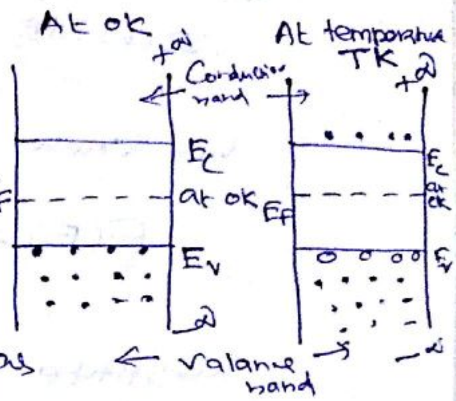
Carrier Concentration in Intrinsic Semiconductors :-



We know, at 0K intrinsic Pure semiconductor behave as insulator. But as temperature increases Some electrons move from valance band to conduction band as shown in figure.

There fore both electrons in conduction band and holes in valene band will contribute to electrical conductivity. There fore the carrier concentration (or) density of electrons (n_e) and hole (n_h) has to be calculated.

Assume that

$m_e^* \rightarrow$ mass of free electron in conduction

$m_h^* \rightarrow$ mass of hole in valance band.

The electrons energy in conduction band E_C to ∞

The holes energy in valence band lying from $-\infty$ to E_V

In figure, $E_C \rightarrow$ lowest energy level in conduction band

$E_V \rightarrow$ highest energy level in valance band

Density of electrons in conduction band

Density of electrons in conduction band

$$n_e = \int_{E_C}^{\infty} Z(E) \cdot F(E) \cdot dE \qquad \text{— (1)}$$

From Fermi dirac Statistics,

$$Z(E)dE = 2 \cdot \frac{\pi}{4} \left(\frac{8m_e^*}{h^2}\right)^{3/2} E^{1/2}\, dE \qquad \text{— (2)}$$

minimum energy of conduction band as E_C and the maximum energy upto ∞ So, equation (2), equation ② becomes

$$Z(E)\,.dE = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2}(E - E_c)^{1/2}.dE \quad -③$$

We know;

$$F(E) = \frac{1}{1+e^{(E-E_F)/k_BT}} \quad -④$$

Substitute ③ & ④ in ①,

$$n_e = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2}\int_{E_c}^{\infty}\frac{(E-E_c)^{1/2}}{1+e^{(E-E_F)/k_BT}}\,.dE \quad -⑤$$

Since electron move from valence band to conduction band the energy required is greater than $4k_BT$. (i.e),

$$E - E_F >> k_BT, \quad \frac{E-E_F}{k_BT} >> 1$$

$$e^{(E-E_F)/k_BT} >> 1$$

$$1 + e^{(E-E_F)/k_BT} \approx e^{(E-E_F)/k_BT}$$

equation ⑤ ⇒

$$n_e = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2}\int_{E_c}^{\infty}\frac{(E-E_c)^{1/2}}{\cancel{1+}e^{(E-E_F)/k_BT}}\,.dE$$

$$n_e = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2}.\,e^{(E_F-E)/k_BT}\,.dE \quad -⑥$$

Let us assume, $E - E_c = x\,k_BT$

$$E = E_c + x\,k_BT$$

$$dE = k_BT.\,dx$$

Limits,	when $E = E_c$	when $E = \infty$
	$x = 0$	$x = \infty$

So Limits are 0 to ∞

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equation (6) becomes,

$$n_e = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2}\int_0^{\infty}(x\,k_BT)^{1/2}\,e^{\frac{(E_F - xk_BT - E_c)}{k_BT}}\cdot k_BT\cdot dx$$

$$= \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2}\int_0^{\infty}(x)^{1/2}(k_BT)^{3/2}\cdot e^{(E_F-E_c)/k_BT}\cdot e^{-x}\cdot dx$$

$$n_e = \frac{\pi}{2}\left(\frac{8m_e^* k_BT}{h^2}\right)^{3/2} e^{(E_F-E_c)/k_BT}\underbrace{\int_0^{\infty} x^{1/2}\cdot e^{-x}\cdot dx}_{\sqrt{\pi}/2}$$

$$n_e = \frac{\pi}{2}\left(\frac{8m_e^* k_BT}{h^2}\right)^{3/2} e^{(E_F-E_c)/k_BT}\,\frac{\sqrt{\pi}}{2}$$

$$n_e = \frac{1}{4}\left(\frac{8\pi m_e^* k_BT}{h^2}\right)^{3/2} e^{(E_F-E_c)/k_BT} \qquad (2)^{3/2} = 2\sqrt{2}$$

