

Unit - IV

Bubble sort

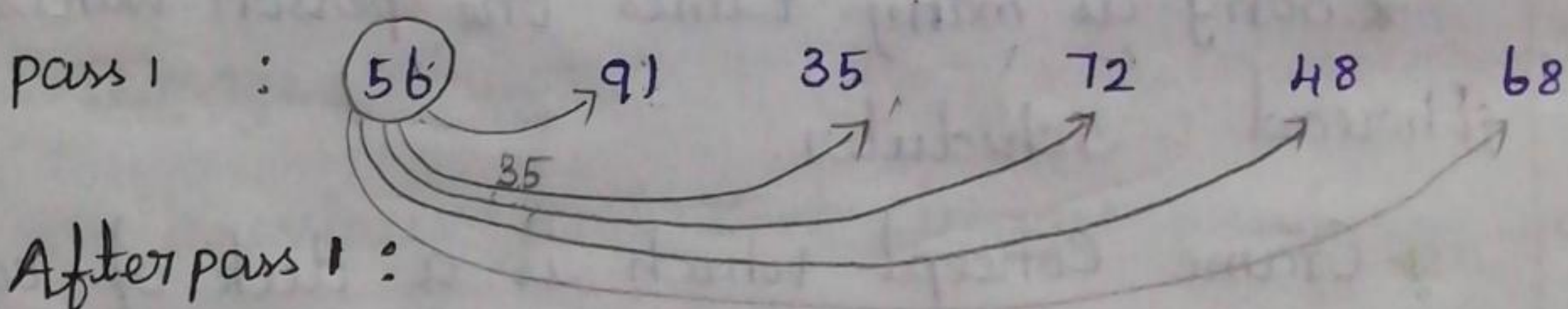
Bubble sort

- * It is the oldest sort
- * It is also referred as sinking sort.

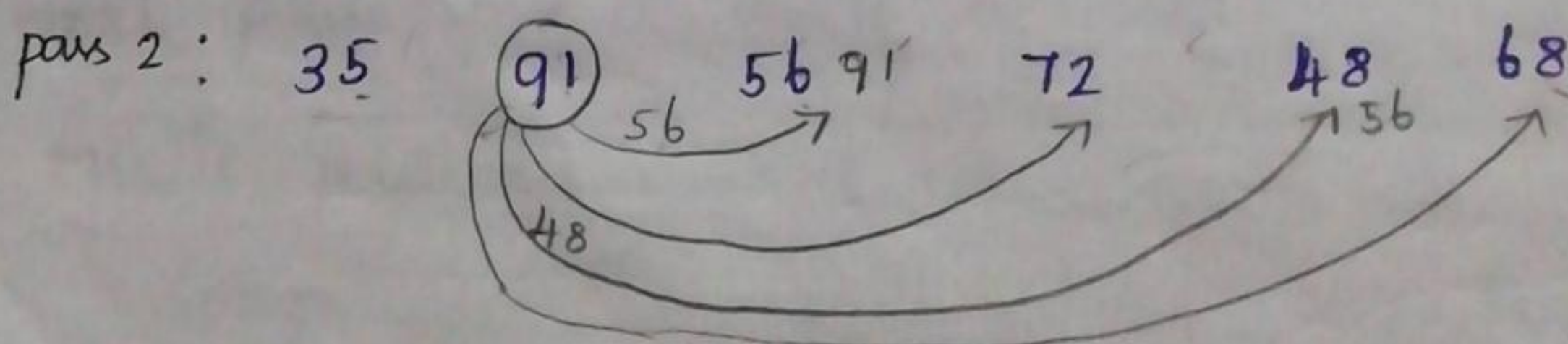
procedure:

- * It Repeatedly move the smallest element to the lowest Index position in the list
- * It Repeatedly Compares two consecutive elements and moves the smallest among them to the left.
- * This process is repeated until all the elements are in the correct order.

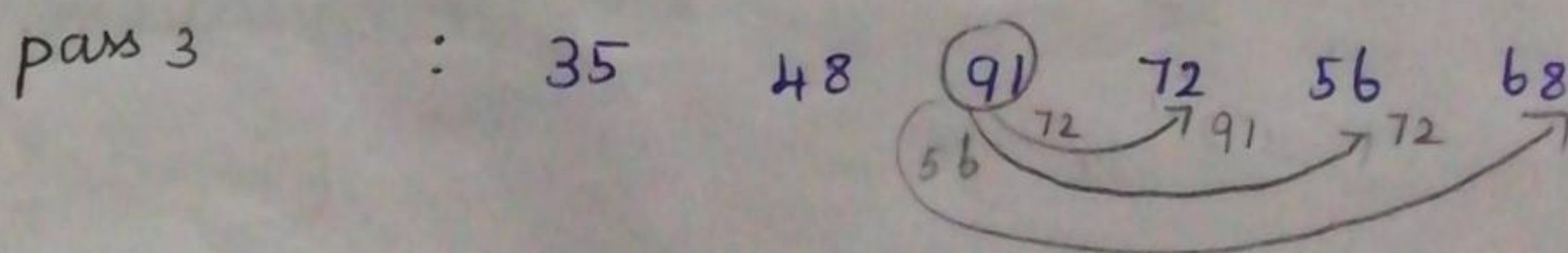
Example: 56 91 35 72 48 68



After pass 1 :



After pass 2 : 35 48 91 72 56 68



After pass 3 : 35 48 56 91 72 68

pass 4 : 35 48 56 68 91 72

After pass 4 : 35 48 56 68 72 91

Solution : 35 48 56 68 72 91

Write Explanation in Exam

Algorithm :

```
void bubblesort (int a [], int n)
```

```
{
```

```
    int i, j, tmp;
```

```
    For (i=0; i<n; i++)
```

```
    {
```

```
        For (j=i+1; j<n; j++)
```

```
        {
```

```
            if (a[i] > a[j])
```

```
            {
```

```
                tmp = a[i];
```

```
                a[i] = a[j];
```

```
                a[j] = tmp;
```

```
            }
```

```
        }
```

```
    }
```


Insertion sort

Let's take this array

5	1	6	2	4	3
---	---	---	---	---	---

As we can see here, in insertion sort, we pick up a key and compares it with elements a head (ahead) of it, and puts the key in right place.

- * File has nothing before it.
- * One is compared to file and is inserted before file.
- * Six is greater than 5 and 1
- * 2 is smaller than 6 and 5, but greater than 1, so it's inserted after 1 and this goes on.

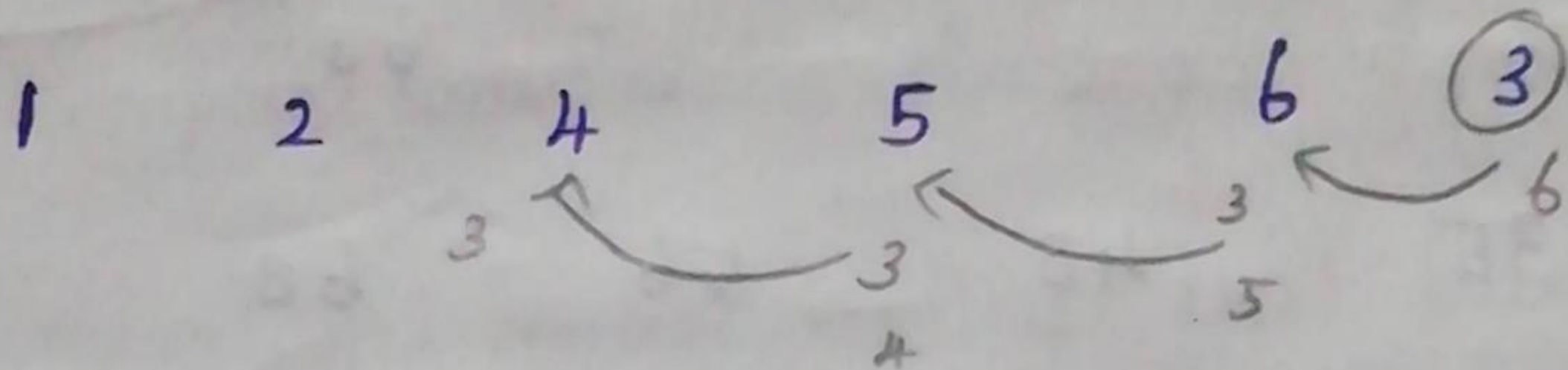
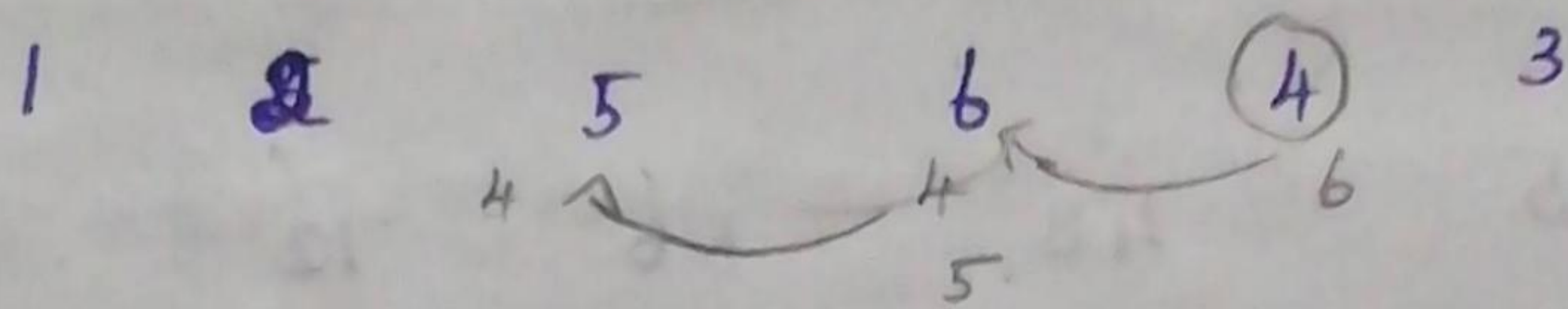
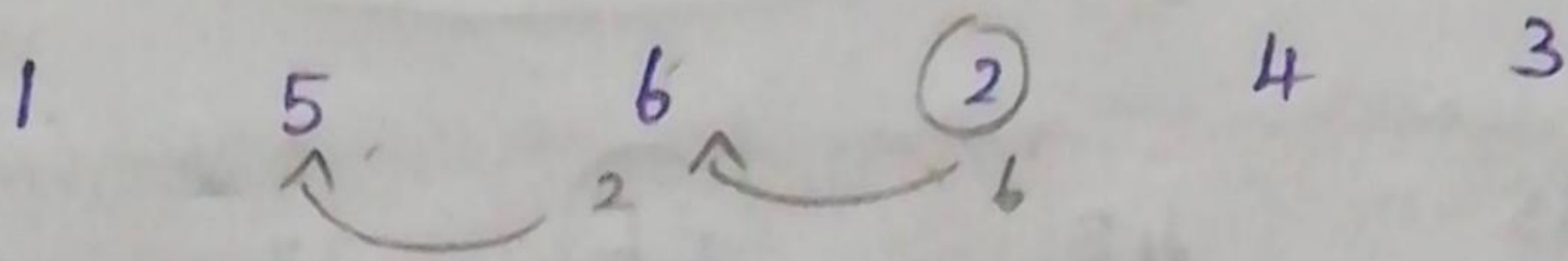
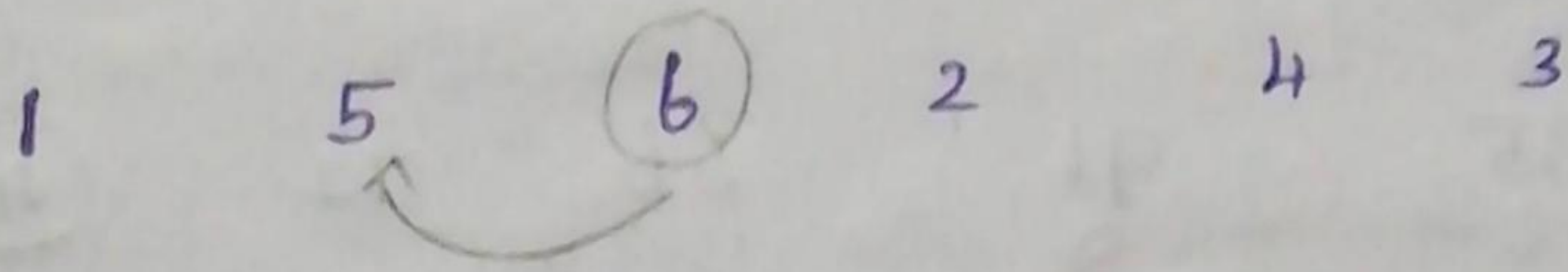
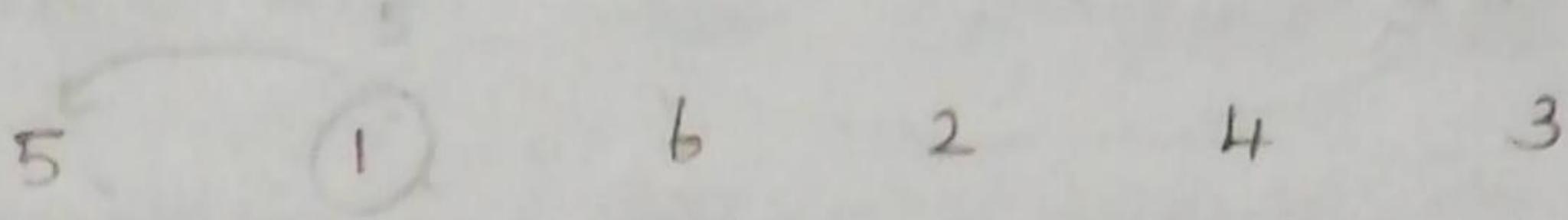
Example:

5	1	6	2	4	3
---	---	---	---	---	---

Always we start with the second element as key

Example

2nd element
as key



solution: 1 2 3 4 5 6

Insertion sort algorithm:

```
void insertion_sort (int a[], int n)
```

```
{  
    int i, p, tmp;
```

```
    for (p=1; p<n; p++)
```

```
    {  
        tmp = a[p];
```

```
        for (j=p; j>0 && a[j-1]>tmp; j--)
```

```
            a[j] = a[j-1];
```

```
        a[j] = tmp;
```

```
    }
```


Selection sort :

It is one of the most basic sorting techniques.

It works on the principals of identifying the smallest elements in the list and moving it to beginning of list this process is repeated untill all the elements in the list are sorted.

Example :

(Minimum element)


```

    a[j] = a[i];
    a[i] = min;
}
}
}

```

Efficiency And selection sort

* The no. of. comparisons made during 1st pass } = n-1

No. of. comparisons made during 2nd pass } = n-2

No. of. comparisons made during last pass } = 1

$$\begin{aligned}
 \text{Total NO. of. comparisons} &= (n-1) + (n-2) \dots \dots + 1 \\
 &= n * (n-1) \\
 &= O(n^2)
 \end{aligned}$$

efficiency of selection sort = $O(n^2)$.

Quick sort

- * It's sort any list very quickly
- * It is not a stable search, but it is very fast and requires very less additional space.
- * It is based on the rule of divide and conquer (partition exchange sort).
- * This algorithm divides the list in two three main parts.

1. elements less than the pivot element
2. pivot element (central element)
3. Elements greater than the pivot element.

Complexity Analysis :

- * Worst case time complexity = $O(n^2)$
- * Best case time complexity = $O(n \log n)$
- * Space complexity. order of $n \log n$.

