeV}$$

The positive Q -value of the reaction shows that the reaction is exothermic.

46)

Half life of the fuel of the fission reactor, $t_{(1/2)} = \text{years}$

$$= 5 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

We know that in the fission of 1 g of ${}_{92}^{235}\text{U}$ nucleus, the energy released is equal to 200 MeV .

1 mole , i.e., 235 g of ${}_{92}^{235}\text{U}$ contains 6.023×10^{23} atoms.

$$\therefore 1 \text{ g } {}_{92}^{235}\text{U} \text{ contains } \frac{6.023 \times 10^{23}}{235}$$

The total energy generated per gram of ${}_{92}^{235}\text{U}$ is calculated as:

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV} / \text{g}$$

$$= (200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19})$$

The reactor operates only 80% of the time.

Hence, the amount of ${}_{92}^{235}\text{U}$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$= \frac{5 \times 80 \times 60 \times 60 \times 365 \times 24 \times 1000 \times 10^6}{100 \times 8.20 \times 10^{10}}$$

$$\approx 1538 \text{ kg}$$

$$\therefore \text{Initial amount of } {}_{92}^{235}\text{U} = 2 \times 1538 = 3076 \text{ kg}$$

47) The given fusion reaction is:

$${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + {}_0^1\text{n} + 3.27 \text{ MeV}$$

Amount of deuterium, $m = 2 \text{ kg}$

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

\therefore 2.0 kg of deuterium contains $= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

\therefore Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^6$$

$$= 1.576 \times 10^{14} \text{ J}$$

Power of the electric lamp, $P = 100 \text{ W} = 100 \text{ J/s}$

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576 \times 10^{14}}{100} \text{ s}$$

$$\frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365} \approx 4.9 \times 10^4 \text{ year}$$

48)

When two deuterons collide head-on, the distance between their centres, d is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = $2 \times 10^{-15} \text{ m}$

$$\therefore d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15} \text{ m}$$

Charge on a deuteron nucleus = Charge on an electron = $e = 1.6 \times 10^{-19} \text{ C}$

Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4 \pi \epsilon_0 d}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4 \pi \epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} \text{ J}$$

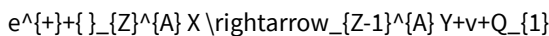
$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} \times (1.6 \times 10^{-19}) \text{ eV}$$

$$= 360 \text{ keV}$$

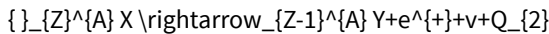
Hence, the height of the potential barrier of the two-deuteron system is 360 keV

49)

Let the amount of energy released during the electron capture process be Q_1 . The nuclear reaction can be written as:



Let the amount of energy released during the positron capture process be Q_2 . The nuclear reaction can be written as:



$$m_N \left({}_Z^A X \right) = \text{Nuclear mass of } {}_Z^A X$$

$$m_N \left({}_{Z-1}^A Y \right) = \text{Nuclear mass of } {}_{Z-1}^A Y$$

$$m \left({}_Z^A X \right) = \text{Atomic mass of } {}_Z^A X$$

$$m \left({}_{Z-1}^A X \right) = \text{Nuclear mass of } {}_{Z-1}^A X$$

$$m_e = \text{Mass of an electron}$$

$$c = \text{Speed of light}$$

Q-value of the electron capture reaction is given as:

$$Q_1 = \left[m_N \left({}_Z^A X \right) + m_e - m_N \left({}_{Z-1}^A Y \right) \right] c^2$$

$$= \left[m \left({}_Z^A X \right) - Z m_e + m_e - m \left({}_{Z-1}^A Y \right) + (Z-1) m_e \right] c^2$$

$$= \left[m \left(\begin{array}{c} c \\ A \\ Z \end{array} X \right) - m \left(\begin{array}{c} l \\ A \\ z-1 \end{array} \right) \right] c^2$$

Q-value of the positron capture reaction is given as:

$$Q_2 = \left[m_N \left({}_Z^A X \right) - m_N \left({}_{Z-1}^A Y \right) - m_e \right] c^2$$

$$= \left[m \left(\begin{array}{c} c \\ A \\ Z \end{array} X \right) - m \left(\begin{array}{c} l \\ A \\ z-1 \end{array} \right) + (Z-1) \right. \\ \left. m_e - m_e \right] c^2$$

$$= \left[m \left({}_Z^A X \right) - m \left({}_{Z-1}^A Y \right) - 2 m_e \right] c^2 \dots (4)$$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

In other words, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically-allowed nuclear reaction.

