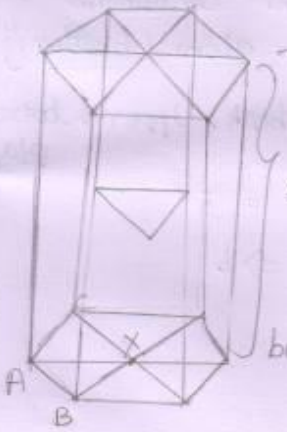
Hexagonal closed packed Structure



The unit cell has one atom at each of the 12 corners

The unit cell has one atom at the center of the two faces

The unit cell has 3 atoms arranged in body.

It consist of 3 layer.

At bottom layer, the central atom has 6 nearest neighbouring atoms in the plane.

The middle layer has 3 atoms and it is separated by the distance c/2 from bottom layer.

The top layer is similar to the bottom layer and at a distance c from the bottom layer.

No. of atoms per unit cell

No. of atom in unit cell from the Contribution of corner atoms (TOP) $= \frac{1}{8} \times 8 = 1$

No. of atom in unit cell from the Contribution of corner atoms (bottom) $= \frac{1}{8} \times 8 = 1$

No. of central atoms in both Upper & bottom planes $\frac{1}{2} \times 2 = 1$

No. of middle layered atom = 3

$1+1+1+3 \Rightarrow 6$

Co-ordination number

bottom

The atom 'x' is consist of

bottom layer atom = 6
above layer = 3
below layer = 3

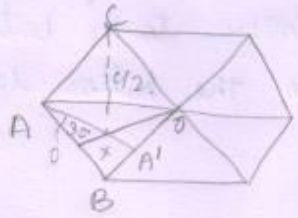
$6+3+3=12$

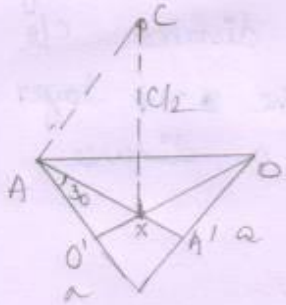
atomic radius

$a = 2r$

$r = a/2$

Calculation of c/a ratio





height of the unit cell = C

ΔABy

$\cos 30^\circ = \frac{Ay}{AB}$

$Ay = AB \cos 30^\circ$

$\boxed{Ay = \frac{a\sqrt{3}}{2}}$

X is center for ΔABO,

$AX = \frac{2}{3} Ay$

$AX = \frac{2}{3} \frac{a\sqrt{3}}{2} = \frac{a\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$\boxed{AX = \frac{a}{\sqrt{3}}}$

ΔAXC

$AC^2 = AX^2 + CX^2$

We know $AC = a$, $AX = a/\sqrt{3}$, $CX = C/2$

$a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left[\frac{c}{2}\right]^2$