Quick sort conditions :

1. i value \rightarrow small /
 j value \rightarrow large / pivot

else
 Swap(i, j)

2. i value \rightarrow large (or) $i \geq j$
 j value \rightarrow small

Swap(i, j)

Example : pivot element

50 30 10 90 80 20 40 70
 pivot element 50, comparing i & j value 30 and 70

Step 1: 50 30 10 90 80 20 40 70
 p i condition is True NO needed swap j

Step 2: 50 30 10 90 80 20 40 70
 p i j

In step 2 p is pivot 50 i value is 10
 j value is 70 Condition is True no need

step 3 : 50 30 10 90 80 20 40 70
P i x j ✓

pivot value (p) is 50, i condition is True
j condition false.

step 4 : 50 30 10 90 80 20 40 70
P i x 80 90 j x
(swap i, j)

pivot value is 50, Then comparing i and j
value condition is false. so swap is needed.

step 5 : 50 30 10 40 80 20 90 70
P i (greater) j move

i value should be increment. j value should
not increment

step 6 : 50 30 10 40 80 20 90 70
P j not obey j swap(i, j)
is false pivot value (p) is 50, i and j condition

step 7 : 50 30 10 40 20 80 90 70
P i j
Then ij value is 20

step 8 : 50 30 10 40 20 80 90 70
P i greater i move x

i is greater than j so we can implement
2nd condition and swapping pivot element and i

step 9 : 50 30 10 40 20 80 90 70
P 20 j swap(p, j)
50

step 10 : 20 30 10 40 50 80 90 70
(i) (j) (i)

When the pivot element move to the center,
divide the array left & right

Step 11 :	20	30	10	40	50	80	90	70
	(p)	(i)*		j		p	i	j
Step 12 :	20	30	10	40		80	70	90
	p	i*	j*	Swap (i, j)		p	i	j
Step 13 :	20	10	30	40		80	70	90
	p	i	j✓			p		(i, j)
Step 14 :	20	10	30	40		80	70	90
	p		(i, j)			p	j	i
Step 15 :	20	10	30	40		70	80	90
	p	j*	i*					
Step 16 :	10	20	30	40	50	70	80	90
			Swap (i, j)	i ≥ j				

Algorithm :

i) if $l < r$

$s \rightarrow$ partition ($A(l \dots r)$)

Quicksort ($A(l \dots s-1)$)

Quicksort ($A(s+1 \dots r)$)

ii) $l \leftarrow A[l]$

$i \leftarrow l, j \leftarrow r+1$

repeat

repeat $i \leftarrow i+1$ until $A[i] \geq p$

repeat $j \leftarrow j-1$ until $A[j] \leq p$

Swap [$A[i], A[j]$]

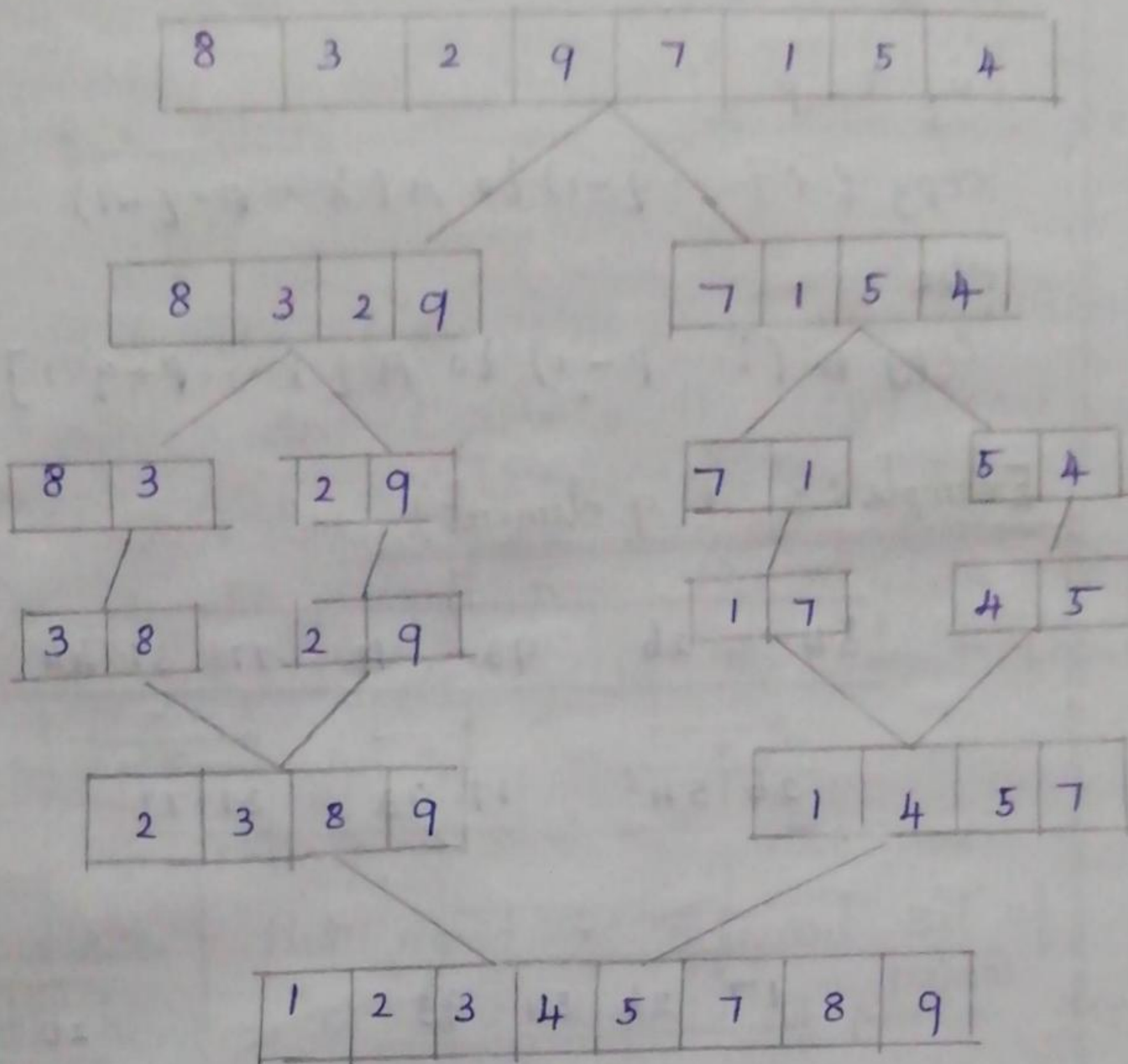
until $i \geq j$

Swap ($A[i], A[j]$)

Swap ($A[l], A[i]$)

Merge Sort

Example



Algorithm

i. if $n > 1$

copy $A[0 \dots (n/2) - 1]$ to $B[0 \dots (n/2) - 1]$

copy $A[n/2 \dots n - 1]$ to $C[0 \dots (n/2) - 1]$

mergesort ($B[0 \dots (n/2) - 1]$)

mergesort ($C[0 \dots (n/2) - 1]$)

merge (B, C, A)

ii. $i \leftarrow 0, j \leftarrow 0, k \leftarrow 0$

while $i < p \ \& \ j < q$ do

if $B[i] \leq C[j]$

$A[k] \leftarrow B[i];$

$i \leftarrow i + 1$

$A[k] \leftarrow [j];$

$j \leftarrow j+1$

$k \leftarrow k+1$

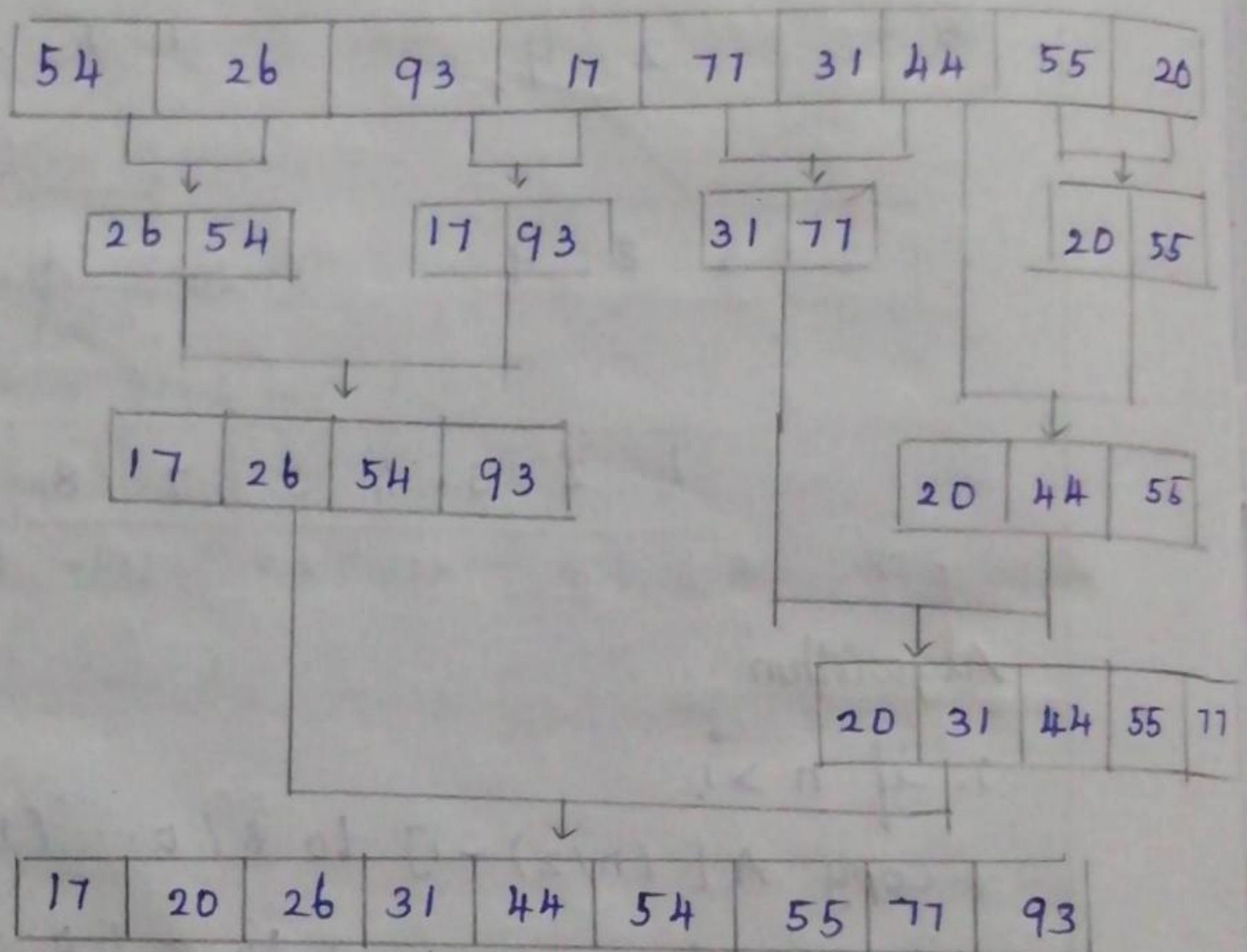
if $i = p$

copy $(j \dots q-1)$ to $A[k \dots p+q-1]$

else

copy $B(i \dots p-1)$ to $A[k \dots p+q-1]$

Example: 2 (q element)



Worst case time complexity (Big-O): $O(n \cdot \log n)$

Best case time complexity (Big- Ω): $O(n \cdot \log n)$

Average case time complexity (Big- Θ): $O(n \cdot \log n)$

Space case time complexity : $O(n)$

Radix sort

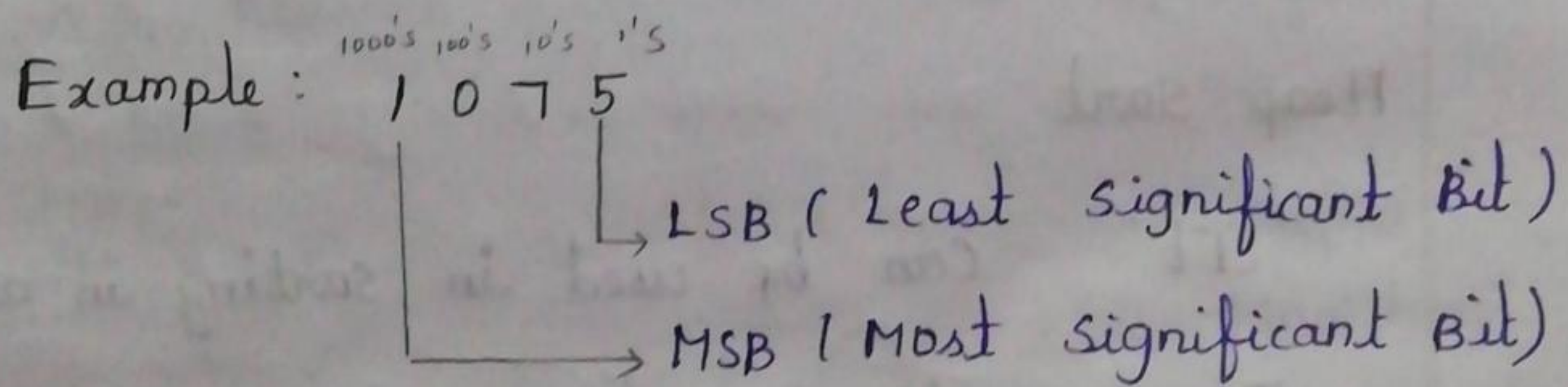
It is a generalized form of bucket sort. It distributes the list of elements across different buckets from 0 to 9

~~process~~^{du}: procedure

* In first pass, all elements are sorted according to the LSB (Least Significant Bit)

* In second pass, the numbers are arranged according to the second Least Significant Bit

* This process is repeated until it reaches the most Significant Bit of all numbers in the number of ~~process~~^{passes} depends upon number of digits



Example 1:

64, 8, 216, 512, 27, 729, 0, 1, 343, 125

pass 1: (LSB)

	0	1	512	343	64	125	216	27	8	729
After	0	1	2	3	4	5	6	7	8	9
pass	<u>00</u>	<u>01</u>	<u>512</u>	<u>343</u>	<u>64</u>	<u>125</u>	<u>216</u>	<u>27</u>	<u>08</u>	<u>729</u>

pass 2: (Second LSB)

8										
1			729							
	216		27							
0	512		125		343			64		
0	1	2	3	4	5	6	7	8	9	

After pass 2

<u>00</u> 8			<u>7</u> 29							
<u>00</u> 1	<u>2</u> 16		<u>0</u> 27							
<u>000</u>	<u>5</u> 12		<u>1</u> 25		<u>3</u> 43			<u>0</u> 64		

pass 3: (Third Least Significant Bit)

	64									
	27									
	8									
	1									
0	125	216	343		512			729		
0	1	2	3	4	5	6	7	8	9	

0, 1, 8, 27, 64, 125, 216, 343, 512, 729

Heap sort

It can be used in sorting in array

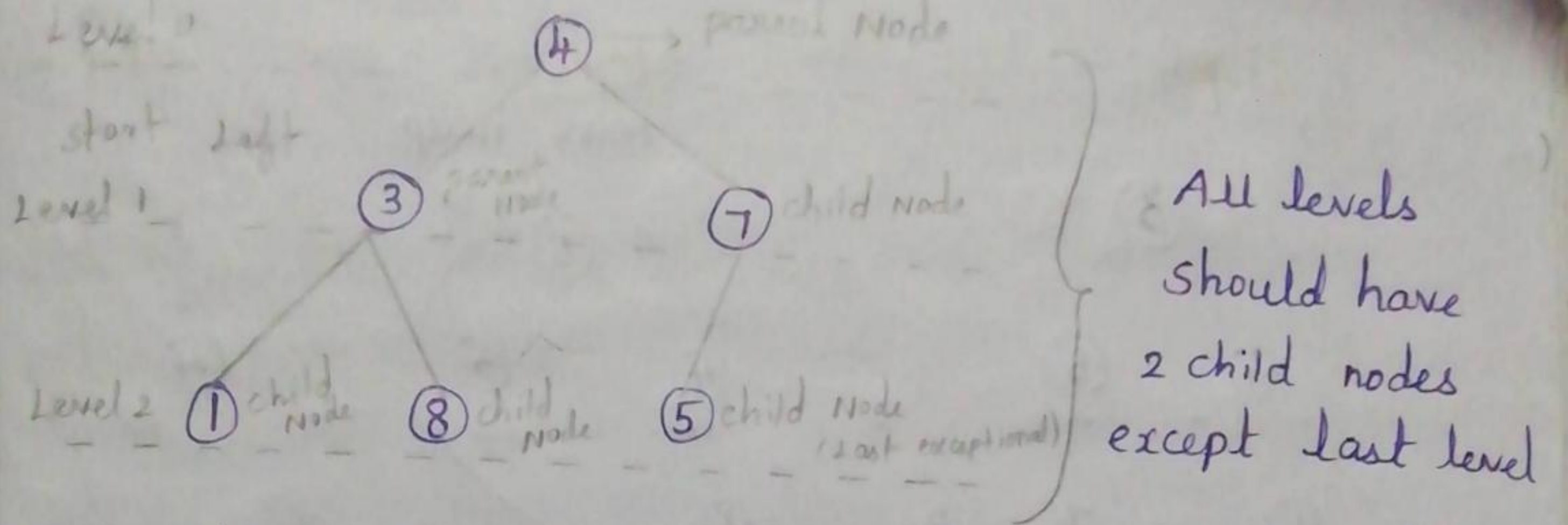
It may be Ascending order or descending order.

1. Complete Binary Tree (CBT)

2. Max heap or min heap

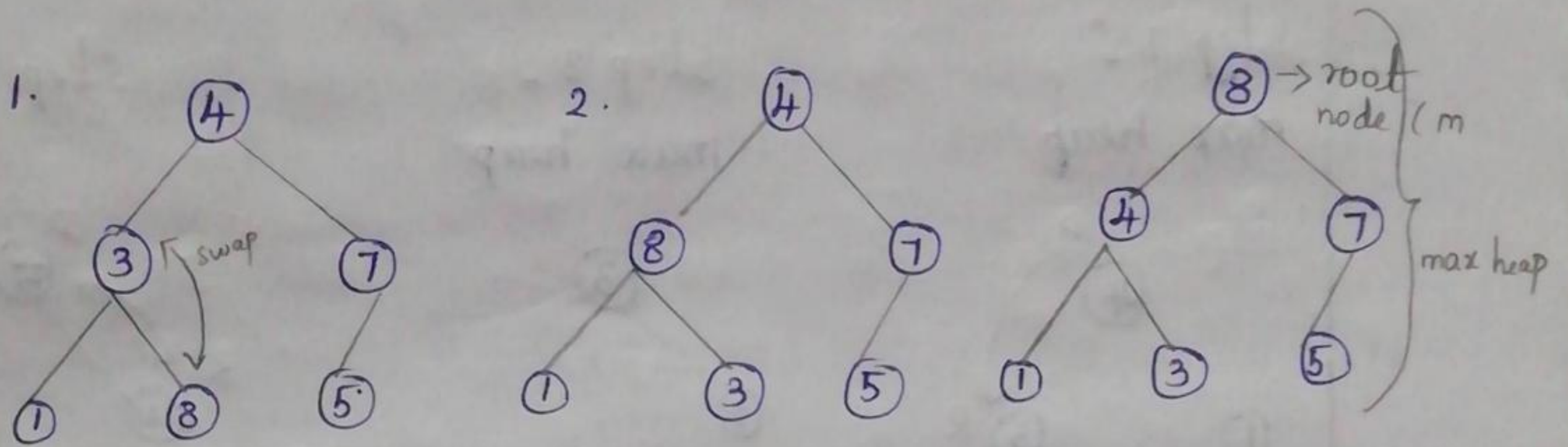
CBT (complete Binary Tree)

Example: 4, 3, 7, 1, 8, 5



Max heap :