50)

Average atomic mass of magnesium, $m = 24.312 \text{ u}$

Mass of magnesium isotope ${}_{12}^{24} \text{Mg}$, $m_1 = 23.98504 \text{ u}$

Mass of magnesium isotope ${}_{12}^{25} \text{Mg}$, $m_2 = 24.98584 \text{ u}$

Mass of magnesium isotope ${}_{12}^{26} \text{Mg}$, $m_3 = 25.98259 \text{ u}$

Abundance of ${}_{12}^{24} \text{Mg}$, $\eta_1 = 78.99\%$

Abundance of ${}_{12}^{25} \text{Mg}$, $\eta_2 = x\%$

Hence, abundance of ${}_{12}^{26} \text{Mg}$, $\eta_3 = 100 - x - 78.99\% = (21.01 - x)\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1 \eta_1 + m_2 \eta_2 + m_3 \eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$2431.2 = 1894.5783096 + 24.98584 x + 545.8942159 - 25.98259 x$$

$$0.99675x = 9.2725255$$

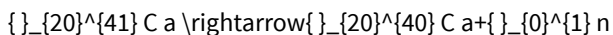
$$\therefore x = 9.3\%$$

$$\text{And } 21.01 - x = 11.71\%$$

Hence, the abundance of ${}_{12}^{25} \text{Mg}$ is 9.3% and that of ${}_{12}^{26} \text{Mg}$ is 11.71%.

51)

For ${}_{20}^{41}\text{Ca}$: Separation energy = 8.363007 MeV For ${}_{13}^{27}\text{Al}$: Separation energy = 13.059 MeV A neutron (${}_0^1\text{n}$) is removed from a ${}_{20}^{41}\text{Ca}$ nucleus. The corresponding nuclear reaction can be written as:



It is given that:

$$\text{Mass } m({}_{20}^{40}\text{Ca}) = 39.962591 \text{ u}$$

$$\text{Mass } m({}_{20}^{41}\text{Ca}) = 40.962278 \text{ u}$$

$$\text{Mass } m({}_0^1\text{n}) = 1.008665 \text{ u}$$

The mass defect of this reaction is given as:

$$\Delta m = m\left({}_{20}^{40}\text{Ca}\right) + m\left({}_0^1\text{n}\right) - m\left({}_{20}^{41}\text{Ca}\right)$$

$$= 39.962591 + 1.008665 - 40.962278 = 0.008978 \text{ u}$$

$$\text{But } 1 \text{ u} = 931.5 \text{ MeV} / c^2$$

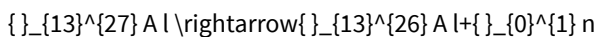
$$\therefore \Delta m = 0.008978 \times 931.5 \text{ MeV} / c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta m c^2$$

$$= 0.008978 \times 931.5 = 8.363007 \text{ MeV}$$

For ${}_{13}^{27}\text{Al}$ the neutron removal reaction can be written as:



it is given that.

$$\text{Mass } m({}_{13}^{27}\text{Al}) = 26.981541 \text{ u}$$

$$\text{Mass } m({}_{13}^{26}\text{Al}) = 25.986895 \text{ u}$$

The mass defect of this reaction is given as:

$$\Delta m = m\left({}_{13}^{26}\text{Al}\right) + m\left({}_0^1\text{n}\right) - m\left({}_{13}^{27}\text{Al}\right)$$

$$= 25.986895 + 1.008665 - 26.981541$$

$$= 0.014019 \text{ u}$$

$$= 0.014019 \times 931.5 \text{ MeV} / c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta m c^2$$

$$= 0.014019 \times 931.5 = 13.059 \text{ MeV}$$

52)

Half life of ${}_{32}^{15}\text{P}$, $T_{1/2} = 14.3$ days

Half life of ${}_{33}^{15}\text{P}$, $T'_{1/2} = 25.3$ days

${}_{33}^{15}\text{P}$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of ${}_{33}^{15}\text{P}$ nucleus and 90% of ${}_{32}^{15}\text{P}$ nucleus.

