



Q]. Solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 4y = \log x \sin(\log x)$

Soln.

Given $[x^2 D^2 + xD + 4] y = \log x \sin(\log x)$
 $\hookrightarrow (1)$

Take

$$x = e^z$$

$$\log x = z$$

$$xD = D' ; x^2 D^2 = D'(D'-1) \\ = D'^2 - D'$$



$$(1) \Rightarrow [D'^2 - D' + D' + 4]y = x \sin x$$

$$[D'^2 + 4]y = x \sin x$$

AE

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$CF = A \cos 2x + B \sin 2x$$

$$PI = \frac{1}{D'^2 + 4} x \sin x$$

$$= x \frac{1}{D'^2 + 4} \sin x - \frac{2D'}{(D'^2 + 4)^2} \sin x$$

$$= x \frac{1}{-1 + 4} \sin x - \frac{2 \cos x}{(-1 + 4)^2}$$

$$D'^2 \rightarrow -a^2$$

$$= -1^2$$

$$= -1$$

$$= \frac{x \sin x}{3} - \frac{2 \cos x}{9}$$

The soln. is

$$y = CF + PI$$

$$y = A \cos 2x + B \sin 2x + \frac{x \sin x}{3} - \frac{2 \cos x}{9}$$

$$= A \cos 2(\log x) + B \sin 2(\log x)$$

$$+ \frac{\log x \sin (\log x)}{3} - \frac{2}{9} \cos (\log x)$$

HJ. Solve $(x^2 D^2 - xD + 1)y = \log x$

Soln.

Given $(x^2 D^2 - xD + 1)y = \log x \rightarrow (1)$

Take $x = e^z$

$$z = \log x$$

$$xD = D'$$

$$x^2 D^2 = D'(D' - 1) = D'^2 - D'$$



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$$(1) \Rightarrow (D'^2 - D' - D' + 1) y = x$$

$$(D'^2 - 2D' + 1) y = x$$

AE

$$m^2 - 2m + 1 = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1, 1$$

$$\therefore CF = (A + Bx) e^x$$

$$PI = \frac{1}{D'^2 - 2D' + 1} x$$

$$= [1 + (D'^2 - 2D')]^{-1} x$$

$$= [1 - (D'^2 - 2D') + (D'^2 - 2D')^2 - \dots] x$$

$$= x - D'^2 x + 2D'(x)$$

$$PI = x + 2$$

\therefore The soln. is, $y = CF + PI$

$$y = (A + Bx) e^x + x + 2$$

$$y = (A + B \log x) x + \log x + 2$$