Example: 4, 3, 7, 1, 8, 5



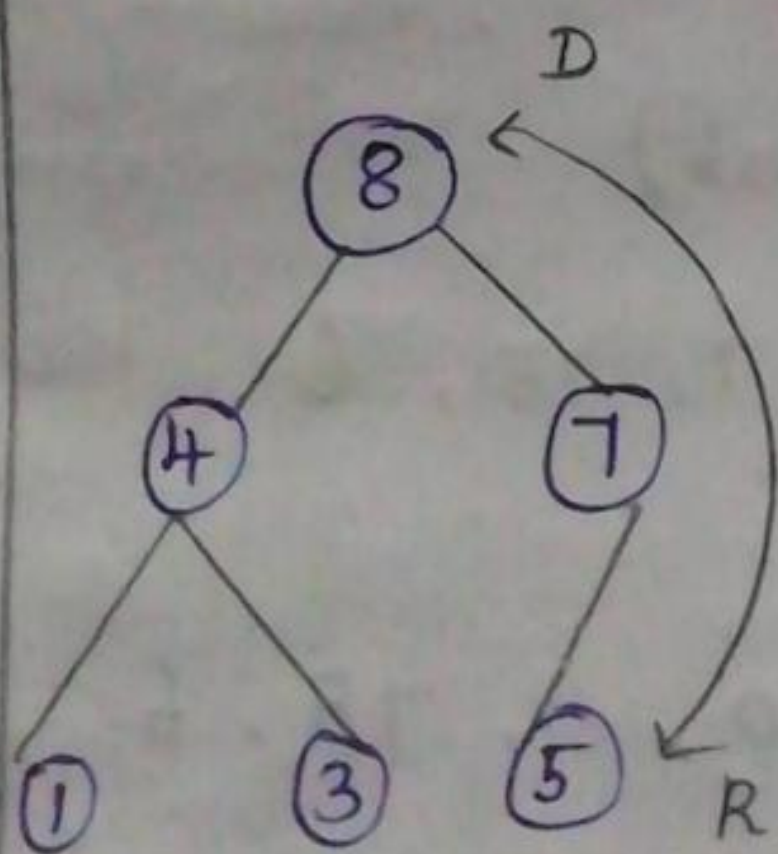
* In max heap, Maximum element will always be at the root

Max heap :

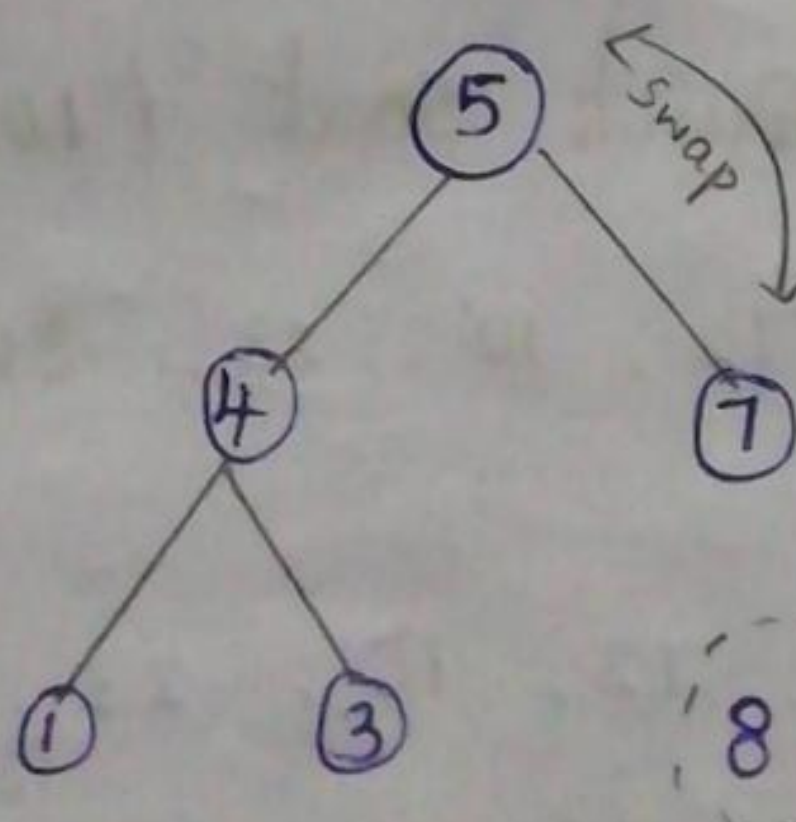
* Replace & delete the 1st node from the Tree.

Step 1:

Initial

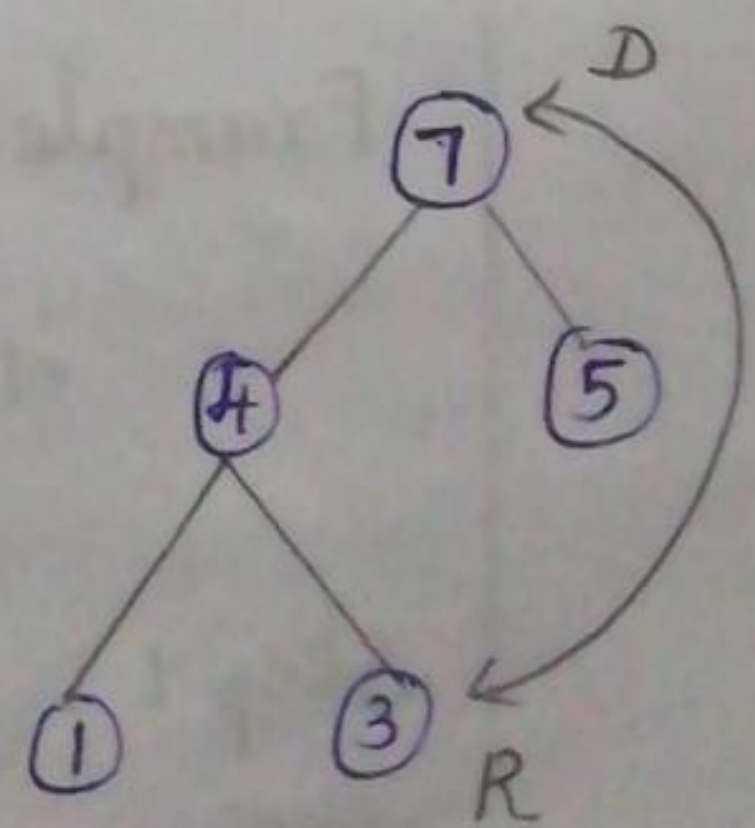


Step 2:



Step 3:

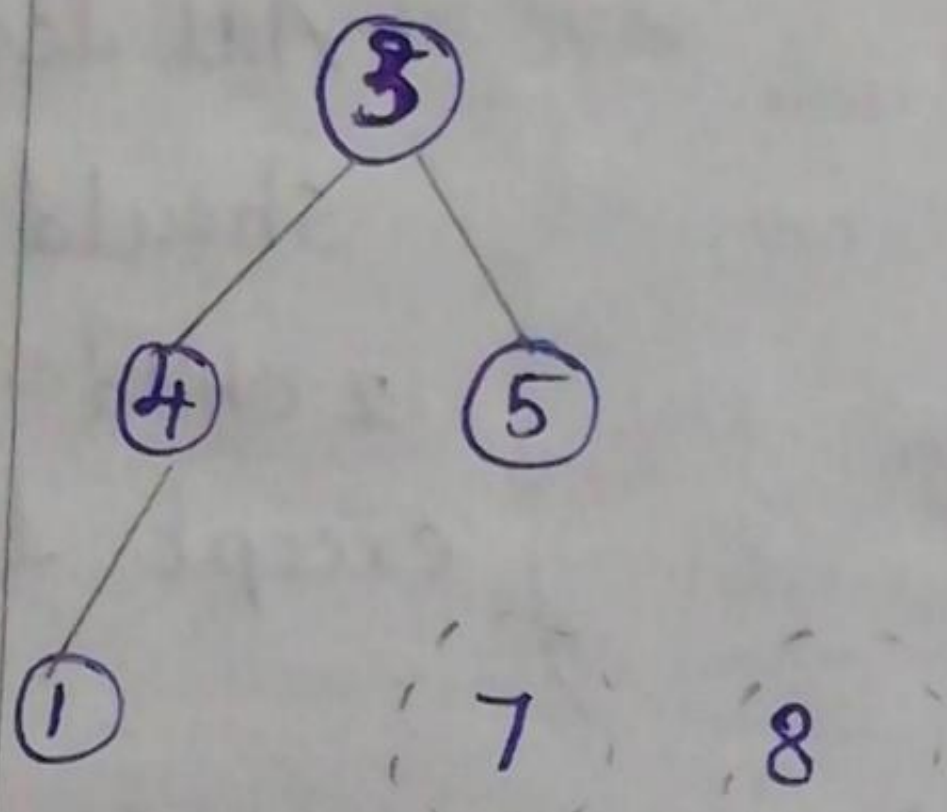
Max heap



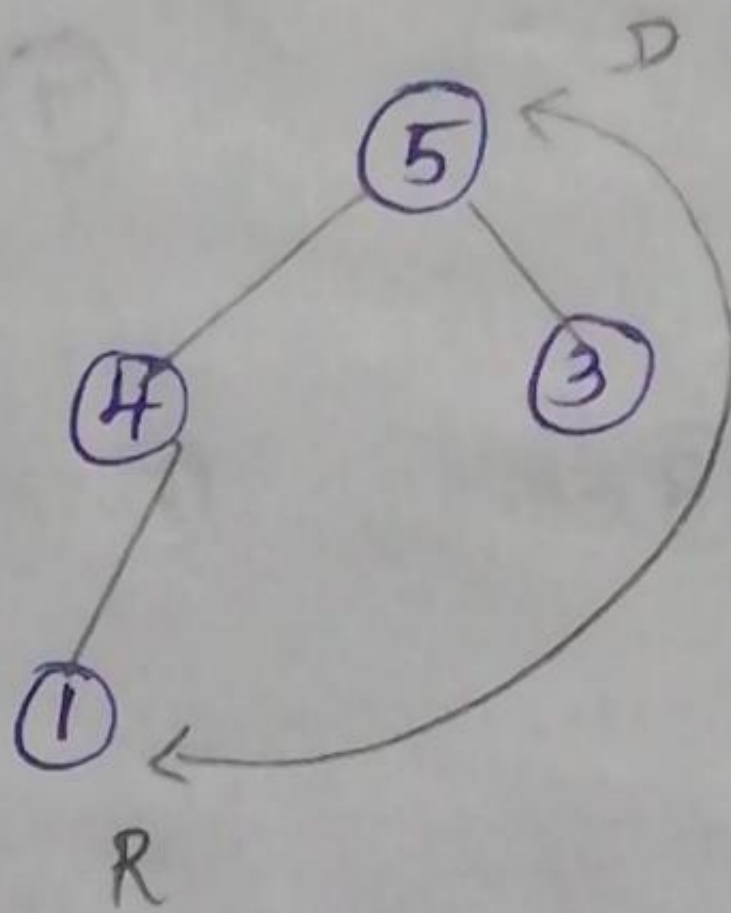
* R - Replace

* D - Delete

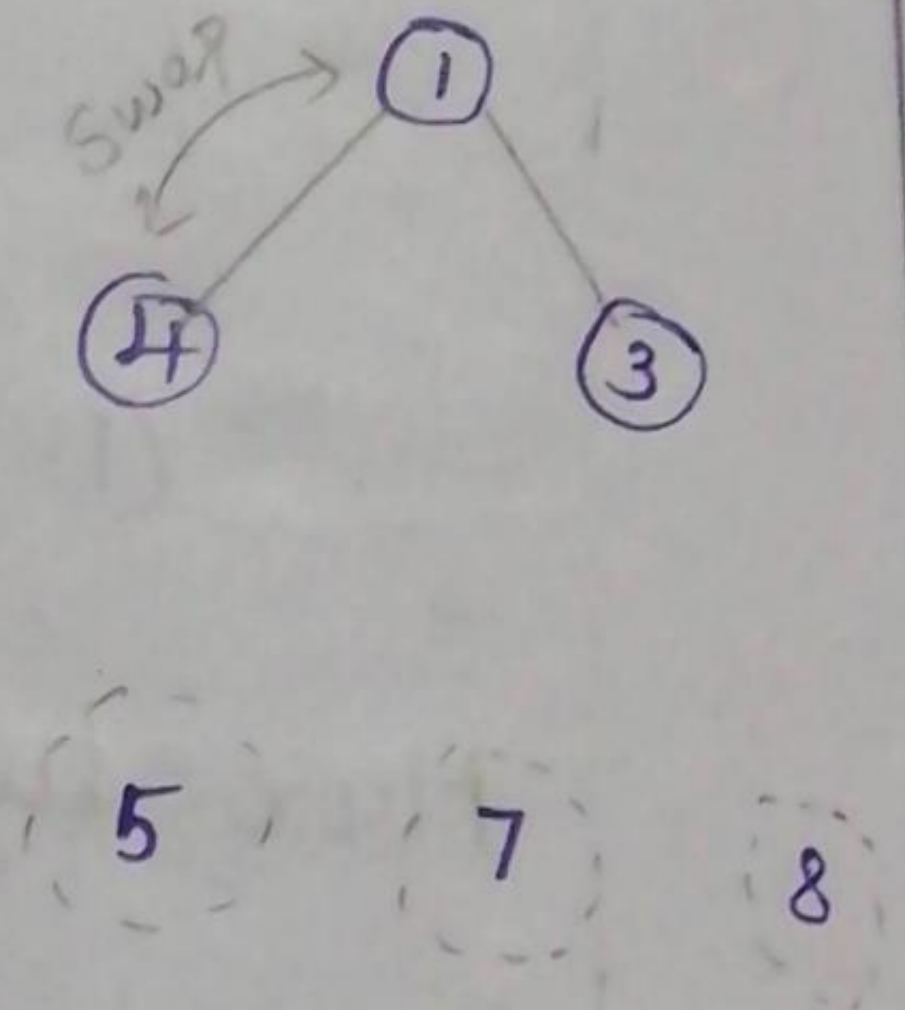
Step 4:



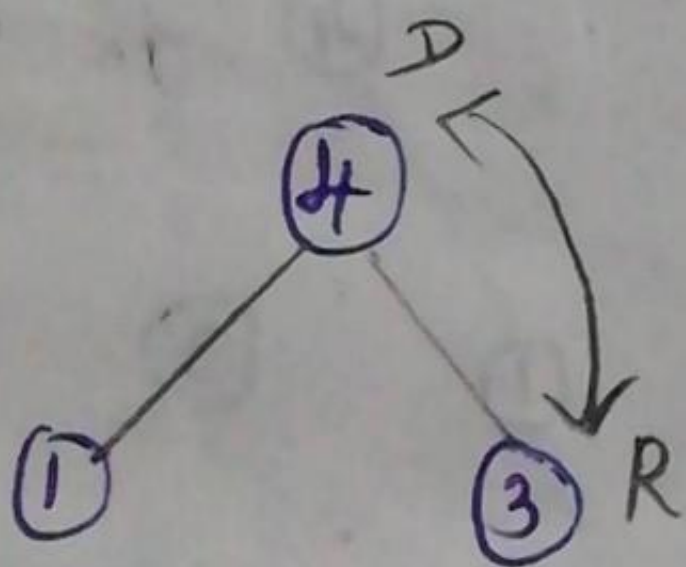
Step 5:
max heap



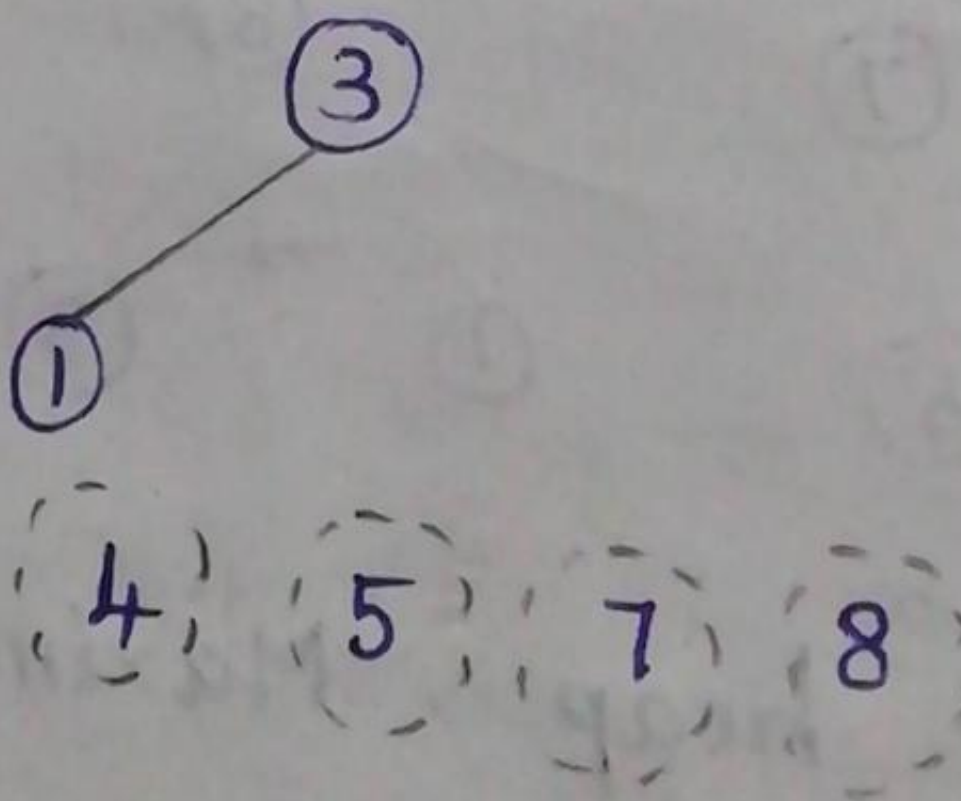
Step 6:



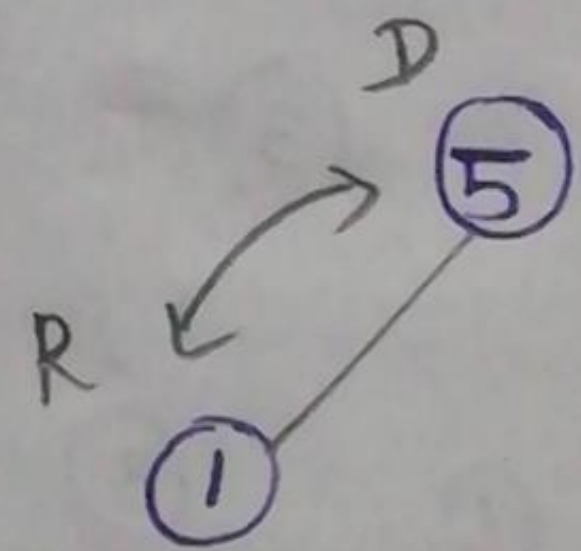
Step 7:
max heap



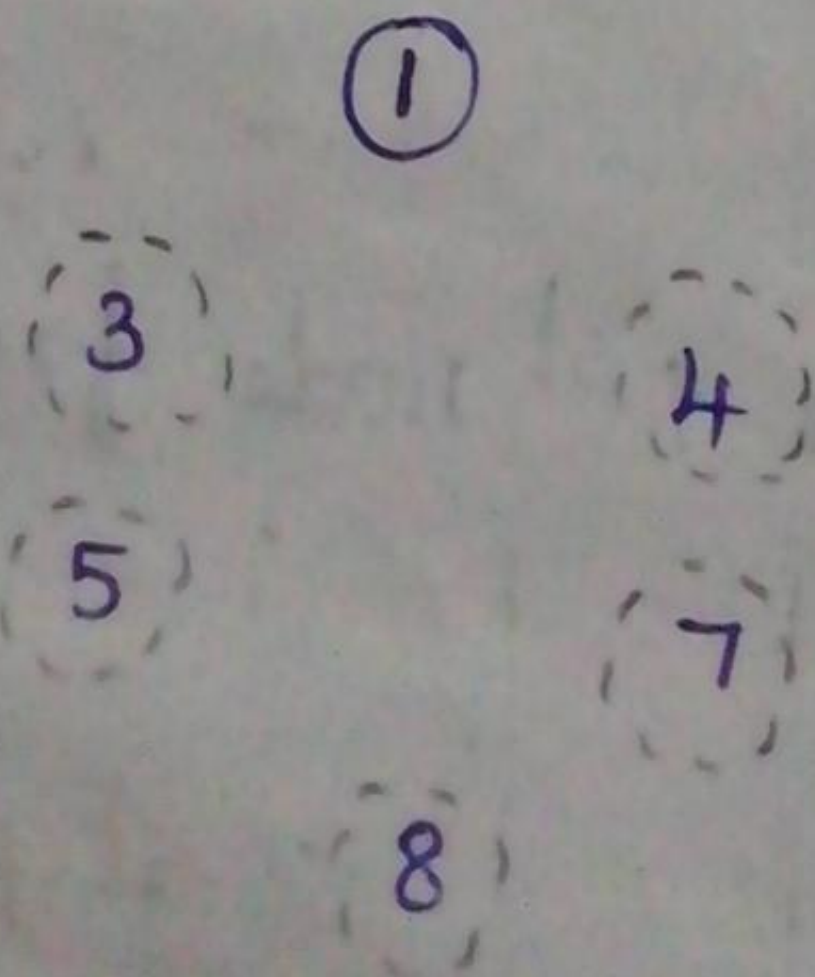
Step 8:
max heap



Step 9:



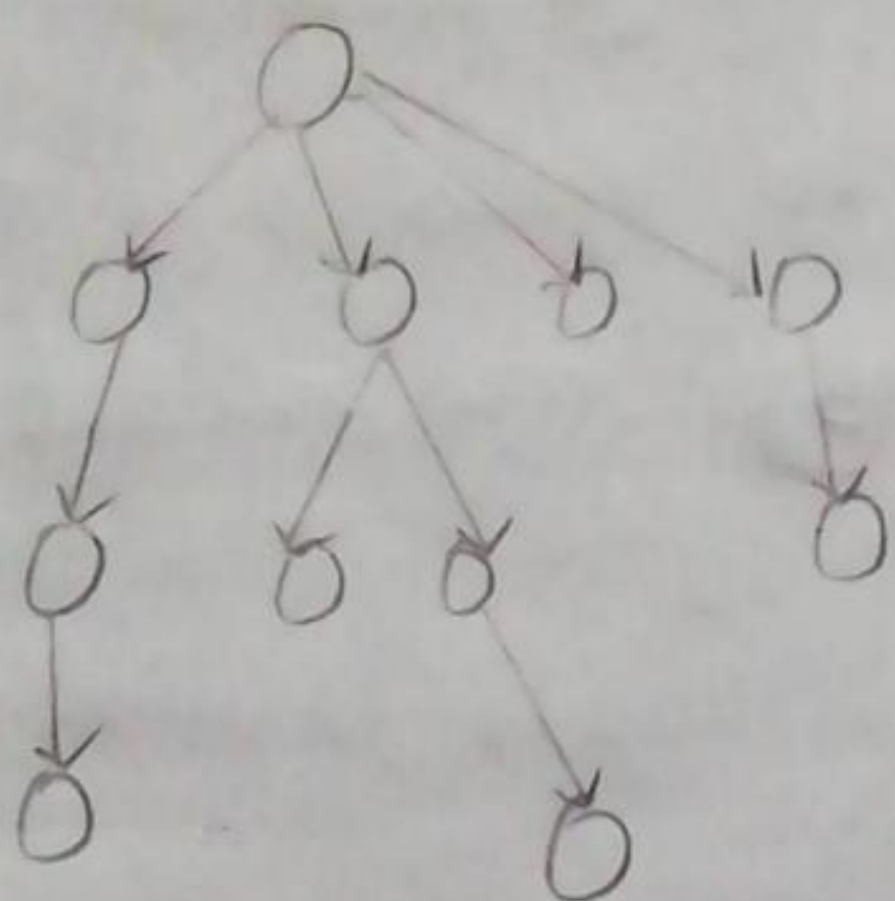
Step 10:



Sorted Array 1, 3, 4, 5, 7, 8

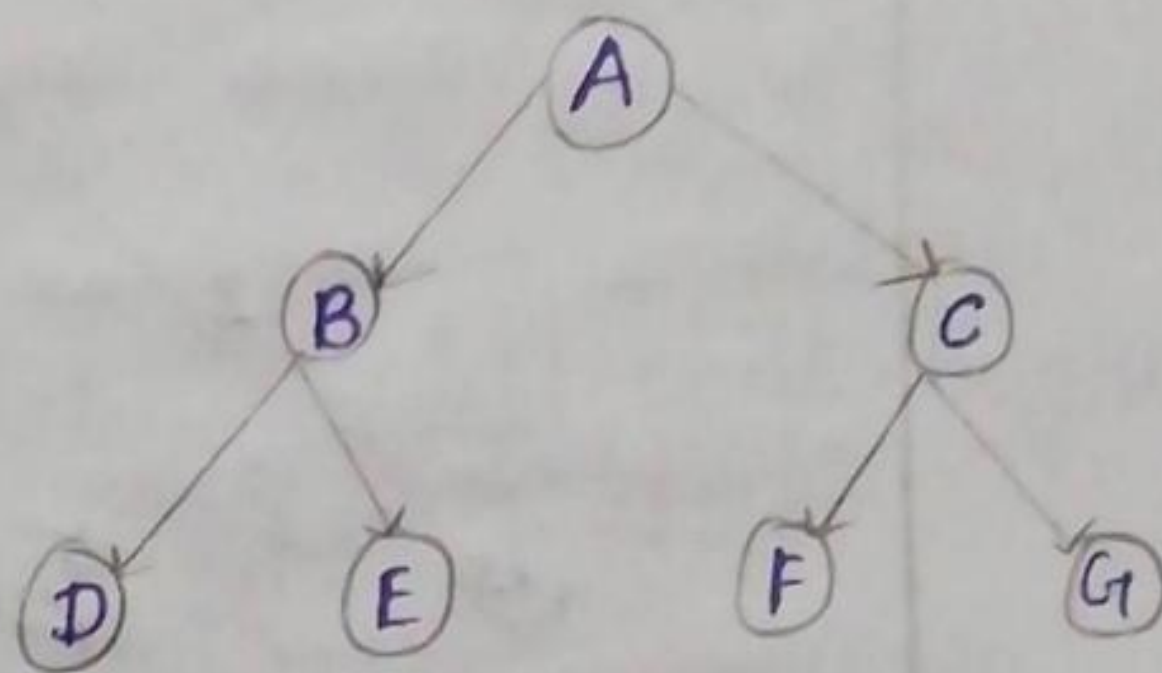
Binary Tree

- * Non Linear
- * maximum of 2 children



General Tree

VS



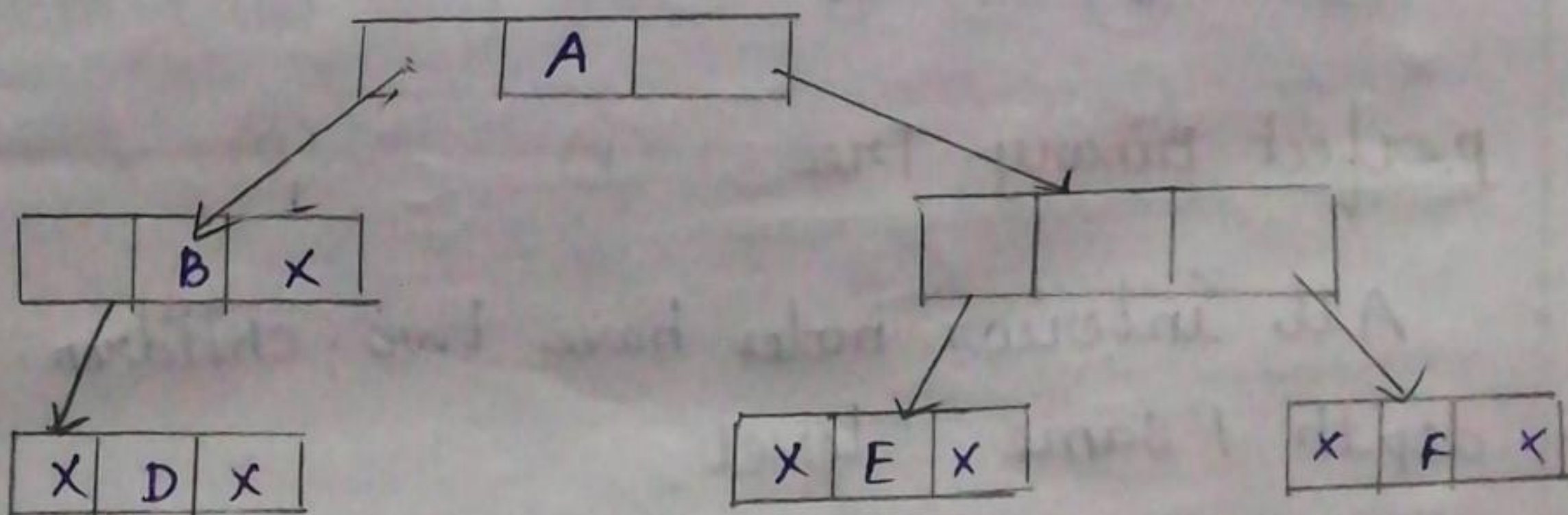
Binary Tree

- * Left and Right

Binary Tree components

3 components

- * Delete element
- * pointer to left sub tree
- * pointer to right sub tree



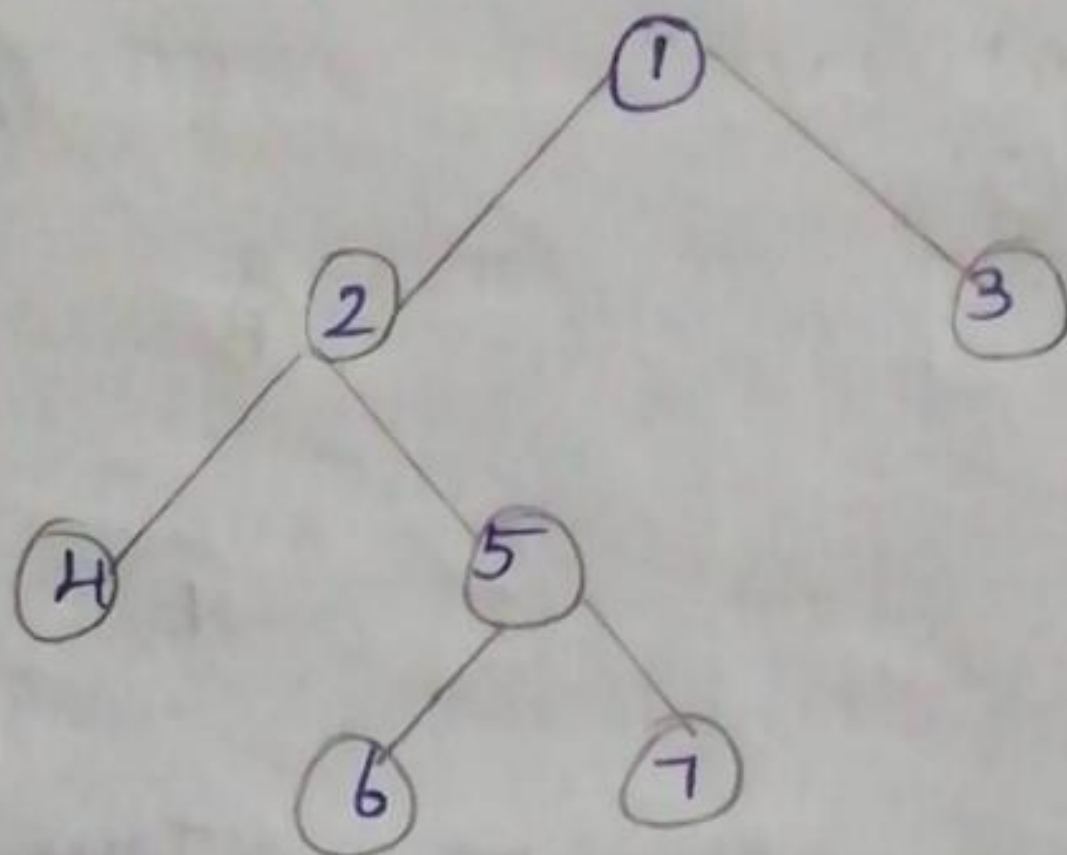
Types of Binary Trees

- * Full Binary Tree
- * Complete Binary Tree
- * perfect Binary Tree
- * Balanced Binary Tree
- * Degenerated Binary Tree

Full Binary Tree

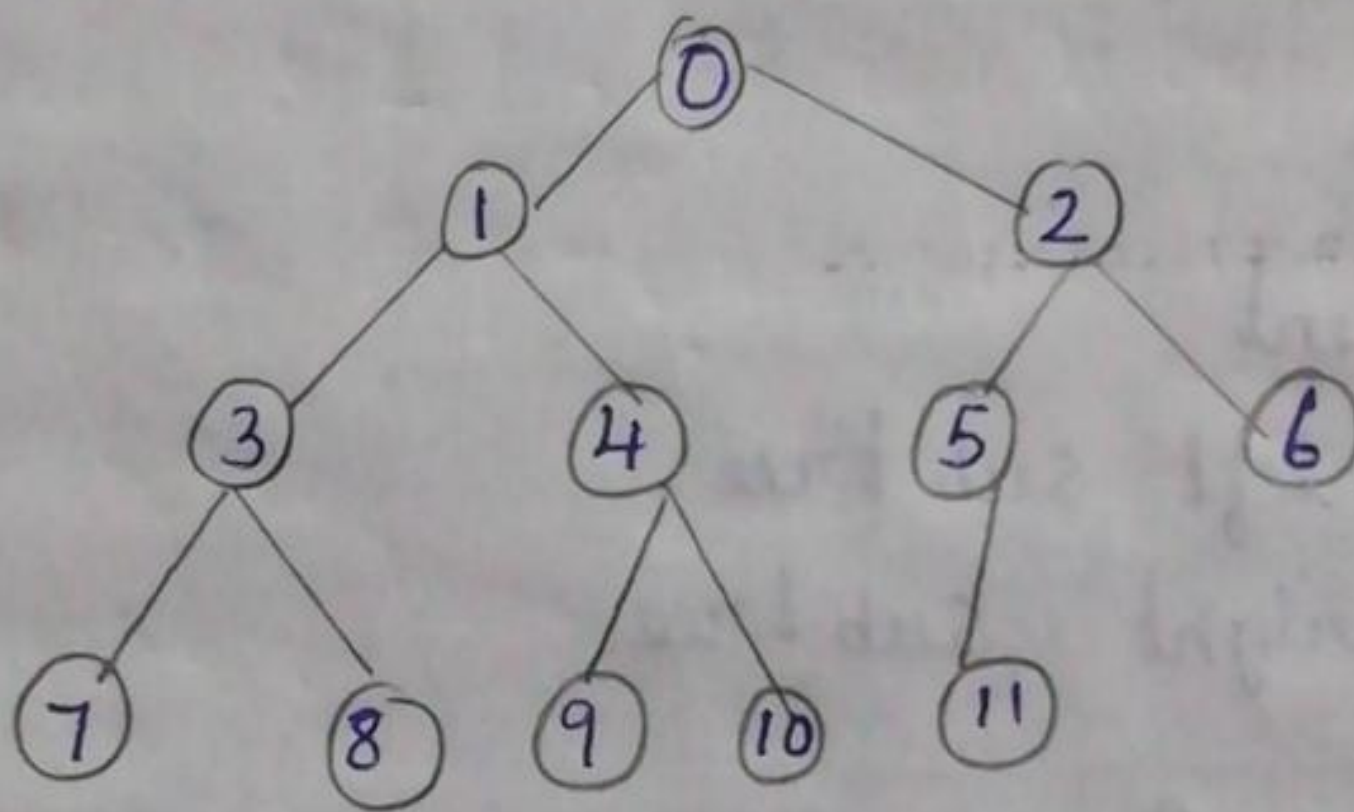
(Every parent node)