Suppose after t days, the source has 10% of ${}_{32}^{15}\text{P}$ nucleus and 90% of ${}_{33}^{15}\text{P}$ nucleus.

Initially:

Number of ${}_{33}^{15}\text{P}$ nucleus = N

Number of ${}_{32}^{15}\text{P}$ nucleus = $9N$

Finally:

Number of ${}_{33}^{15}\text{P}$ nucleus = N'

Number of ${}_{32}^{15}\text{P}$ nucleus = $9N'$

For ${}_{32}^{15}\text{P}$ nucleus, we can write the number ratio as:

$$N' \frac{9N}{9N} = \frac{\left(\frac{1}{2}\right)^t}{T_{1/2}}$$

$$N' = 9N \left(2\right)^{-\frac{t}{14.3}}$$

For ${}_{32}^{15}\text{P}$ we can write the number ratio as:

$$9N' = 9N \left(2\right)^{-\frac{t}{25.3}} \dots (2)$$

On dividing equation (1) by equation (2), we get:

$$\frac{1}{9} = 2^{\frac{t}{25.3} - \frac{t}{14.3}}$$

$$\frac{1}{9} = 2^{-\left(11 \frac{t}{25.3 \times 14.3}\right)}$$

$$\log 1 - \log 9 = -11t / (25.3 \times 14.3) \log 2$$

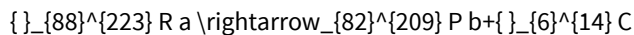
$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = (25.3 \times 14.3 \times 1.908) / 11 \times 0.301 \sim 208.5 \text{ day}$$

Hence, it will take about 208.5 days for 90% decay of ${}_{33}^{15}\text{P}$.

53)

Take a ${}_{6}^{14}\text{C}$ emission nuclear reaction:



We know that:

$$\text{Mass of } {}_{88}^{223}\text{Ra}, m_1 = 223.01850 \text{ u}$$

$$\text{Mass of } {}_{82}^{209}\text{Pb}, m_2 = 208.98107 \text{ u}$$

$$\text{Mass of } {}_{6}^{14}\text{C}, m_3 = 14.00324 \text{ u}$$

Hence, the Q-value of the reaction is given as:

$$Q = (m_1 - m_2 - m_3) c^2$$

$$= (223.01850 - 208.98107 - 14.00324) c^2$$

$$= (0.03419 c^2) \text{ u}$$

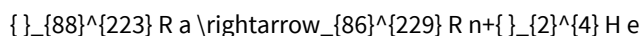
$$\text{But } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$$\therefore Q = 0.03419 \times 931.5$$

$$= 31.848 \text{ MeV}$$

Hence, the Q-value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a ${}_{2}^4\text{He}$ emission nuclear reaction:



We know that:

$$\text{Mass of } {}_{88}^{223}\text{Ra}, m_1 = 223.01850$$

$$\text{Mass of } {}_{86}^{229}\text{Rn}, m_2 = 219.00948$$

$$\text{Mass of } {}_{2}^4\text{He}, m_3 = 4.00260$$

Q-value of this nuclear reaction is given as:

$$Q = (m_1 - m_2 - m_3) c^2$$

$$= (223.01850 - 219.00948 - 4.00260) c^2$$

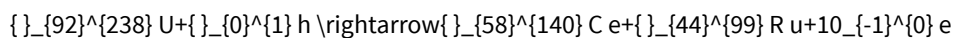
$$= (0.00642 c^2) \text{ u}$$

$$= 0.00642 \times 931.5 = 5.98 \text{ MeV}$$

Hence, the Q value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically allowed.

54)

In the fission of ${}_{92}^{238}\text{U}$, $10\beta^-$ particles decay from the parent nucleus. The nuclear reaction can be written as:



It is given that:

Mass of a nucleus ${}_{92}^{238}\text{U}$ $m_1 = 238.05079\text{ u}$

Mass of a nucleus ${}_{58}^{140}\text{Ce}$ $m_2 = 139.90543\text{ u}$

Mass of a nucleus ${}_{44}^{99}\text{Ru}$, $m_3 = 98.90594\text{ u}$

Mass of a neutron $m_4 = 1.008665\text{ u}$

Q-value of the above equation,

$$Q = [m({}_{92}^{238}\text{U}) + m({}_0^1\text{n}) - m({}_{58}^{140}\text{Ce}) - m({}_{44}^{99}\text{Ru}) - 10m_e] c^2$$

