5.10 MILLER INDICES

Miller introduced a set of three numbers to designate a plane in a crystal. This set of three numbers are known as *Miller indices* of the concerned plane.

Miller indices is defined as the reciprocal of the intercepts made by the plane on the crystallographic axes which are reduced to smallest numbers.

Explanation

The orientation of planes or faces in a crystal may be described in terms of their intercepts on the three axes.

For example, the plane ABC of Fig.(5.11) has intercepts of 2 axial units on X-axis, 1 axial units on Y-axis and 1 axial units on Z-axis.

In other words, the numerical parameters of the faces are 2, 1 and 1. Hence its orientation is (2 1 1)

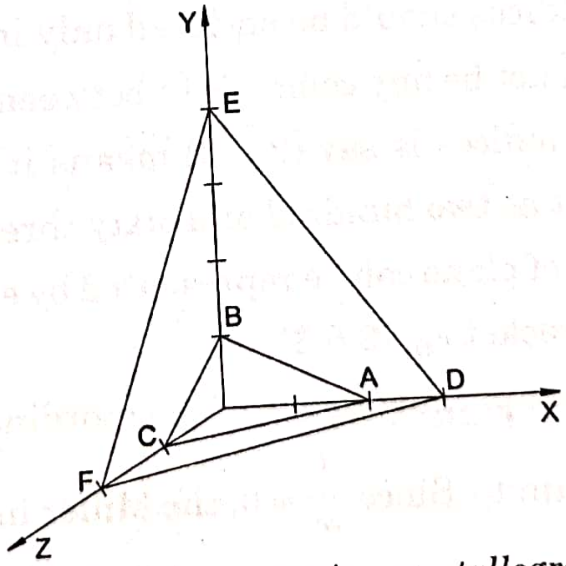


Fig. 5.11 Different planes cutting crystallographic axes

Miller suggested that it is more useful to describe the orientation of a plane by the reciprocal of its numerical parameters rather than by its linear parameters.

These reciprocals are converted into whole numbers. They are called Miller indices. Hence Miller indice of a plane ABC of Fig.(5.11) are $\left(\frac{1}{2}\ \frac{1}{1}\ \frac{1}{1}\right)$ or simply (1 2 2). Similarly Miller indice of a plane DEF are $\left(\frac{1}{3}\ \frac{1}{4}\ \frac{1}{2}\right)$ or simply (4 3 6)

Miller indices is also defined as the three smallest possible integers, which have the same ratio as the reciprocals of the intercepts of the plane concerned along the 3 axes.

Procedure for finding Miller indices

To find the Miller indices for a given plane, the following steps are to be followed.

i. The intercepts made by the plane along X, Y and Z axes are noted.

ii. The co-efficients of the intercepts are noted separately.

iii. Inverse is to be taken.

iv. The fractions are multiplied by LCM so that all the fractions become integers.

v. Write the integers within the parentheses.

Points to remember

While finding the Miller indices of a plane, following points should be kept in mind.

i. The Miller indices should be enclosed only in this bracket (i.e.,) ( )

ii. There should not be any comma's in between the numbers.

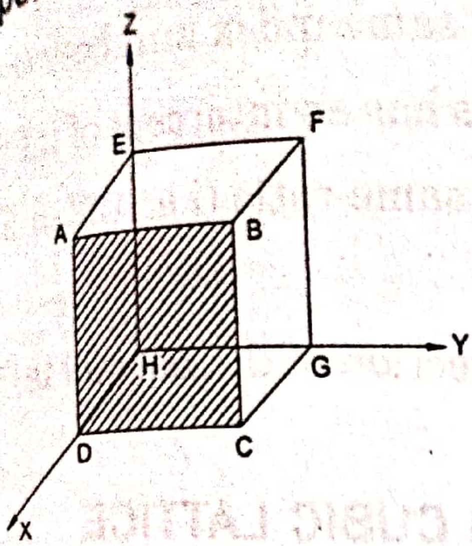
iii. If the Miller indices is say (2 6 3) means it should be read as two six three, and not as two hundred and sixty three.

iv. The direction of plane can be represented by enclosing the Miller indices in a square bracket eg. [2 6 3].

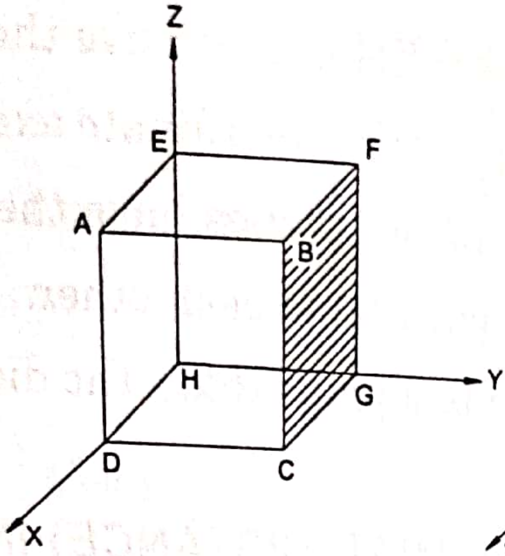
v. When a plane is parallel to one of the coordinate axes, it is said to meet that axis at infinity. Since $\frac{1}{\infty} = 0$, the Miller indices for that axis is zero.

vi. When the intercept of a plane is on the negative part of any axis, the Miller indices is distinguished by a bar put directly over it. Eg. $(h\ \bar{k}\ l)$.