Internal node has either 2 / no child



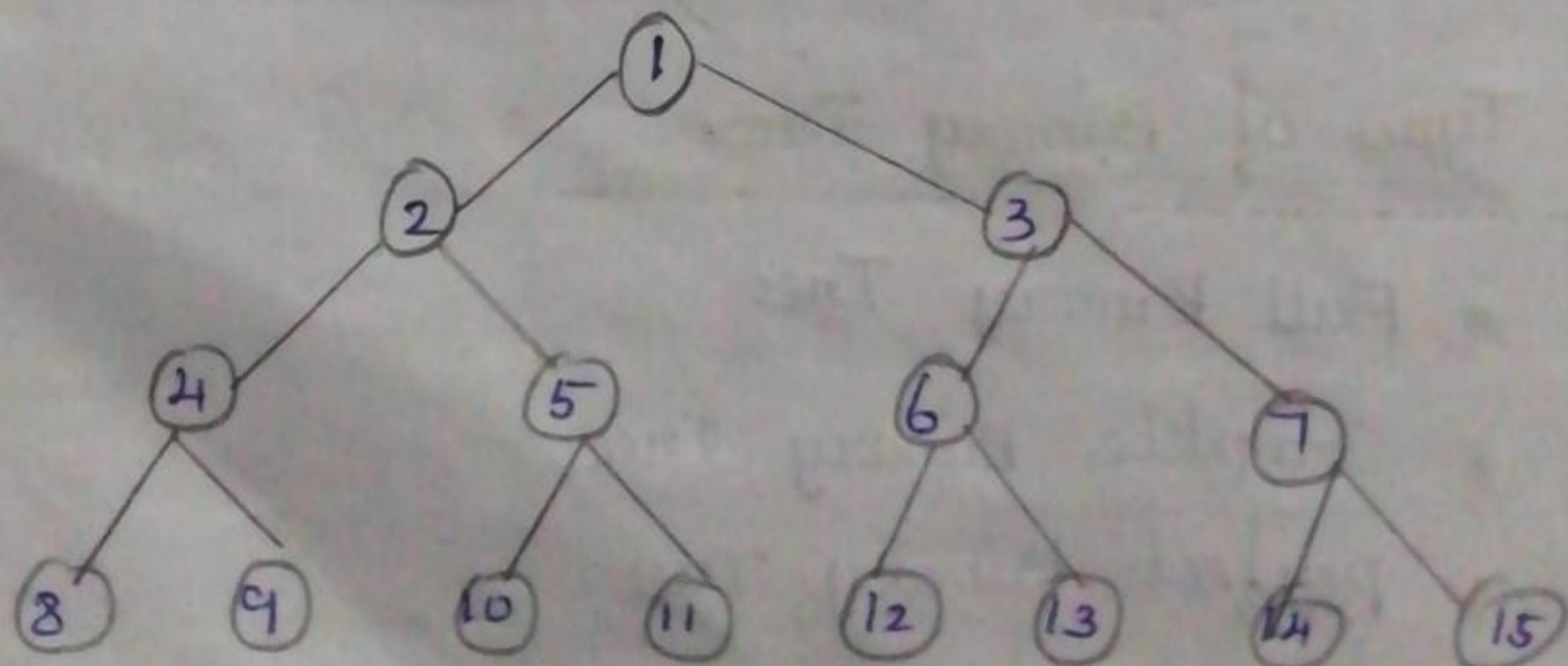
Complete Binary Tree

Which all the levels are completely filled filled from the left.



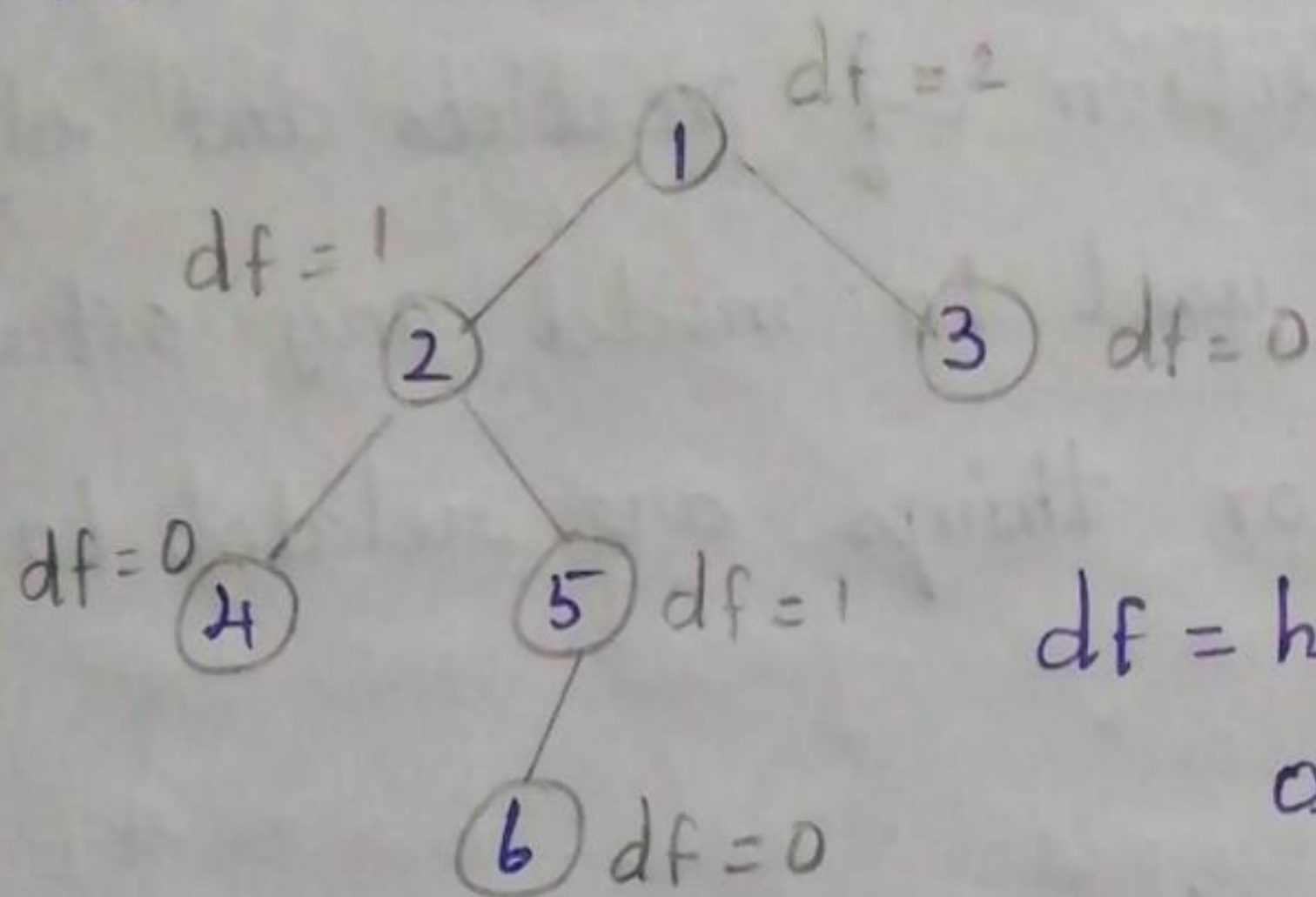
Perfect Binary Tree

All interior nodes have two children, same depth / same level

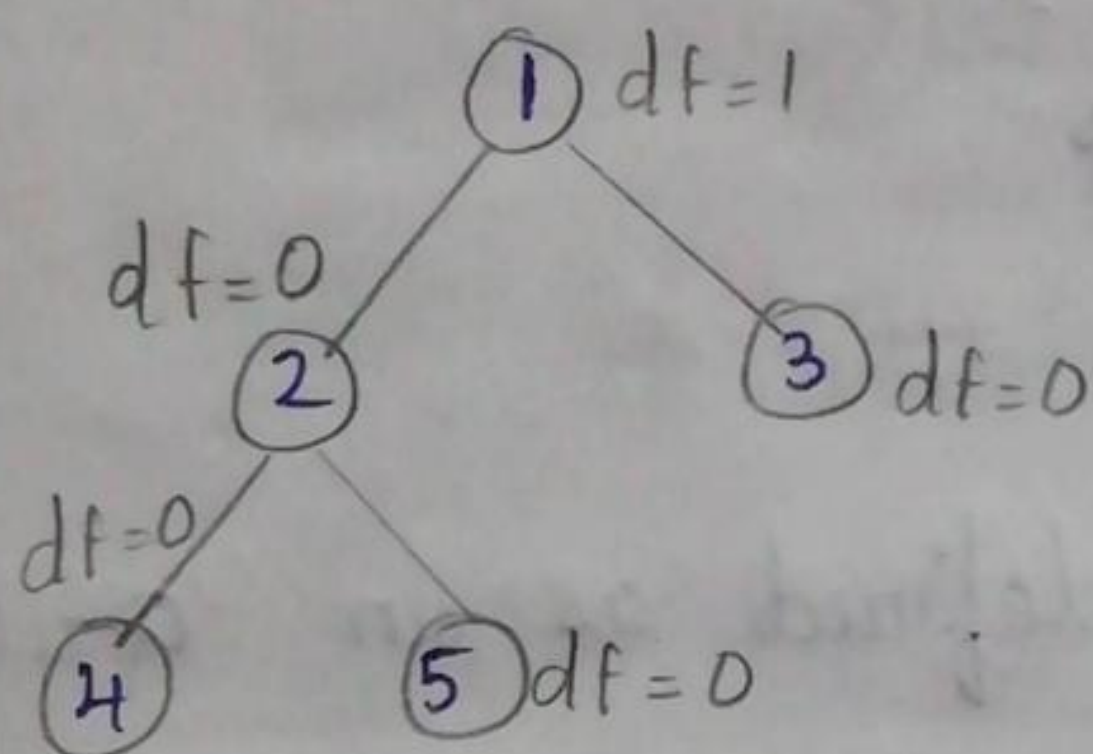


Balanced Binary Tree

left and Right Subtrees - differ in height by no more than 1



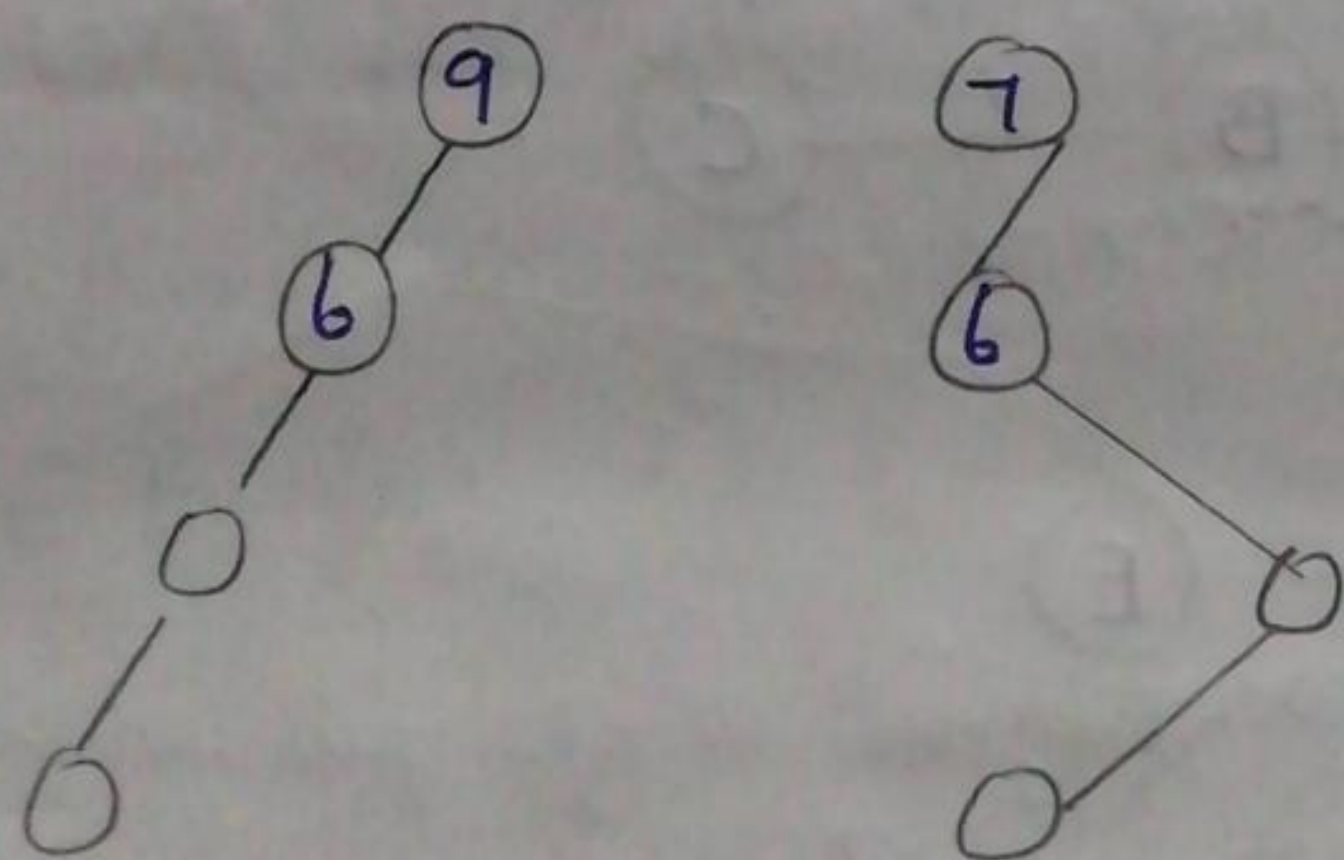
df = height of left child - height of right child.



Degenerate Binary Tree

Each parent node, there is only one associated child node linked list

*Both trees only has one internal node



Tree Traversal: (Operation on Binary Tree)

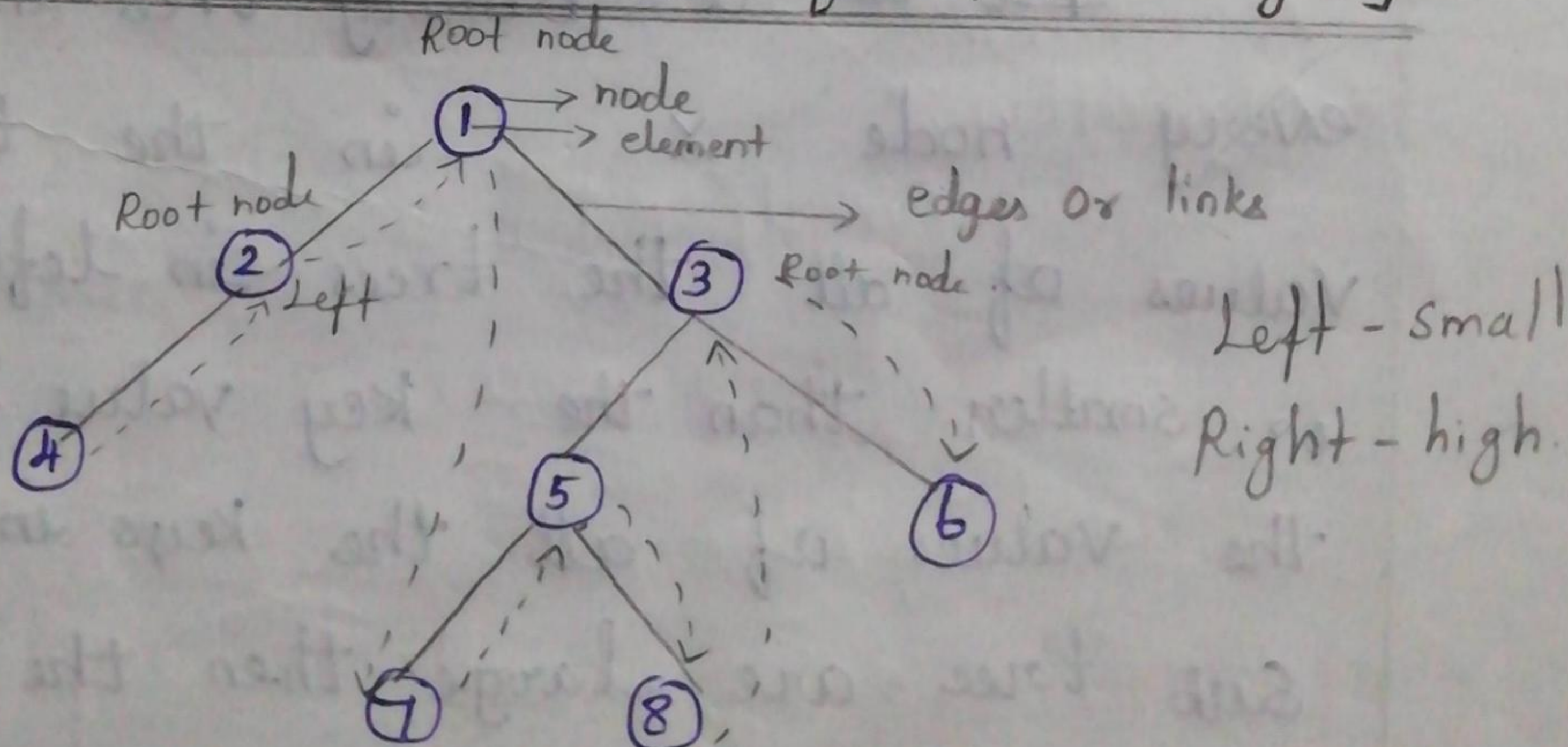
- * Visit all the nodes
- * Connected via edges (links)
- * Three ways