Where,

m' = Represents the corresponding atomic masses of the nuclei

$$m({}_{92}^{238}\text{U}) = m_1 - 92m_e$$

$$m({}_{58}^{140}\text{Ce}) = m_2 - 58m_e$$

$$m({}_{44}^{99}\text{Ru}) = m_3 - 44m_e$$

$$m({}_0^1\text{n}) = m_4$$

$$Q = [m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e] c^2$$

$$m({}_{92}^{238}\text{U}) = m_1 - 92m_e$$

$$m({}_{58}^{140}\text{Ce}) = m_2 - 58m_e$$

$$m({}_{44}^{99}\text{Ru}) = m_3 - 44m_e$$

$$m({}_0^1\text{n}) = m_4$$

$$Q = [m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e] c^2$$

$$= [m_1 + m_4 - m_2 - m_3] c^2$$

$$= [238.0507 + 1.008665 - 139.90543 - 98.90594] c^2$$

$$= [0.247995] c^2 \text{ u}$$

$$\text{But } 1\text{ u} = 931.5\text{ MeV} \left[\frac{V}{c^2} \right]$$

$$Q = 0.247995 \times 931.5 = 231.007 \text{ MeV}$$

Hence, the Q-value of the fission process is 231.007 MeV.

55)

Take the D-T nuclear reaction: ${}_1^2\text{H} + {}_1^3\text{H} \rightarrow {}_2^4\text{He} + n$

It is given that:

Mass of ${}_1^2\text{H}$ $m_1 = 2.014102 \text{ u}$

Mass of ${}_1^3\text{H}$ $m_2 = 3.016049 \text{ u}$

Mass of ${}_2^4\text{He}$ $m_3 = 4.002603 \text{ u}$

Mass of ${}_0^1\text{n}$ $m_4 = 1.008665 \text{ u}$

Q-value of the given D-T reaction is:

$$Q = [m_1 + m_2 - m_3 - m_4] c^2$$

$$= [2.014102 + 3.016049 - 4.002603 - 1.008665] c^2$$

$$= [0.018883 c^2] \text{ u}$$

$$\text{But } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$$\therefore Q = 0.018883 \times 931.5 = 17.59 \text{ MeV.}$$

Radius of deuterium and tritium, $r \approx 2.0 \text{ fm} = 2 \times 10^{-15} \text{ m}$

Distance between the two nuclei at the moment when they touch each other, $d = r + r = 4 \times 10^{-15} \text{ m}$

Charge on the deuterium nucleus = e

Charge on the tritium nucleus = e

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi\epsilon_0(d)}$$

Where

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14} \text{ J}$$

$$\approx 360 \text{ KeV}$$

$$= \frac{5.76 \times 10^{-14} \text{ J}}{1.6 \times 10^{-19}} = 3.6 \times 10^5 \text{ eV} = 360 \text{ KeV}$$

Hence, $5.76 \times 10^{-14} \text{ J}$ or 360 KeV of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.

However, it is given that:

$$KE = \frac{3}{2} kT$$

Where,

$$k = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$$

T = Temperature required for triggering the reaction

$$T = \frac{KE}{\frac{3}{2} k}$$

$$= \frac{5.76 \times 10^{-14}}{\frac{3}{2} \times 1.38 \times 10^{-23}} = 1.39 \times 10^9 \text{ K}$$

Hence, the gas must be heated to a temperature of $1.39 \times 10^9 \text{ K}$ to initiate the reaction.

56)

It can be observed from the given γ -decay diagram that γ_1 decays from the 1.088 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_1 -decay is given as:

$$E_1 = 1.088 - 0 = 1.088 \text{ MeV}$$

$$h\nu_1 = 1.088 \times 1.6 \times 10^{-19} \times 10^6 \text{ J}$$

Where,

$$h = \text{Planck's constant} = 6.6 \times 10^{-34} \text{ Js}$$

ν_1 = Frequency of radiation radiated by γ_1 -decay

$$\therefore \nu_1 = \frac{E_1}{h}$$

$$= \frac{1.088 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 2.637 \times 10^{20} \text{ Hz}$$

It can be observed from the given γ -decay diagram that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_2 -decay is given as:

$$E_2 = 0.412 - 0 = 0.412 \text{ MeV}$$

$$h\nu_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6 \text{ J}$$

Where,

ν_2 = Frequency of radiation radiated by γ_2 -decay

$$\therefore \nu_2 = \frac{E_2}{h}$$

$$= \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 9.988 \times 10^{19} \text{ Hz}$$

It can be observed from the given γ -decay diagram that γ_3 decays from the 1.088 MeV energy level to the 0.412 MeV energy level.