~~Density of electrons in conduction band is~~

$$8^{3/2} = 8\sqrt{8} = 8\,2\sqrt{2}$$

~~[illegible]~~

$$n_e = \frac{8}{4}\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2} e^{(E_F-E_c)/k_BT}$$

$$\boxed{n_e = 2\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2} e^{(E_F-E_c)/k_BT}} \quad \text{--- (7)}$$

The above equation represents the density of electrons in conduction band.

Density of holes in valence band:-

$$n_h = \int_{-\infty}^{E_V} Z(E)\,[1 - F(E)]\cdot dE \quad -(8)$$

$F(E) \rightarrow$ probability of filled state

$1 - F(E) \Rightarrow$ Probability of unfilled state

we know, $$Z(E)\cdot dE = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} (E_V - E)^{1/2}\cdot dE \quad -(9)$$

$$1 - F(E) = 1 - \frac{1}{1 + e^{(E-E_F)/K_BT}}$$

$$= \frac{e^{(E-E_F)/K_BT}}{1 + e^{(E-E_F)/K_BT}}$$

Here, $E - E_F << K_BT$; $\frac{E-E_F}{K_BT} << 1$; $e^{\frac{(E-E_F)}{K_BT}} << 1$

So, $1 + e^{(E-E_F)/K_BT} \simeq 1$

$$\therefore 1 - F(E) = e^{(E-E_F)/K_BT} \quad -(10)$$

substitute equations (9) and (10) in (8)

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2}\cdot e^{(E-E_F)/K_BT}\cdot dE \quad -(11)$$

Let $E_V - E = x K_BT$

$E = E_V - x K_BT$

$dE = -dx\, K_BT$

Limits ~~are~~

$\Rightarrow$ when $E = -\infty$, $x = \infty$ | $E = E_V$, $x = 0$

So, Limits are ~~xx~~ ∞ to 0,

So equation (11) becomes,

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_{\infty}^{0} (x K_BT)^{1/2} e^{\frac{(E_V - xK_BT - E_F)}{K_BT}}\cdot(-dx) K_BT$$

$$= \frac{\pi}{2}\left(\frac{8m_h^*}{h^2}\right)^{3/2} \int_0^{\infty} x^{1/2} (K_BT)^{3/2} e^{(E_V - E_F)/K_BT}\, e^{-x}\cdot dx$$

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^* K_BT}{h^2}\right)^{3/2}\cdot e^{(E_V - E_F)/K_BT}\cdot \underbrace{\int_0^{\infty} x^{1/2} e^{-x}\cdot dx}_{\sqrt{\pi}/2}$$

$$n_h = \frac{\pi}{2}\left(\frac{8m_h^* k_BT}{h^2}\right)^{3/2} \cdot e^{(E_v-E_F)/k_BT} \cdot \frac{\sqrt{\pi}}{2}$$

$$= \frac{1}{4}\left(\frac{8\pi m_h^* k_BT}{h^2}\right)^{3/2} \cdot e^{(E_v-E_F)/k_BT}$$

$$n_h = \frac{8}{4}\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} \cdot e^{(E_v-E_F)/k_BT}$$

$8^{3/2} = 8\sqrt{8}$
$= 8\times 2\sqrt{2}$
$8\times(2^{3/2})$

$$\boxed{n_h = 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} e^{(E_v-E_F)/k_BT}} \quad \text{--- (12)}$$

equation (12) is the density of holes in valance-band.