1. In Order Traversal

2. pre order Traversal

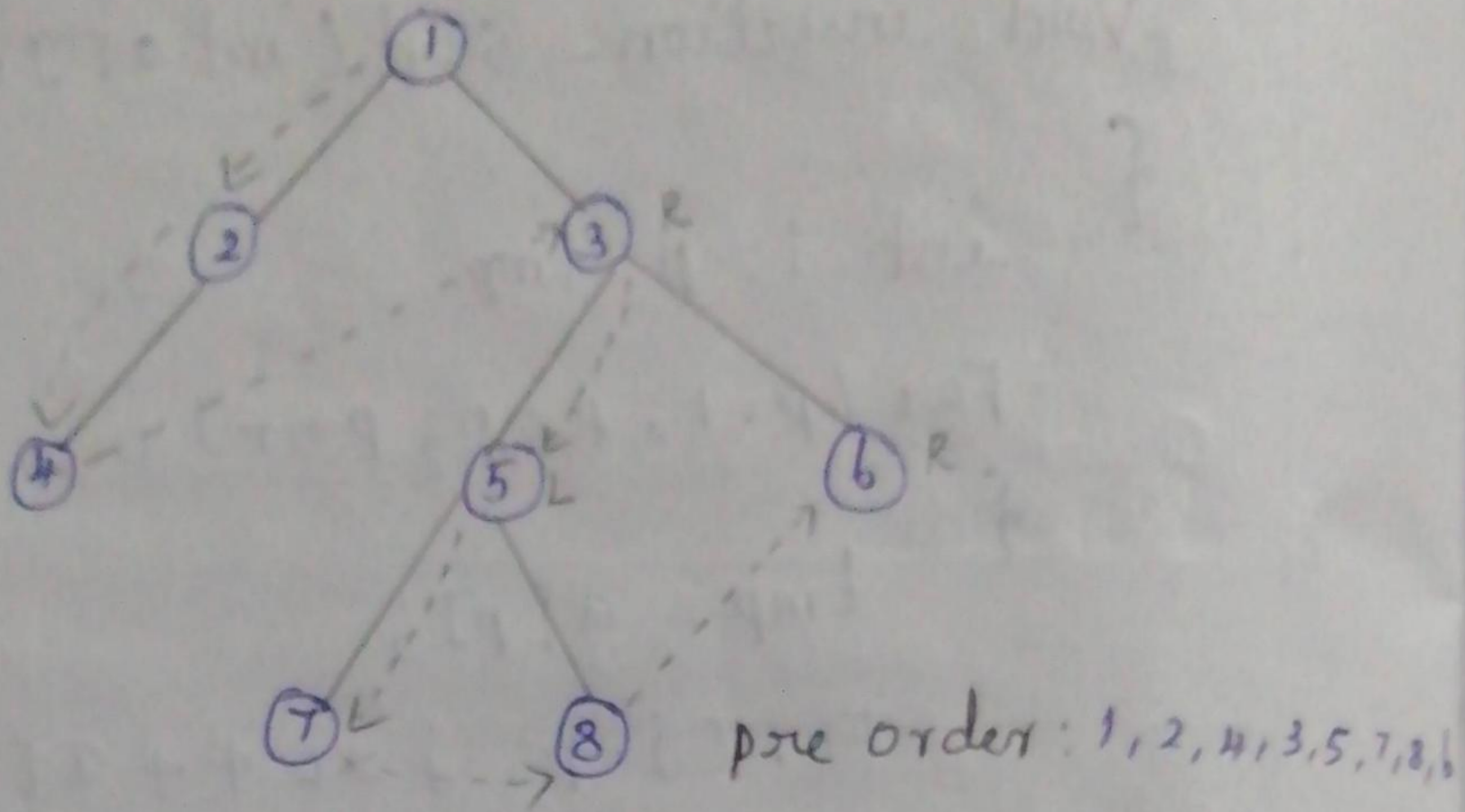
3. post order Traversal

Inorder Traversal: [Left - Root - Right]

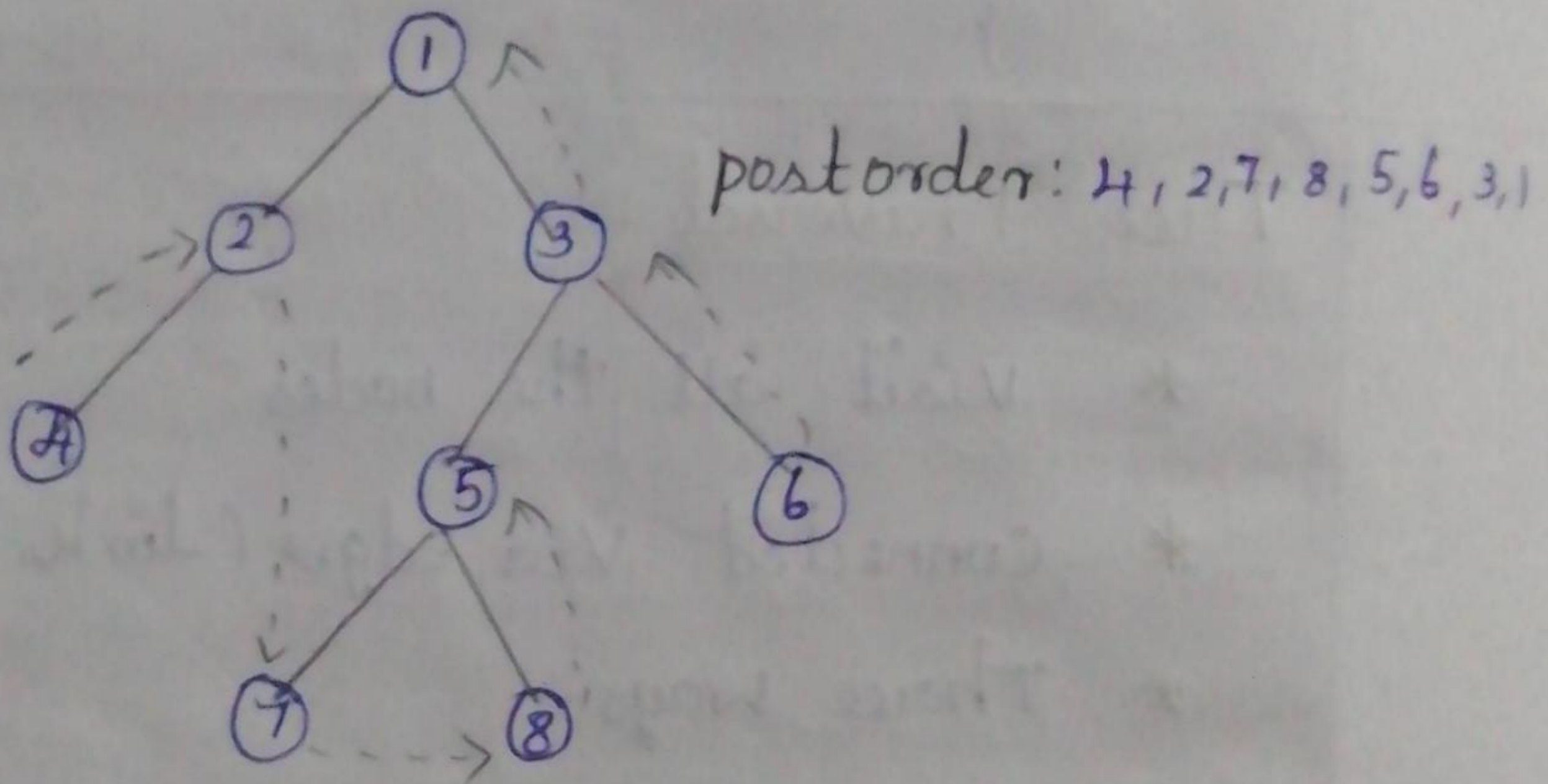


In order: 4, 2, 1, 7, 5, 8, 3, 6.

pre-order Traversal: [Root - Left - Right]



post order Traversal: [Left - Right - Root]



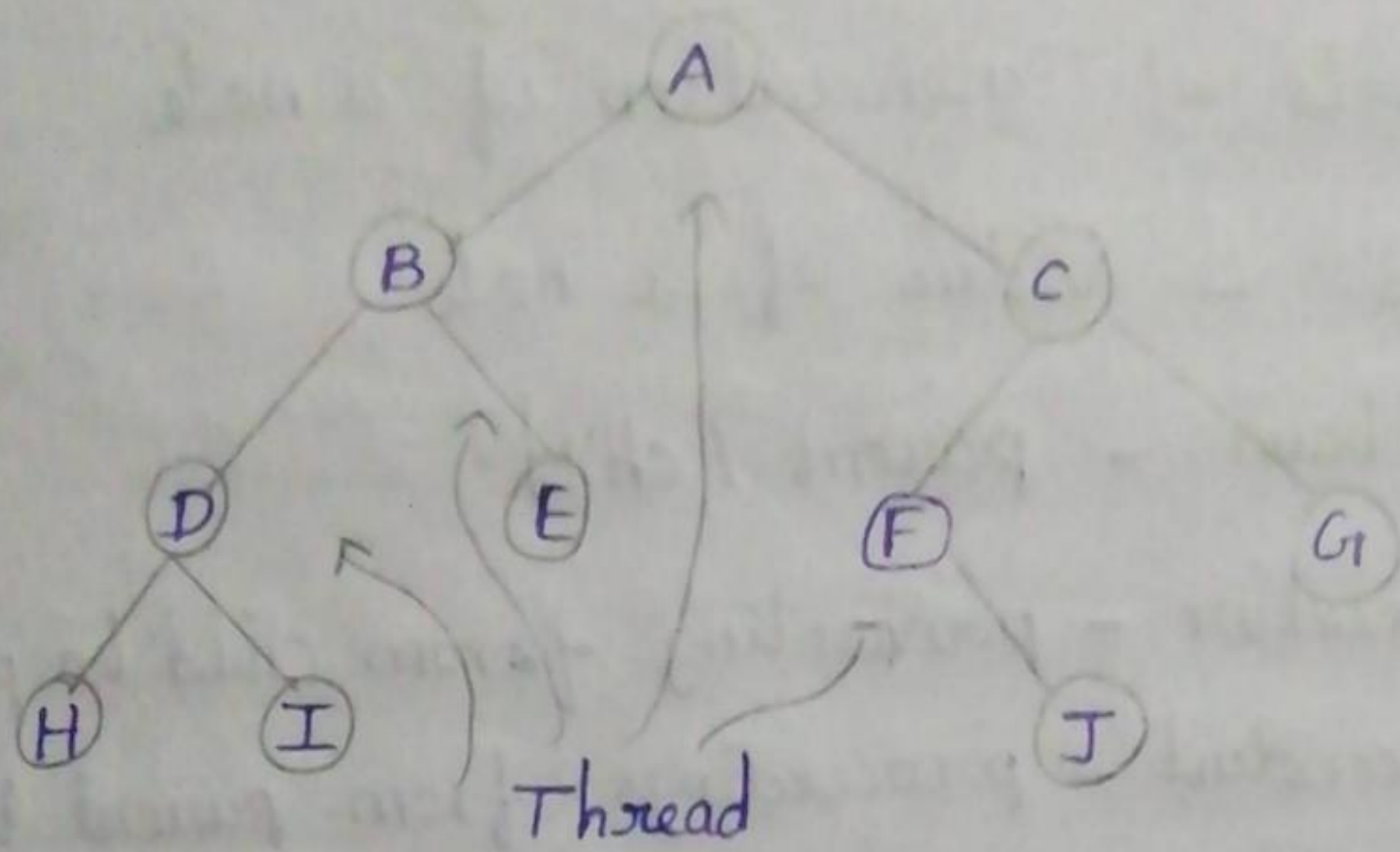
Example: 10, 12, 5, 3, 4, 11, 2, 6, 7, 8 (Inorder, pre order, post order)

Threaded Binary Tree

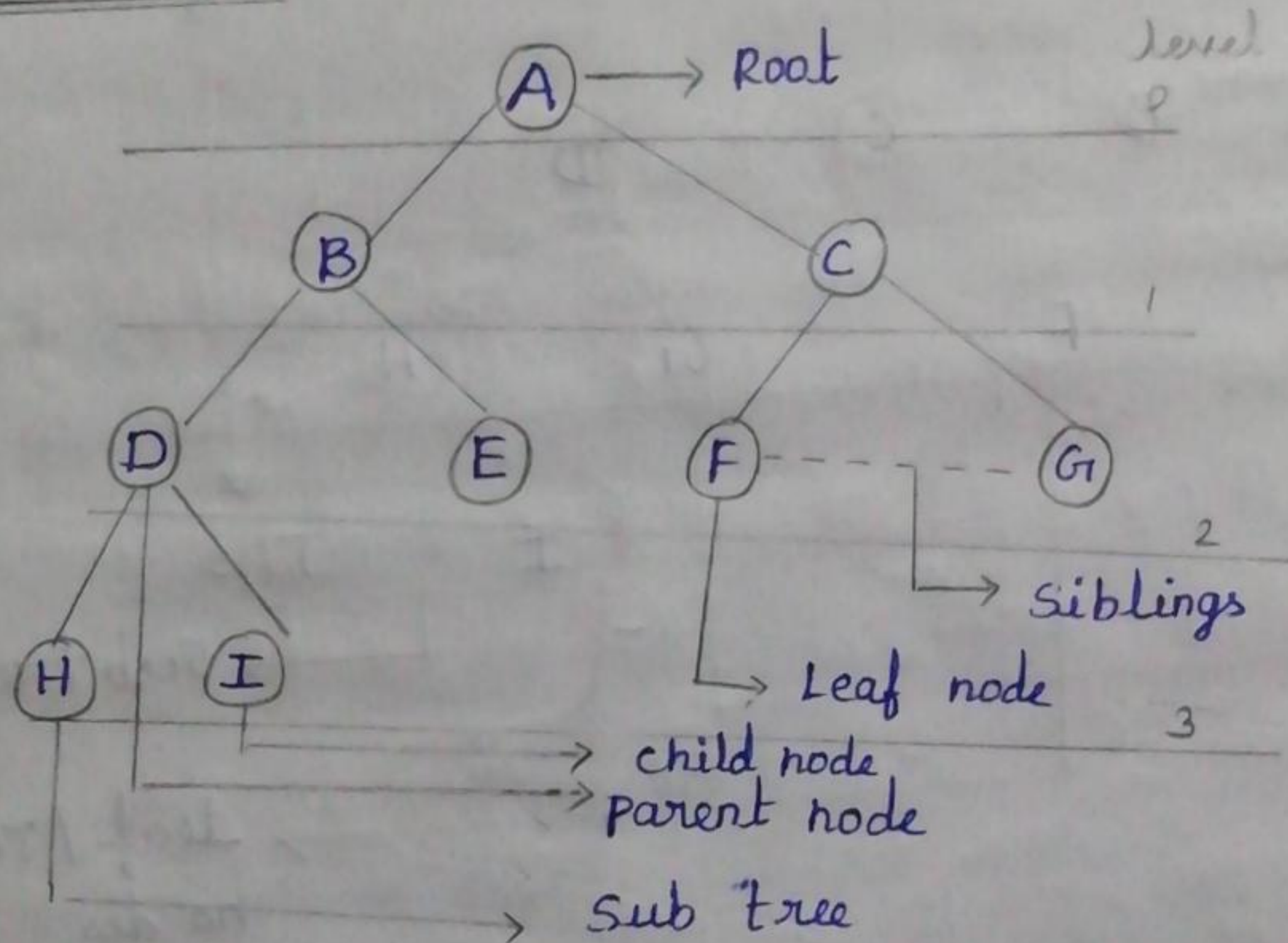
- * It is also Binary Tree in which all left child ~~parents~~^{pointer} that are NULL (in LL rep) points to its in order predecessor, all right child ~~parents~~^{pointer} that are NULL (in LL rep) points to its in order successor.
- * If there is no in order predecessor or in order successor, then it points to the root node.

Example

H - D - I - B - E - A - F - J - C - G



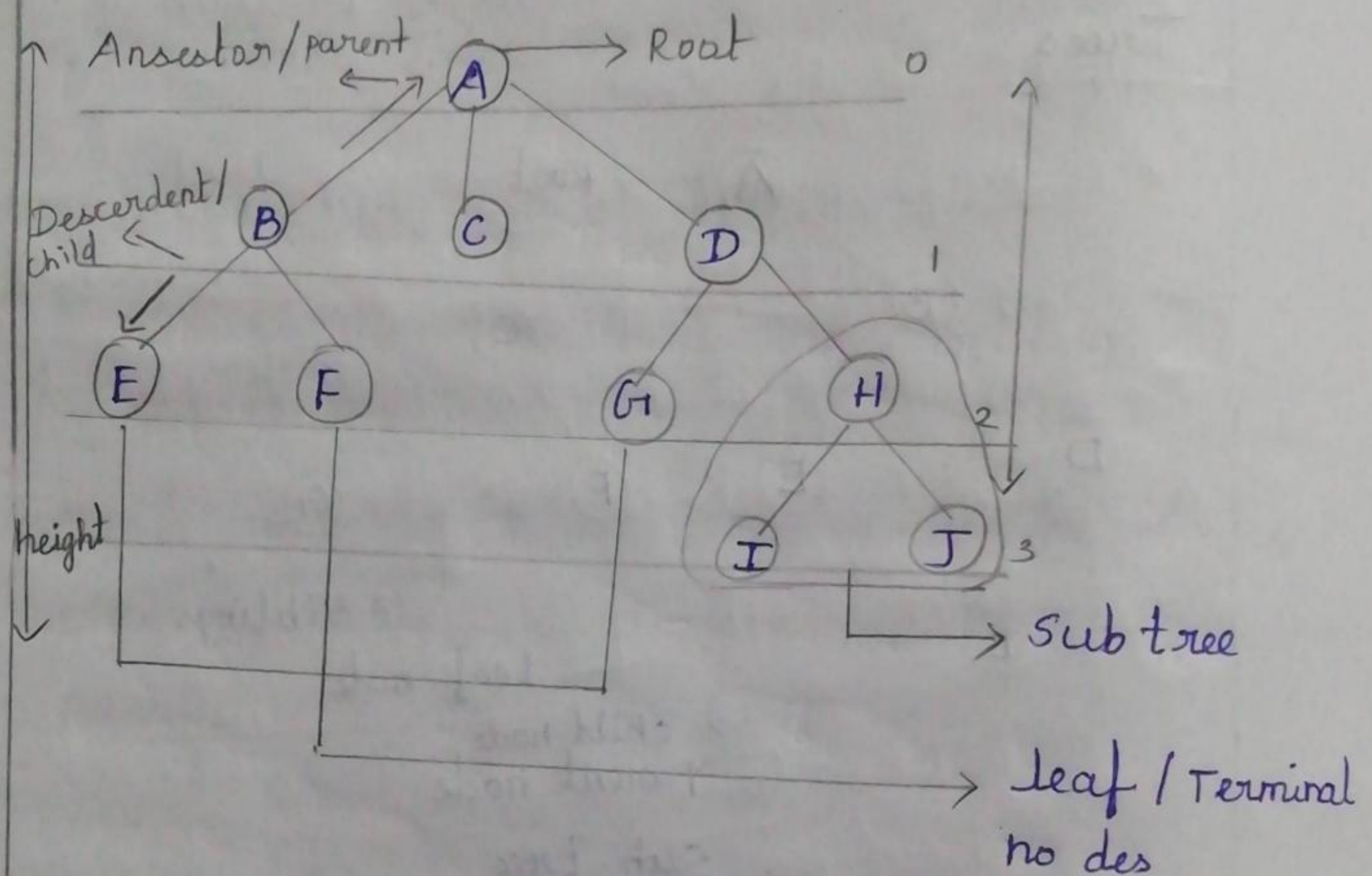
Trees



Important terms with respect to tree

1. path - along the edges
2. Root - only one root / tree, node @ the top
3. parent - edge upward
4. child - edge downward
5. Leaf - does not have any child
6. Subtree - descendants of a node
7. Visiting - checking the value of a node
8. Traversing - passing through nodes

- 9. Levels - generation of a node
- 10. Keys - value of a node
- 11. Neighbour - parent / child
- 12. Ancestor - proceeding from child to parent
- 13. Descendent - proceedings from parent to child



- 14. Distance - shortest path
- 15. width - NO. of nodes in a level
- 16. Breadth - NO. of leaves
- 17. Forest - Set of $n \geq 0$ disjoint trees
- 18. Ordered tree A rooted tree in which an ordering is specified for the children of each vertex
- 19. Size of a tree - NO. of nodes in the tree

Binary Search Tree (BST)

It is a Binary tree in which for every node x in the tree, the values of all the trees in left sub tree or smaller than the key value in x and the values of all the keys in its right sub tree are larger than the key value in x .