Hence, the energy corresponding to γ_3 -decay is given as:

$$E_3 = 1.088 - 0.412 = 0.676 \text{ MeV}$$

$$h\nu_3 = 0.676 \times 10^{-19} \times 10^6 \text{ J}$$

Where,

ν_3 = Frequency of radiation radiated by γ_3 -decay

$$\therefore \nu_3 = \frac{E_3}{h}$$

$$= \frac{0.676 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 1.639 \times 10^{20} \text{ Hz}$$

Mass of ${}_{78}^{198}\text{Au}$ = 197.968233 u

Mass of ${}_{80}^{198}\text{Hg}$ = 197.966760 u

$$1 \text{ u} = 931.5 \text{ MeV}/c^2$$

Energy of the highest level is given as:

$$E = [m({}_{78}^{198}\text{Au}) - m({}_{80}^{198}\text{Hg})]c^2$$

$$= 197.968233 - 197.966760 = 0.001473 \text{ u}$$

$$= 0.001473 \times 931.5 = 1.3720955 \text{ MeV}$$

β_1 decays from the 1.3720955 MeV level to the 1.088 MeV level

∴ Maximum kinetic energy of the β_1 particle = 1.3720955 - 1.088

$$= 0.2840995 \text{ MeV}$$

β_2 decays from the 1.3720955 MeV level to the 0.412 MeV level

∴ Maximum kinetic energy of the β_2 particle = 1.3720955 - 0.412

$$= 0.9600995 \text{ MeV.}$$

57)

(a) Amount of hydrogen, $m = 1 \text{ kg} = 1000 \text{ g}$

1 mole, i.e., 1 g of hydrogen (H) contains 6.023×10^{23} atoms.

$\therefore 1000 \text{ g}$ of H contains $6.023 \times 10^{23} \times 1000$ atoms.

Within the sun, four H nuclei combine and form one ${}^{24}\text{He}$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg H is:

$$E_1 = 6.023 \times 10^{23} \times 26 \times 10^3 \text{ J}$$

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^3}{4}$$

$$= 39.1495 \times 10^{26} \text{ J} \approx 39.15 \times 10^{26} \text{ J}$$

(b) Amount of ${}^{235}\text{U}$ = 1 kg = 1000 g

1 mole, i.e., 235 g of ${}^{235}\text{U}$ contains 6.023×10^{23} atoms.

$\therefore 1000 \text{ g}$ of ${}^{235}\text{U}$ contains

$$\frac{6.023 \times 10^{23} \times 1000}{235} \text{ atoms}$$

It is known that the amount of energy released in the fission of one atom of ${}^{235}\text{U}$ is 200 MeV.

Hence, energy released from the fission of 1 kg of ${}^{235}\text{U}$ is:

$$E_2 = \frac{6 \times 10^{23} \times 1000 \times 200}{235}$$

$$= 5.106 \times 10^{26} \text{ J} \approx 5.11 \times 10^{26} \text{ J}$$

$$\therefore \frac{E_1}{E_2} = \frac{39.1495 \times 10^{26}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

58)

Amount of electric power to be generated, $P = 2 \times 10^5 \text{ MW}$

10% of this amount has to be obtained from nuclear power plants.

$$\therefore \text{Amount of nuclear power, } P_1 = \frac{10}{100} \times 2 \times 10^5$$

$$= 2 \times 10^4 \text{ MW}$$

$$= 2 \times 10^4 \times 10^6 \text{ J/s}$$

$$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365 \text{ J/y}$$

Heat energy released per fission of a ${}^{235}\text{U}$ nucleus, $E = 200 \text{ MeV}$

