



2]. Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

Soln.

Given $[(3x+2)^2 D^2 + 3(3x+2)D - 36]y = 3x^2 + 4x + 1$

Take $3x+2 = e^x \Rightarrow 3x = e^x - 2 \Rightarrow x = \frac{e^x - 2}{3} \rightarrow (1)$

$\log(3x+2) = x$

$(3x+2)D = 3D'$

$(3x+2)^2 D^2 = 9D'(D'-1)$

(1) $\Rightarrow [9D'(D'-1) + 3(3D') - 36]y = 3\left[\frac{e^x - 2}{3}\right]^2 + 4\left[\frac{e^x - 2}{3}\right] + 1$

$[9D'^2 - 9D' + 9D' - 36]y = \frac{3}{9}[e^{2x} + 4 - 4e^x] + \frac{4}{3}e^x - \frac{8}{3} + 1$

$[9D'^2 - 36]y = \frac{e^{2x}}{3} + \frac{4}{3} - \frac{4e^x}{3} + \frac{4}{3}e^x - \frac{8}{3} + 1$

$= \frac{e^{2x}}{3} - \frac{1}{3}$

$9(D'^2 - 4)y = \frac{1}{3}[e^{2x} - 1]$

$(D'^2 - 4)y = \frac{1}{27}[e^{2x} - 1]$

CF: $m^2 - 4 = 0 \Rightarrow m^2 = 4$
 $m = \pm 2$

CF = $Ae^{2x} + Be^{-2x}$

PI = $\frac{1}{D^2 - 4} \left[\frac{1}{27}(e^{2x} - 1) \right]$

$= \frac{1}{27} \left[\frac{1}{D^2 - 4} e^{2x} - \frac{1}{D^2 - 4} e^{0x} \right]$

$= \frac{1}{27} \left[x \frac{1}{2D'} e^{2x} - \frac{1}{(-4)} e^{0x} \right] \quad \begin{matrix} 1 \rightarrow D' \rightarrow 2 \\ 2 \rightarrow D' \rightarrow 0 \end{matrix}$

$= \frac{1}{27} \left[\frac{x}{4} e^{2x} + \frac{1}{4} \right]$

$= \frac{1}{108} [xe^{2x} + 1]$

The soln. is, $y = CF + PI = Ae^{2x} + Be^{-2x} + \frac{1}{108} [xe^{2x} + 1]$



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$y = A(3x+2)^2 + B(3x+2)^{-2} + \frac{1}{108} [\log(3x+2)[3x+2]^2 + 1]$