Operation perform in BST are:

1. Make empty
2. Insertion
3. deletion
4. Find
5. Find max
6. Find min.

Insertion:

* To insert x into tree T , ^{proceed.} proceed the

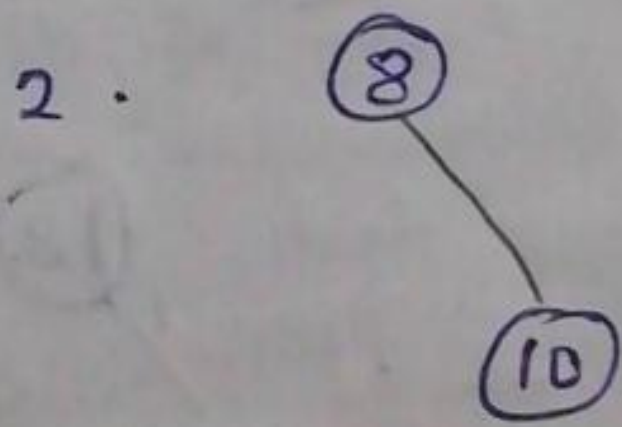
Find Function

* If x is found, do nothing otherwise insert x at the last spot on the path

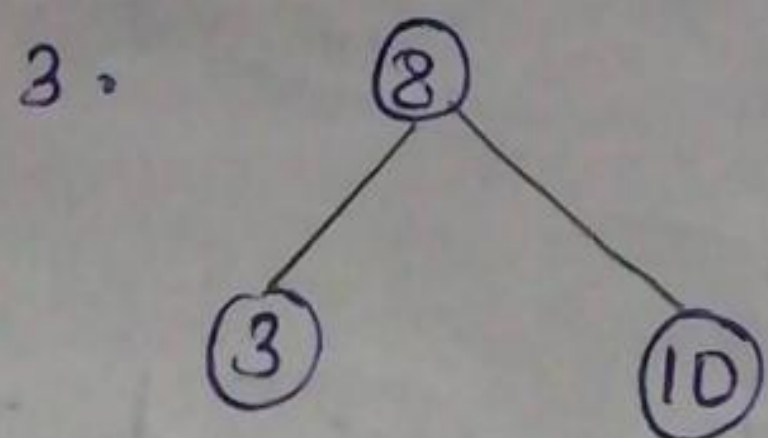
Travel. Example: 8, 10, 3, 2, 18, 6, 5, 13, 11