Efficiency of a reactor = 25%

Hence, the amount of energy converted into the electrical energy per fission is calculated

$$\frac{25}{100} \times 200 = 50 \text{ MeV}$$

$$= 50 \times 1.6 \times 10^{-19} \times 10^6 = 8 \times 10^{-12} \text{ J}$$

Number of atoms required for fission per year:

$$\frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24} \text{ atoms}$$

1 mole, i.e., 235 g of ${}^{235}\text{U}$ contains 6.023×10^{23} atoms.

$$\therefore \text{Mass of } 6.023 \times 10^{23} \text{ atoms of } {}^{235}\text{U} = 235 \text{ g} = 235 \times 10^{-3} \text{ kg}$$

$$\therefore \text{Mass of } 78840 \times 10^{24} \text{ atoms of } {}^{235}\text{U}$$

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24}$$

$$= 3.076 \times 10^4 \text{ kg}$$

Hence, the mass of uranium needed per year is $3.076 \times 10^4 \text{ kg}$.

59)

In fact the number of protons and number of neutrons are same before and after nuclear reaction, but the binding energies of nuclei present before and after a nuclear reaction are different. This difference is called mass defect. This mass defect appears as energy of reaction. In this sense, a nuclear reaction is an example of mass-energy inter conversion.

60)

Density of nucleus (of water),

$$\rho = \frac{3m}{4\pi R_0^3} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times \frac{4}{3} \times (1.2 \times 10^{-15})^3}$$

$$= \frac{7 \times 3 \times 1.67 \times 10^{18}}{88 \times 1.2 \times 1.2 \times 1.2}$$

$$= 2.307 \times 10^{17} \text{ kg} / \text{m}^3$$

Density of water, $\rho' = 10^3 \text{ kg} / \text{m}^3$

$$\therefore \frac{\rho}{\rho'} = \frac{2.307 \times 10^{17}}{10^3} = 2.307 \times 10^{14}$$

61)

In a nucleus of ${}_{20}\text{Ca}^{40}$,

Number of protons = 20

Number of neutrons = 40 - 20 = 20

Total mass of 20 protons and 20 neutrons

$$= 20m_p + 20m_n = 20(m_p + m_n)$$

$$= 20(1.007825 + 1.008665)$$

$$= 40.3298 \text{ u}$$

Mass defect, $\Delta m = 40.3298 - 39.962589 = 0.367211 \text{ u}$ Total binding energy = $0.367211 \times 931 = 341.873441 \text{ MeV}$ E_b per nucleon, $E_{\text{bn}} = \frac{341.873441}{40}$

$$= 8.547 \text{ MeV/nucleon}$$

62)

Given, half-life period, $T_{1/2} = 3.8 \text{ days}$, $t = 38 \text{ days}$,initial concentration, $N_0 = 15 \text{ mg}$, final concentration, $N = ?$ Number of half-lives in 38 days, $n = \frac{38}{3.8} = 10$ We know that, $N = N_0 \left(\frac{1}{2}\right)^n = 15 \left(\frac{1}{2}\right)^{10} = 0.015 \text{ mg}$

63)

Let N_0 and N be the values of the number of atoms in a radioactive sample at $t = 0$ and at $t = 40$ years, respectivelyHere, $N = \frac{1}{16} N_0$, $t = 40 \text{ years}$

∴ Decay constant

$$\lambda = \frac{2.303}{t} \log_{10} \left(\frac{N_0}{N} \right) = \frac{2.303}{40} \log_{10} \left(\frac{N_0}{N_0/16} \right)$$

$$= \frac{2.303}{40} \times \log_{10} 16 = \frac{2.303}{40} \times 1.2041 = 0.0693 \text{ year}^{-1}$$

64)

Let the particle emitted in this case be represented as ${}_Z^AX$. Therefore, ${}_{86}\text{Rn}^{222} \rightarrow {}_{84}$ ${}_{\text{Po}}^{218} + {}_Z^AX^A$

Using the law of conservation of mass number and charge number, we get

$$222 = 218 + A \text{ and } 86 = 84 + Z$$

$$\rightarrow A = 4 \quad Z = 2$$

Now, $A = 4$ and $Z = 2$ correspond to an α -particle i.e.Therefore, emitted particle is an α -particle and the equation is

$${}_{86}\text{Rn}^{222} \rightarrow {}_{84}\text{Po}^{218} + {}_2^4\text{He}$$