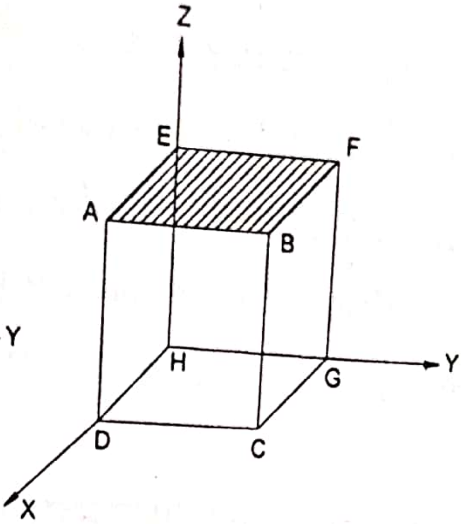
Important features of Miller indices



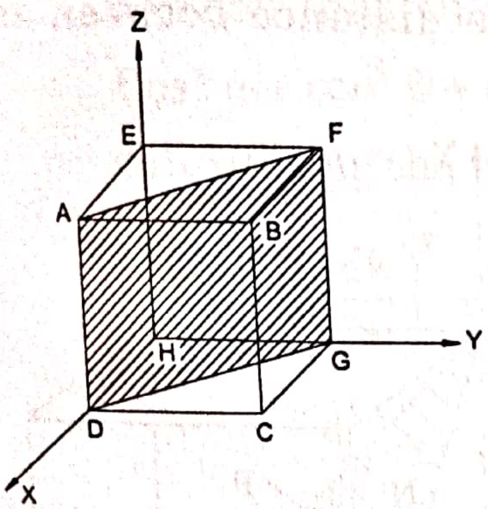
(a) plane (100)



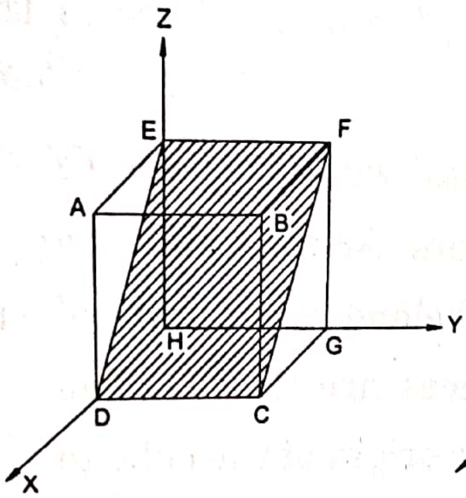
(b) plane (010)



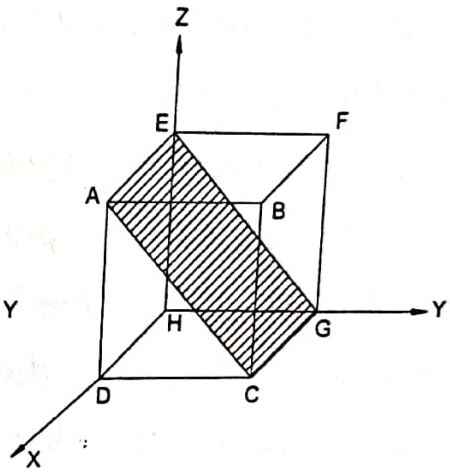
(c) plane (001)



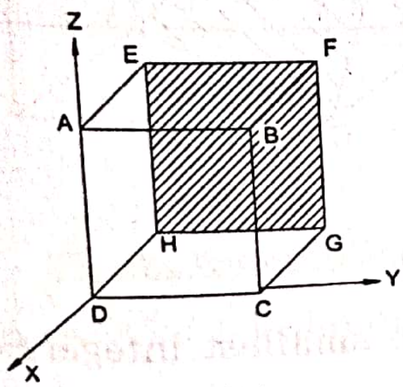
(a) plane (110)



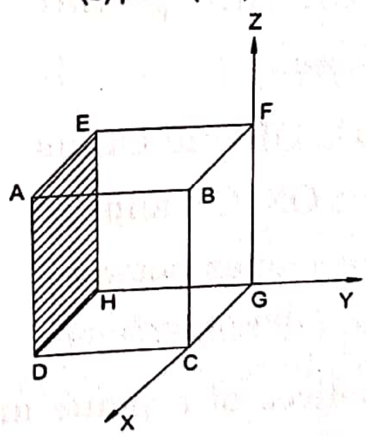
(b) plane (101)