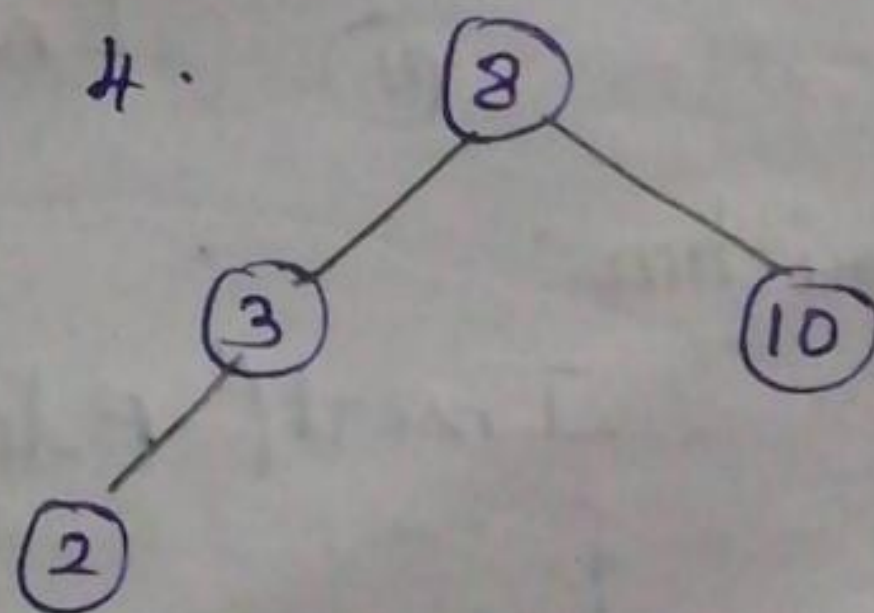
insert 8



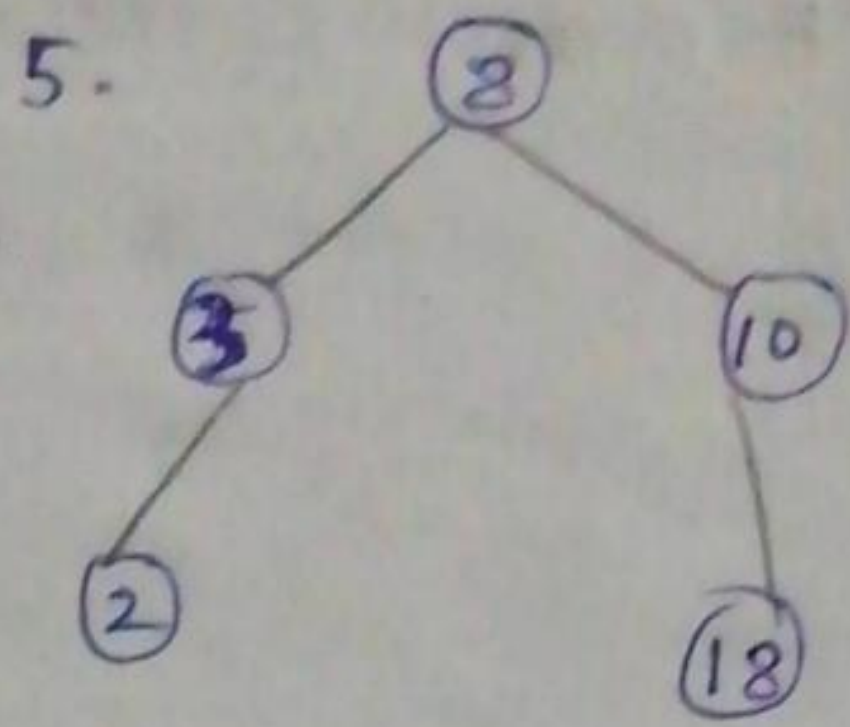
insert 10



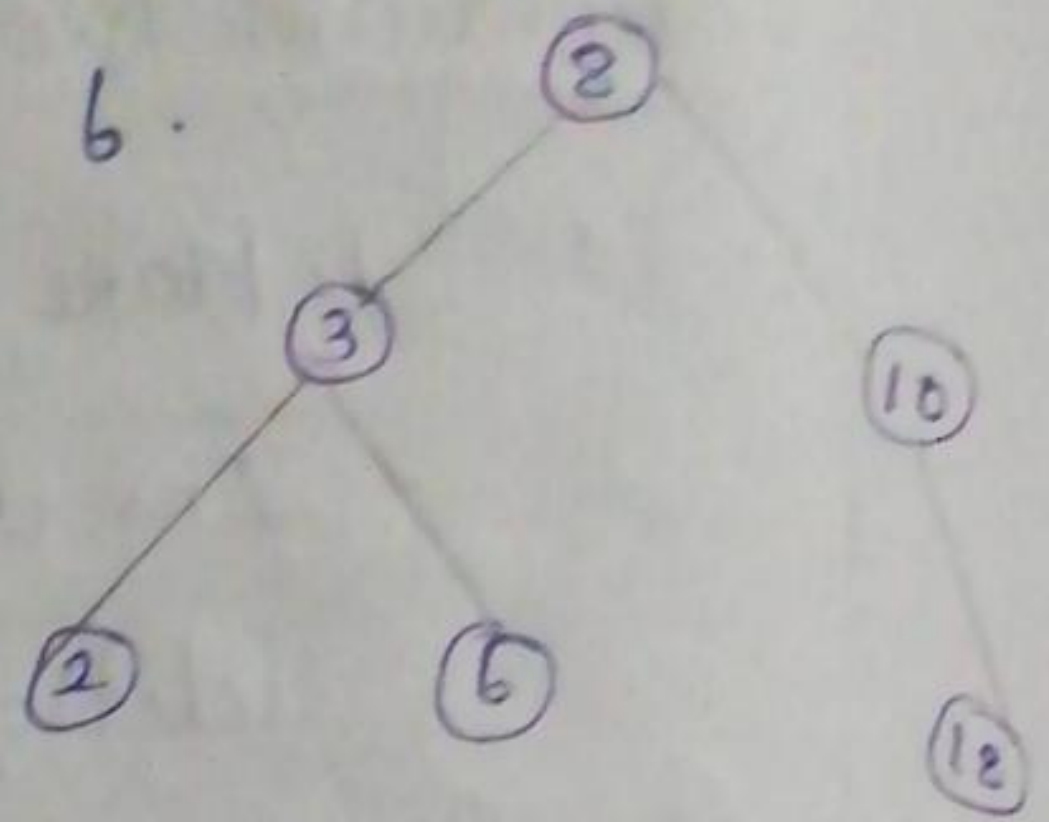
insert 3



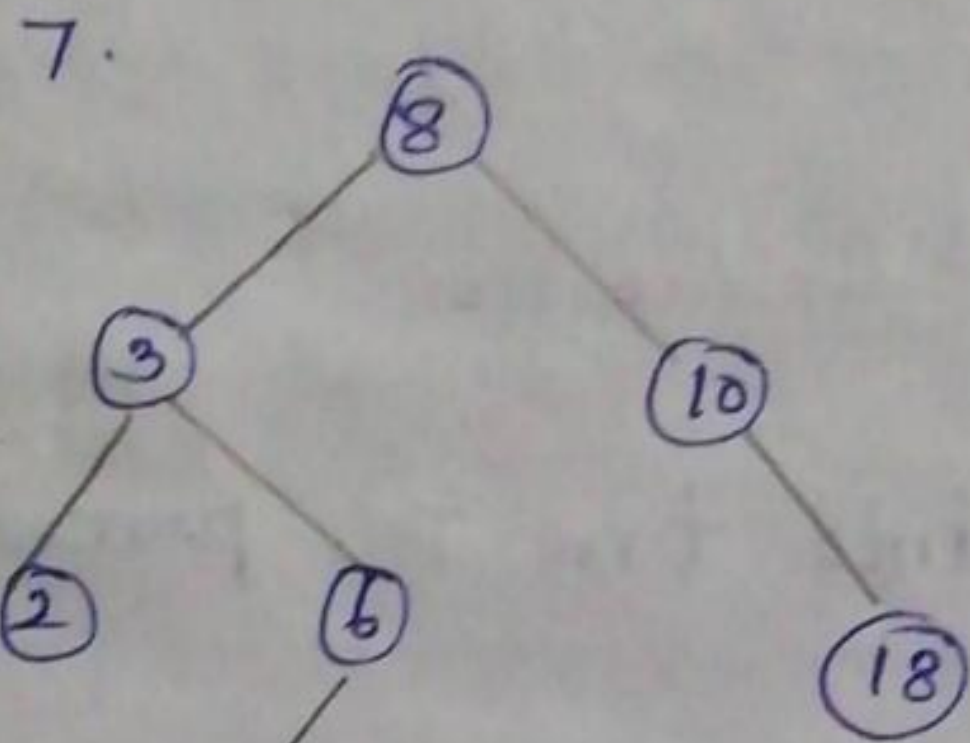
insert 2



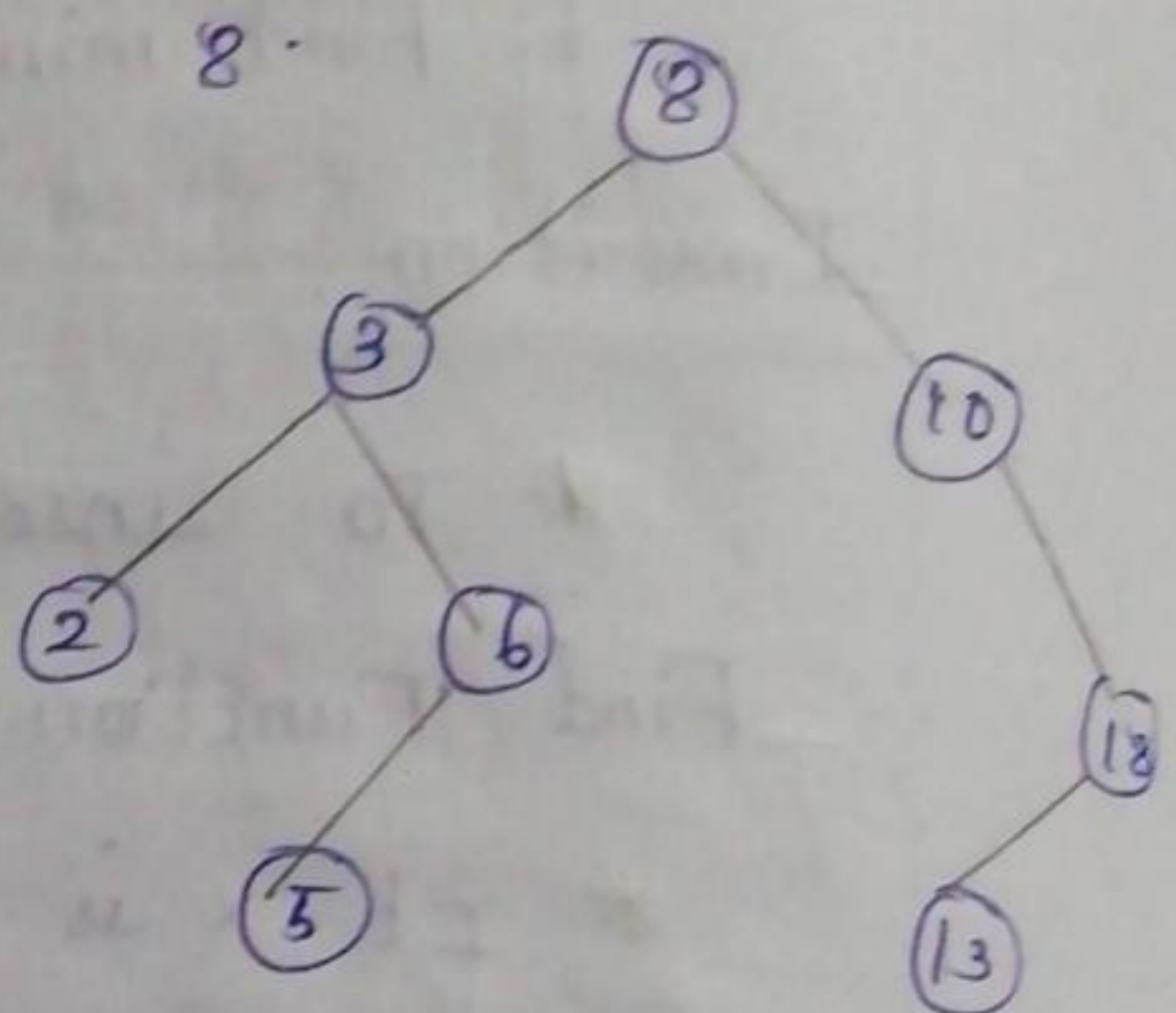
Insert 18



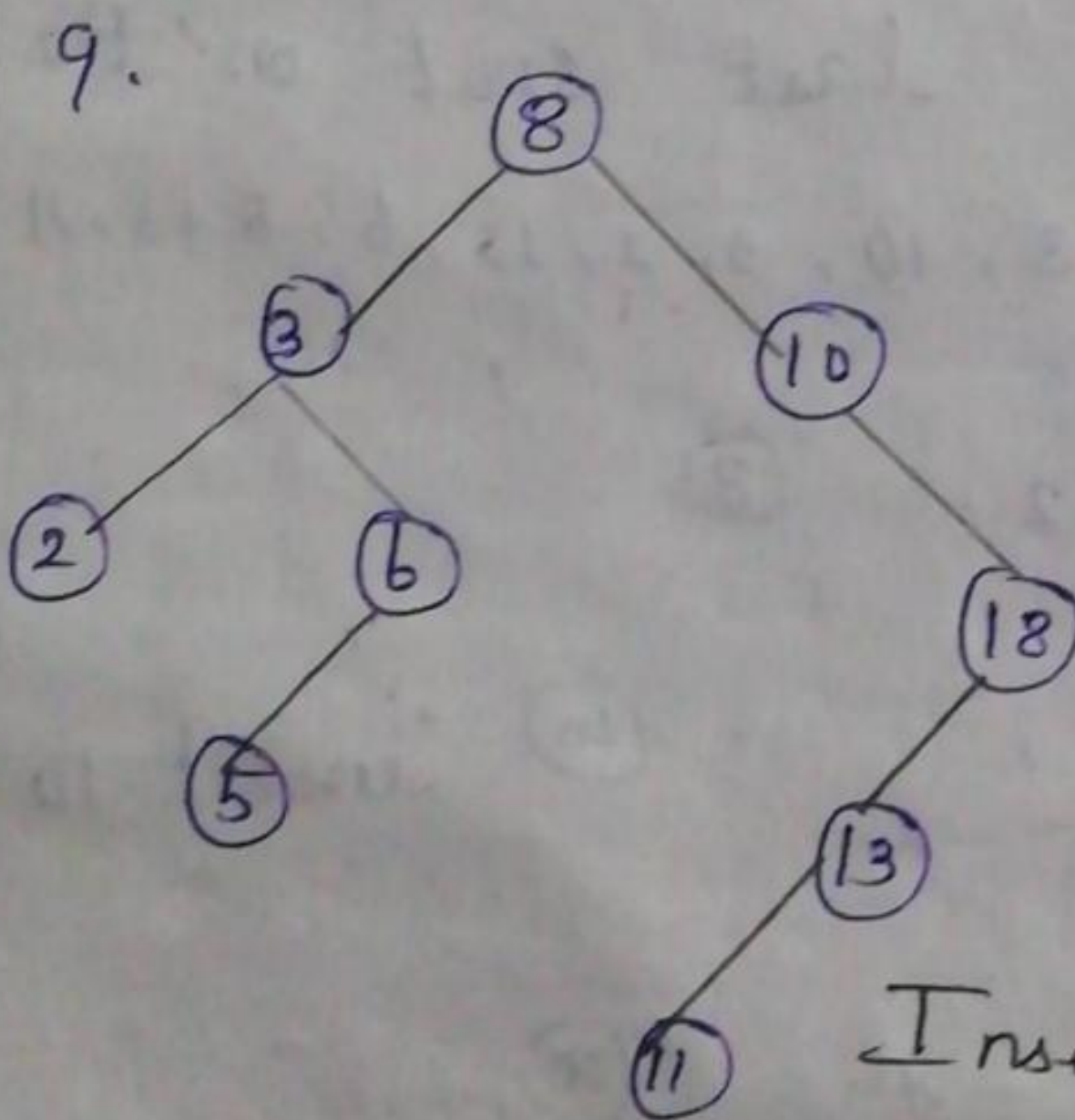
Insert 6



Insert 5



Insert 13



Insert 11

Algorithm:

Insert(Element Type x, Search Tree T)

{ if (T == NULL)

{ T = malloc (size of (struct Tree node));

if (T == NULL)

Fatal Error (" out of space ");

else

{


```

T => left = NULL;
T -> Right = NULL;
}
else
if (x > T -> Element)
T -> Right = Insert(x, T -> Right);
if (x < T -> Element)
T -> Left = Insert(x, T -> Left);
}
return T;

```

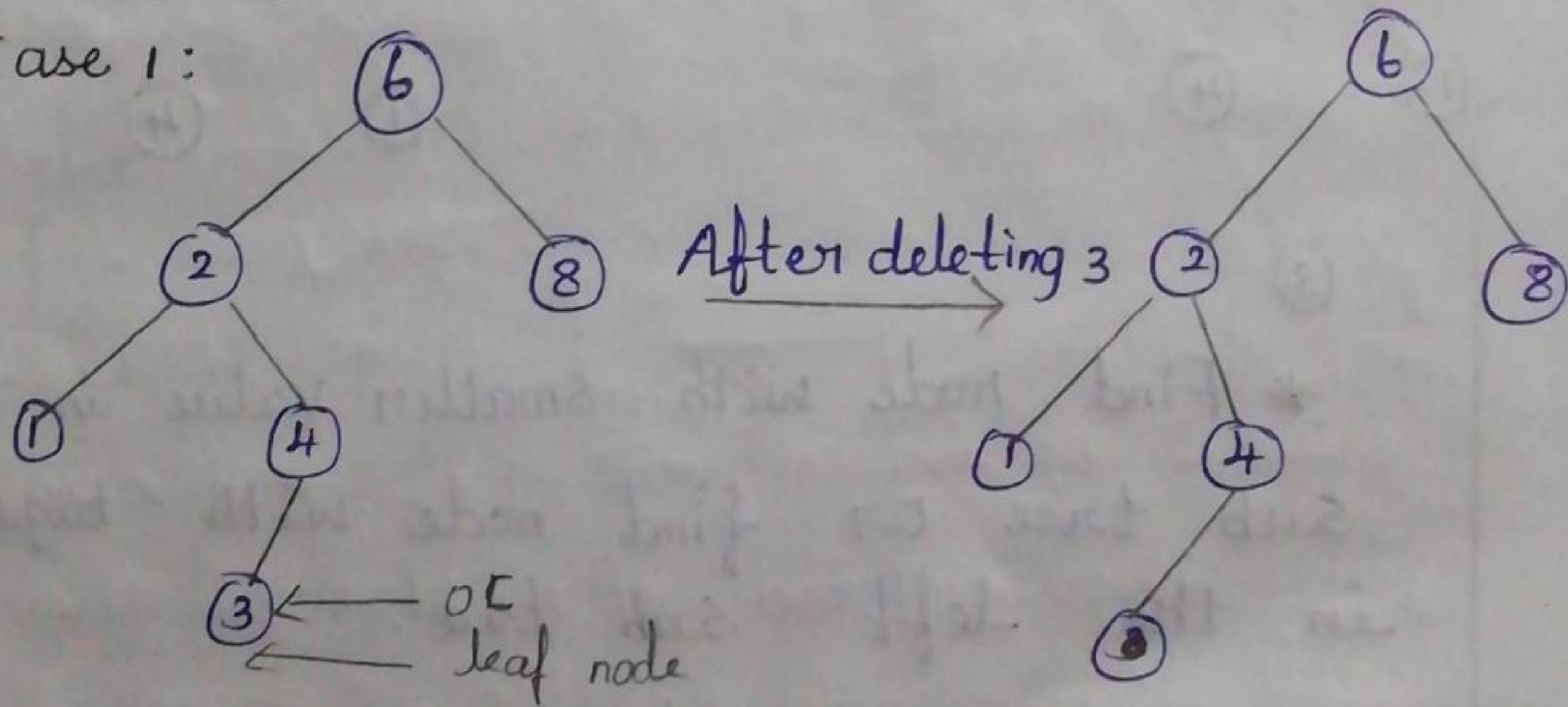
Deletion:

Case 1: Node with no children

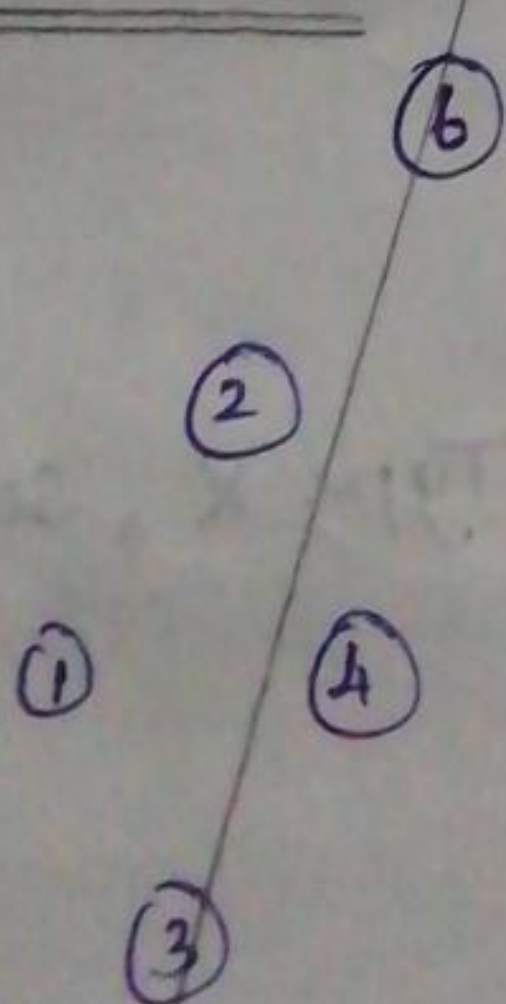
If the node is a leaf. It can be deleted immediately.

Example:

(case 1:



Case 2:

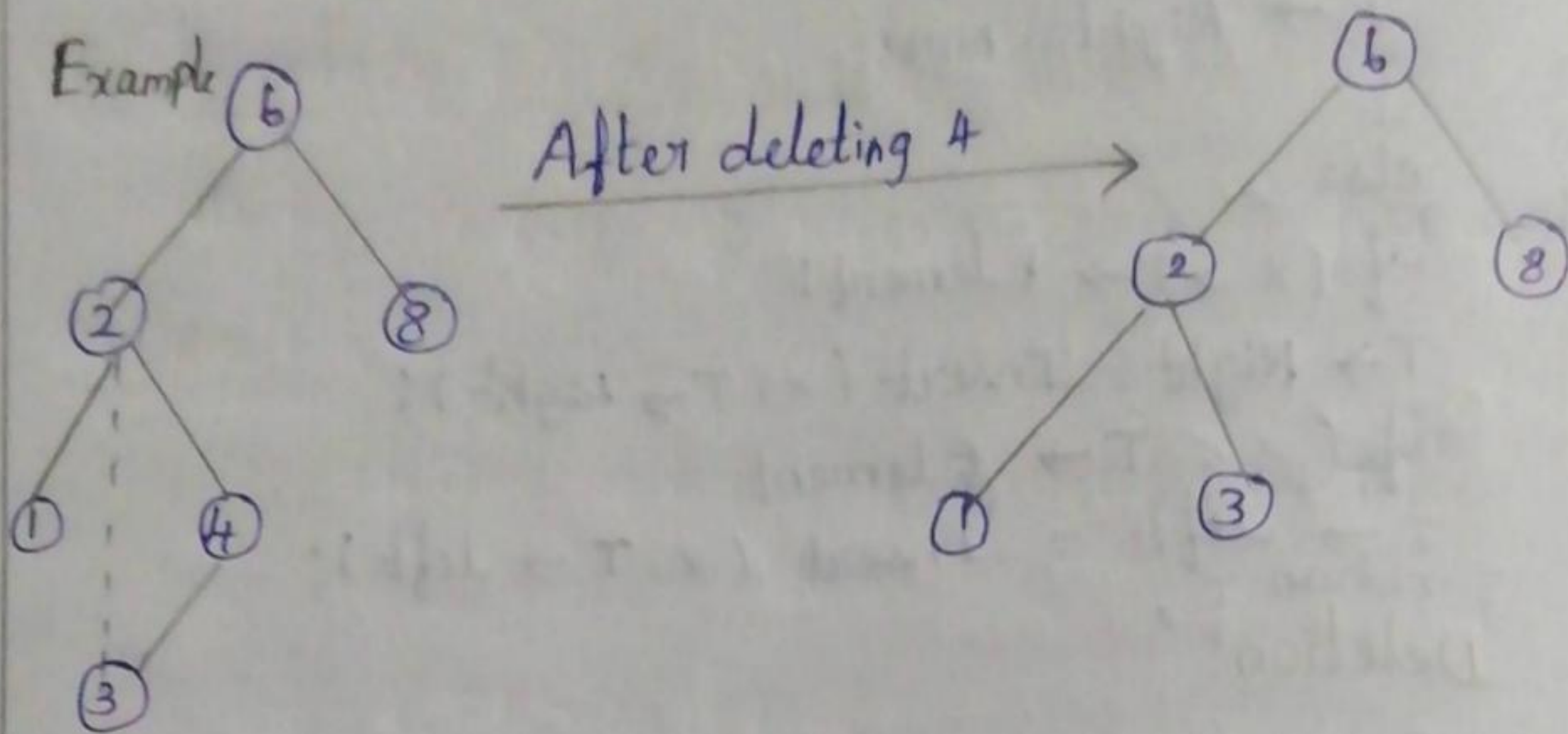


case 2: node with one children.

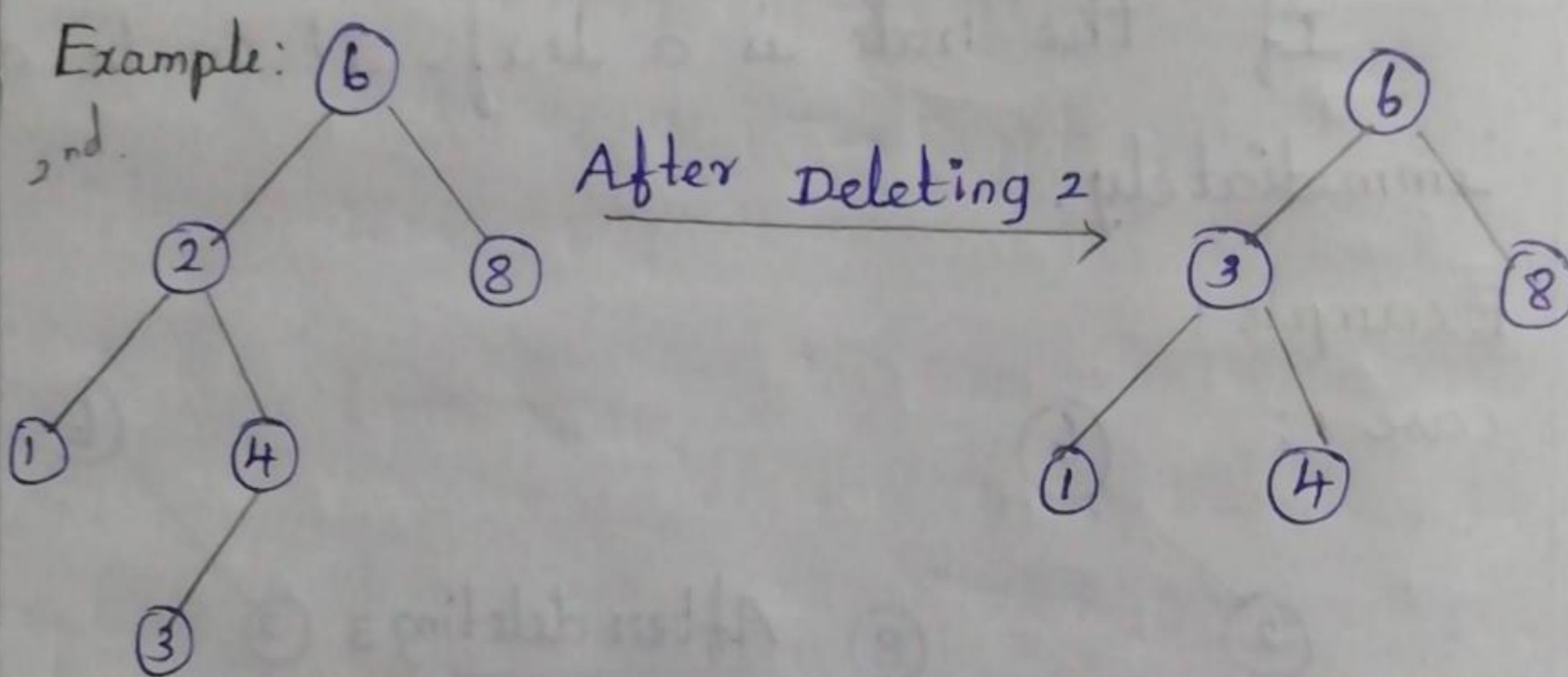
⑧

The node has one child, the node can be deleted after its parent, a pointed to by pass the node.

Case 2: 1 children (Node with one children)



Case 3: 2 children (Node with 2 children)



1st * Find node with smaller value in the right sub tree or find node with bigger value in the left sub tree.

* Replace the node in place of deleted node.

Algorithm:

Search Tree delete (Element Type X, search Tree)

{
position Tmpcell;

if (T == NULL)

Error ("Element not found");
else


```

if ( x < T → Element )
T → left = delete ( x, T → left )
else
if ( x > T → Element )
T → Right = Delete ( x, T → Right )
else
if ( T → left && T → Right )
{
    Tmpcell = Findmin
    ( T → Right );
    T → Element = Tmpcell → Element ;
    T → Right = Delete
    ( T → Elements, T → Right );
}
else
{
    Tmpcell = T;
    if ( T → left == NULL )
    T = T → Right;
    else
    if ( T → Right == NULL )
    T = T → left;
    Free ( Tmpcell );
}
return T;

```

R.D.

The operation performed in BST

Make empty :

make empty operation is used for initialization

Algorithm

```
SearchTree makeEmpty (SearchTree T)
{
    if (T != NULL)
    {
        makeEmpty (T → Left);
        makeEmpty (T → Right);
        Tree (T);
    }
    return NULL;
}
```

Find

It's operation will find key x in the BST. If key is found then ptr to that node is returned. If the key is not found it will return NULL

Algorithm :

```
position Find (Element Type x, Search Tree T)
{
    if (T == NULL)
        return NULL;
    if (x < T → Element)
        return Find (x, T → left);
}
```



```
else if (x > T → Element)
return Find (x, T → Right)
else
return T;
}
```

Find min :

* It routine return the position of smallest element in the tree.

* To perform a Findmin, start at the root and go left as long as there is a left child. The stopping point is the smallest element.

Algorithm :

```
position Findmin (Search Tree T)
```

```
{
if (T == NULL)
return NULL;
else if (T → Left == NULL)
return T;
else
return Findmin (T → Left);
}
```

Find Max :

* It routine return the position of largest element in the tree.

* To perform a Findmax, start at the root and go right as long as there is a right child. The stopping point is the largest element.

Algorithm

position Findmax (Search Tree T)

{ if (T == NULL)

return NULL;

else if

~~if~~ (T → right == NULL)

return T;

else

return Findmax (T → Right);

}