Variation of Fermi level and Carrier Concentration with temperature in an intrinsic Semiconductor

For intrinsic Semiconductor number of electrons (i.e.,) electron density will be the same as that of number of holes (i.e.,) hole density.

i.e., $n_e = n_h$

From (4) and (12);

$$2\left(\frac{2\pi m_e^* k_BT}{h^2}\right)^{3/2} e^{(E_F-E_C)/k_BT} = 2\left(\frac{2\pi m_h^* k_BT}{h^2}\right)^{3/2} e^{(E_v-E_F)/k_BT}$$

$$\left(\frac{m_h^*}{m_e^*}\right)^{3/2} = \frac{e^{(E_F-E_C)/k_BT}}{e^{(E_v-E_F)/k_BT}}$$

$$\left(\frac{m_h^*}{m_e^*}\right)^{3/2} = e^{(E_F-E_c-E_v+E_F)/k_BT}$$

Taking log on both sides,

$$\frac{3}{2}\log\left(\frac{m_h^*}{m_e^*}\right) = \frac{(2E_F-(E_v+E_c))}{k_BT}$$

$$2E_F = E_v+E_c+\frac{3}{2}k_BT\log\left(\frac{m_h^*}{m_e^*}\right)$$

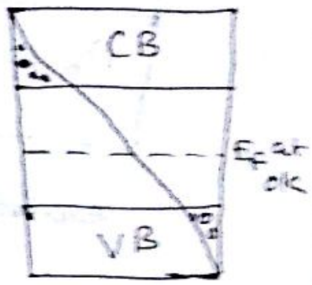
$$\boxed{E_F = \frac{E_c + E_v}{2} + \frac{3}{4} k_B T \log\left(\frac{m_h^*}{m_e^*}\right)} \quad — (13)$$

If $m_h^* = m_e^*$, then, $\log \frac{m_h^*}{m_e^*} = 0$ $[\because \log 1 = 0]$

$$E_F = \frac{E_c + E_v}{2} \quad — (14)$$

i.e., Fermilevel lies in the midway between E_c and E_v (at $T = 0K$). But Fermilevel slightly — increases with increase in temperature.

CB – conduction band
VB – valence band



Density of electrons and holes interms of E_g :-

Interms of Energy gap $(E_g = E_c - E_v)$, we can get the expression of n_e and n_h by substituting the value of E_F interms of E_c and E_v.

We know,

$$n_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} e^{(E_F - E_c)/k_B T}$$

$$n_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \cdot \exp\left[\frac{\frac{E_c + E_v}{2} + \frac{3}{4} k_B T \log\left(\frac{m_h^*}{m_e^*}\right) - E_c}{k_B T}\right]$$

$$= 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[\frac{2E_c + 2E_v + 3 k_B T \log\left(\frac{m_h^*}{m_e^*}\right) - 4E_c}{4 k_B T}\right]$$

$$= 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[\frac{-2E_c + 2E_v + 3k_BT\log\left(\frac{m_h^*}{m_e^*}\right)}{4k_BT}\right]$$

$$= 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[\frac{2(E_v - E_c)}{4k_BT} + \frac{3}{4}\log_e\left(\frac{m_h^*}{m_e^*}\right)\right]$$

We know, $E_g = E_c - E_v$

$$n_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[\frac{-E_g}{2k_BT} + \log_e\left(\frac{m_h^*}{m_e^*}\right)^{3/4}\right]$$

$$n_e = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} \cdot e^{-E_g/2k_BT} \cdot (m_e^*)^{3/2} \cdot \frac{(m_h^*)^{3/4}}{(m_e^*)^{3/4}}$$

$3/2 - 3/4 = \frac{6-3}{4} = 3/4$

$$n_e = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} e^{-E_g/2k_BT} \cdot (m_e^*)^{3/4} \cdot (m_h^*)^{3/4}$$

$$\boxed{n_e = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} \cdot e^{-E_g/2k_BT} \cdot (m_e^* \cdot m_h^*)^{3/4}} \quad \text{— (15)}$$

Illy $$n_h = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} e^{-E_g/2k_BT} (m_h^* \cdot m_e^*)^{3/4} \quad \text{— (16)}$$

It is found that $n_e = n_h = n_i$. In an intrinsic semiconductor density of electrons in conduction band is equal to the density of holes in valence band.

$n_i = n_e = n_h$

$$\boxed{n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} (m_e^* \cdot m_h^*)^{3/2} e^{-E_g/2k_BT}} \quad \text{— (17)}$$

equation (17) is the carrier concentration for intrinsic semiconductor.