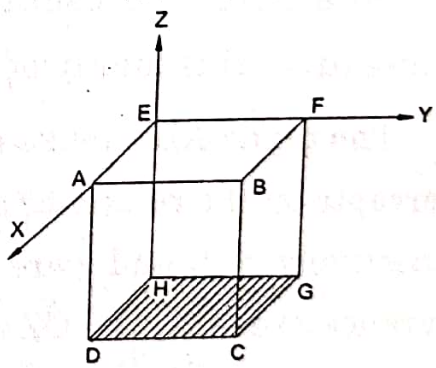
(c) plane (011)



(a) plane $(\bar{1}00)$



(b) plane $(0\bar{1}0)$



(c) plane $(00\bar{1})$

Z

5.11 'd' SPACING (INTERPLANAR DISTANCE) IN CUBIC LATTICE

'd' spacing (or) *interplanar distance* is the distance between any two successive planes.

Consider a cubic crystal with 'a' as length of the cube edge and a plane ABC as show in Fig.(5.13). Let this plane belong to a family of planes whose Miller indices are (h k l). The perpendicular OP from the origin of the cube to the plane ABC represents the interplanar spacing (d) of this family of plane.

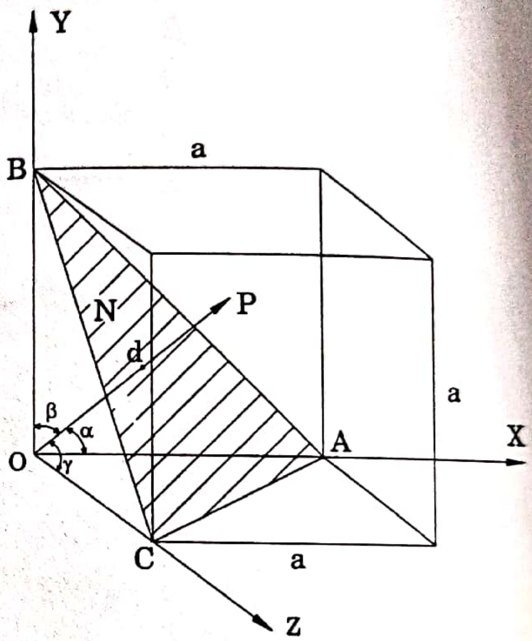


Fig. 5.13

The plane ABC makes OA, OB and OC as intercepts on the reference axes OX, OY and OZ respectively. α, β and γ are the angles between reference axes OX, OY, OZ and OP respectively.

We know that Miller indices of a plane are the smallest integers of the reciprocals of its intercepts. Therefore the intercepts may also be expressed as reciprocals of Miller indices.

$$\text{(ie)}\quad OA : OB : OC = \frac{1}{h} : \frac{1}{k} : \frac{1}{l}$$

$$= \frac{a}{h} : \frac{a}{k} : \frac{a}{l}$$

$$\therefore OA = \frac{a}{h};\ OB = \frac{a}{k} \text{ and } OC = \frac{a}{l}$$

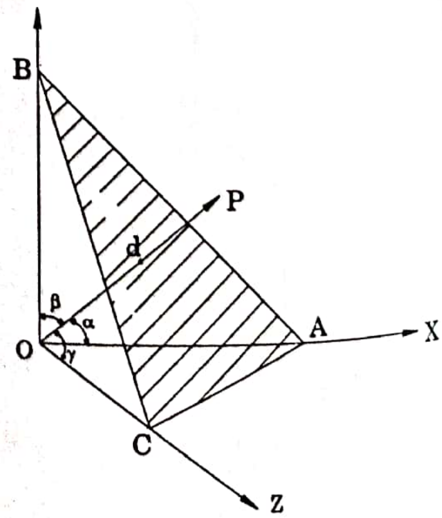


Fig. 5.14

From the geometry of the right angles OAP, OBP and OCP (Fig.5.14).

We have, $\cos\alpha = \frac{OP}{OA} = \frac{d}{\frac{a}{h}} = \frac{dh}{a}$

$$\cos\beta = \frac{OP}{OB} = \frac{d}{\frac{a}{k}} = \frac{dk}{a}$$

$$\cos\gamma = \frac{OP}{OC} = \frac{d}{\frac{a}{l}} = \frac{dl}{a}$$

We know that, the law of direction cosines is

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Substituting the values, we have,

$$\left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$$

$$\frac{d^2}{a^2}(h^2 + k^2 + l^2) = 1$$

$$\therefore \quad d^2 = \frac{a^2}{(h^2 + k^2 + l^2)}$$

$$\boxed{d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}}$$

This is the relation between interplanar spacing 'd', cube edge 'a' and Miller indices (h k l). Extending the planes to cut at 2a, 3a, ... so on

We have $d_2 = \frac{2a}{\sqrt{h^2 + k^2 + l^2}}$

$$d_3 = \frac{3a}{\sqrt{h^2 + k^2 + l^2}}$$