$a^2 = \frac{a^2}{3} + \frac{c^2}{4}$

rearrange the above term

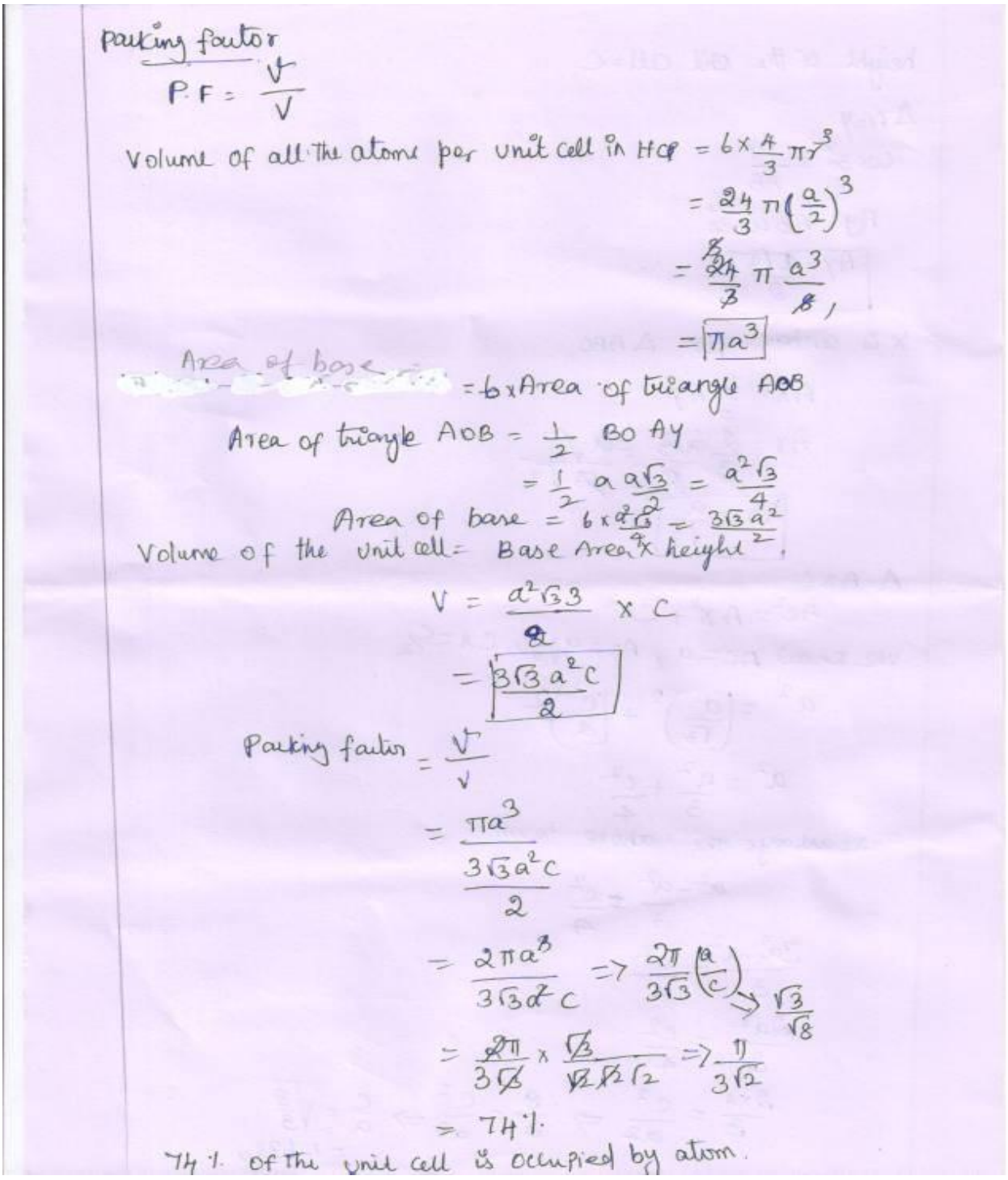
$a^2 - \frac{a^2}{3} = \frac{c^2}{4}$

$\frac{3a^2 - a^2}{3} = \frac{c^2}{4}$

$\frac{2a^2}{3} = \frac{c^2}{4}$

$\frac{2 \times 4}{3} = \frac{c^2}{a^2} \Rightarrow \frac{8}{3} = \frac{c^2}{a^2} \Rightarrow \frac{c}{a} = \sqrt{\frac{8}{3}}$

$= 1.633$ //



Packing factor

$$P.F = \frac{V'}{V}$$

Volume of all the atoms per unit cell in HCP $= 6 \times \frac{4}{3}\pi r^3$

$$= \frac{24}{3}\pi\left(\frac{a}{2}\right)^3$$

$$= \frac{24}{3}\pi\frac{a^3}{8}$$

$$= \boxed{\pi a^3}$$

Area of base $= 6 \times$ Area of triangle AOB

Area of triangle AOB $= \frac{1}{2}$ BO AY

$$= \frac{1}{2} a \frac{a\sqrt{3}}{2} = \frac{a^2\sqrt{3}}{4}$$

Area of base $= 6 \times \frac{a^2\sqrt{3}}{4} = \frac{3\sqrt{3}a^2}{2}$

Volume of the unit cell = Base area $\times$ height.

$$V = \frac{a^2\sqrt{3}3}{2} \times c$$

$$= \boxed{\frac{3\sqrt{3}a^2c}{2}}$$

Packing factor $= \frac{V'}{V}$

$$= \frac{\pi a^3}{\frac{3\sqrt{3}a^2c}{2}}$$

$$= \frac{2\pi a^3}{3\sqrt{3}a^2c} \Rightarrow \frac{2\pi}{3\sqrt{3}}\left(\frac{a}{c}\right) \Rightarrow \frac{\sqrt{3}}{\sqrt{8}}$$

$$= \frac{2\pi}{3\sqrt{8}} \times \frac{\sqrt{3}}{\sqrt{2}\sqrt{2}\sqrt{2}} \Rightarrow \frac{\pi}{3\sqrt{2}}$$

$= 74\%$

74% of the unit cell is occupied by atom.